Relativity and Accelerator Engineering

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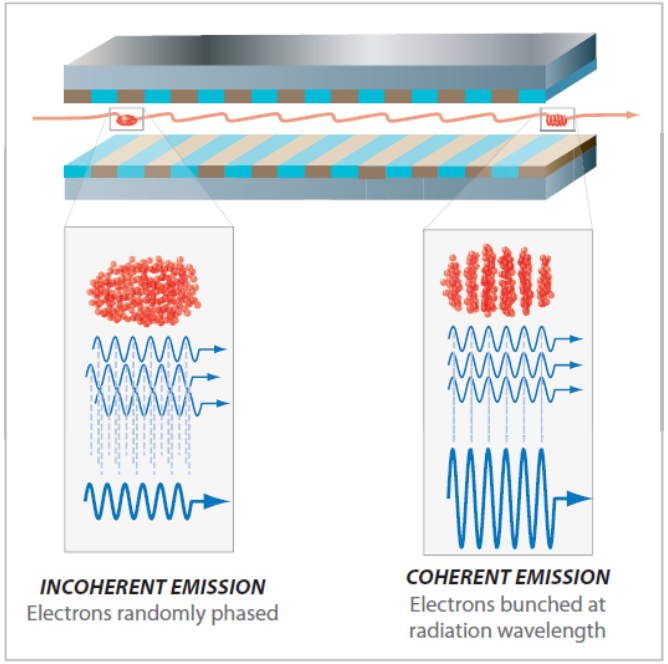
DESY

Relativistic kinematics effects are important for current and future XFEL operation

ultrarelativistic electron bunch modulated at nanometer-scale in XFELs

is in fact

ultrarelativistic macroscopic objects with an internal fine structure.



internal coherent structure

is characterized in terms of a

wave number vector

The dynamics of relativistic particles in the lab frame

It is generally accepted that in order to describe the dynamics of relativistic particles in the lab frame, which we assume inertial, one only needs to take into account the relativistic dependence of the particles momenta on the velocity. In other words, the treatment of relativistic particle dynamics involves only a corrected form of Newton's second law. To quote Feynman, Leiton and Sands "Newton's second law, $d(m\vec{v})/dt = \vec{f}$, was stated with the tacit assumption that m is a constant, but we now know that this is not true, and the mass of a body increases with velocity. (...) For those who want to learn just enough about it so they can solve problems, that is all there is to the theory of relativity - it just changes Newton's laws by introducing a correction factor to the mass."

The dynamics of relativistic particles in the lab frame

Conventional particle tracking $\longrightarrow$
well-defined initial data (Cauchy) problem $\longrightarrow$ time and space are treated differently

$$\frac{d\vec{p}}{dt} = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right),$$

$$\vec{p} = m\vec{v}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \qquad \text{Eq.(1)}$$

Note that this solution of the dynamics problem in the lab frame makes no reference to Lorentz transformations. This approach, that parallels the non relativistic ideas, do not require introduction of four-dimension Minkowski space. In fact, we have no mixture of positions and time. Such approach to relativistic dynamics is commonly accepted as useful method in accelerator and plasma physics and usually forces the physicist to believe that the relativistic particle tracking is possible without detailed knowledge of the theory of relativity.

Conventional (non-covariant) particle tracking

The concept of synchronization is a key concept in the understanding of special relativity. However, it appease that there does not exist a clear exposition of it in the literature. In particular, type of clock synchronization which provides time coordinate "t" in corrected Newton's equation Eq. (1) never discussed in accelerator and plasma physics.

This study of relativistic particles motion looks precisely the same as in non relativistic Newtonian dynamics. Conventional particle tracking treats the space-time continuum in a non-relativistic format, as a (3+1) manifold In fact, we do not have the typical mixture of positions and time that arises from Lorentz transformations

The trajectory of the particle $\vec{x}(t)$, which follows from the solution of the corrected Newton's second law does not include, however, relativistic effects

Result of conventional particle tracking does not include relativistic kinematics effects

Within the lab frame, if the particle motion follows the corrected Newton's second law there cannot be Wigner rotation, Lorentz-Fitzgerald contraction, time dilation, and relativistic corrections in the law of composition of velocities.

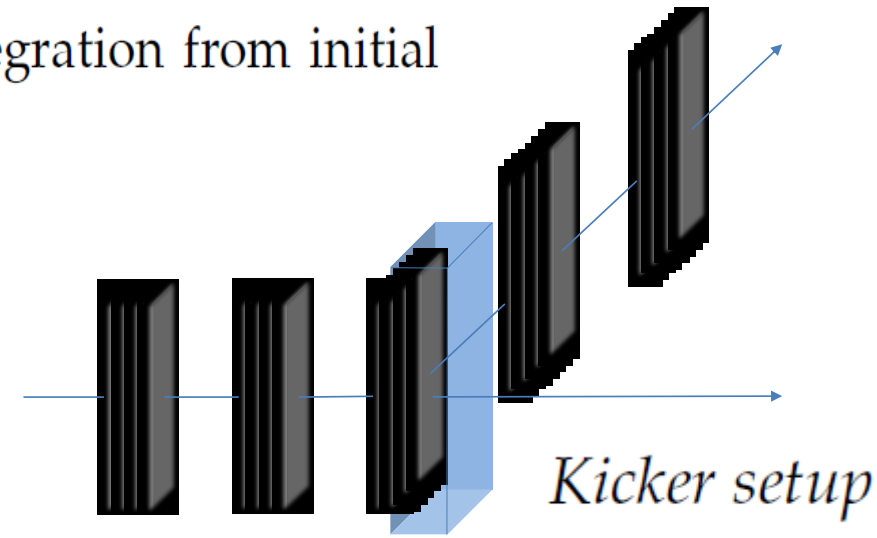
The conventional particle tracking in accelerator and plasma physics based on the use an approach that parallels the non relativistic ideas. Actually the evolution parameter is, like in non relativistic dynamics, the absolute time t. In this non covariant particle tracking the usual Galileo (vectorial) rule for addition of velocities is used.

Conventional (non-covariant) particle tracking

well-known result of classical particle tracking states that after the electron beam is kicked there is a change in the trajectory of the electron beam, while the orientation of the microbunching wavefront remains as before

particle's trajectory was found by integration from initial conditions $x' = x - vt, \; t' = t$

absolute simultaneity convention



approach to relativistic particle dynamics is based on the hidden assumption t
that the type of clock synchronization which provides time coordinate t in the lab frame is based on the use absolute simultaneity convention

Covariant particle tracking

one can define the class of inertial frames and can adopt a Lorentz frame with orthonormal basis vectors for any given inertial frame. Within the chosen Lorentz frame, Einstein's synchronization of distant clocks and Cartesian space coordinates are then automatically enforced, the metric tensor components are the usual $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, and any two Lorentz frames are related by a Lorentz transformation that preserves the metric tensor components, so that in any Lorentz coordinate system the law of motion becomes

$$m\frac{d^2x_\mu}{d\tau^2} = eF^{\mu\nu}\frac{dx_\nu}{d\tau}\,, \qquad \text{Eq.(2)}$$

Here the electromagnetic field is described by the second-rank, antisymmetric tensor with components $F^{\mu\nu}$. The coordinate-independent proper time τ is a parameter describing the evolution of physical system

Covariant particle tracking

world-line in terms of lab ccordinates components

$$[ct(\tau), x_1(\tau), x_2(\tau), x_3(\tau)]$$

we consider the spatial position as a function of the lab frame time t $\longrightarrow$ $x_\mu = [ct, \vec{x}_{cov}(t)]$

in this case relativistic kinematics effects arise $\longrightarrow$

Wigner rotation, Lorentz-Fitzgerald contraction, time dilation, and relativistic corrections in the law of composition of velocities

$\longrightarrow$ $\vec{x}_{cov}(t)$ and $\vec{x}(t)$ differ from each other

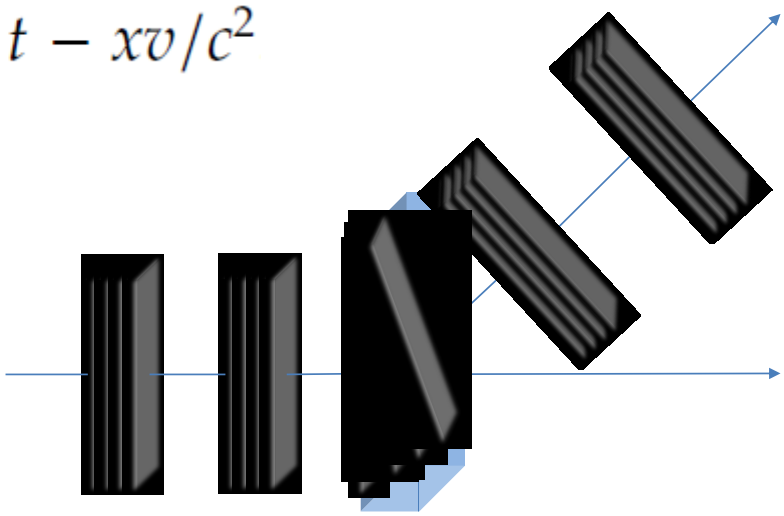
Covariant particle tracking

Wigner rotation $$\delta\Phi = \left(1 - \frac{1}{\gamma}\right)\frac{\vec{v}\times d\vec{v}}{v^2} = \left(1 - \frac{1}{\gamma}\right)\delta\theta$$

which is closely associated to the relativity of simultaneity.

infinitesimal Lorentz transformation

$$x' = x - vt, \quad t' = t - xv/c^2$$



Kicker setup

The electrodynamics problem

Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \,,$$
$$\vec{\nabla} \cdot \vec{B} = 0 \,,$$
$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \,, \qquad \text{Eq.(3)}$$
$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \,.$$

Here the charge density ρ and current density $\vec{j}$ are written as

$$\rho(\vec{x}, t) = \sum_n e_n \delta(\vec{x} - \vec{x}_n(t)) \,,$$
$$\vec{j}(\vec{x}, t) = \sum_n e_n \vec{v}_n(t) \delta(\vec{x} - \vec{x}_n(t)) \,,$$

Error in coupling fields and particles

We showed that $\vec{x}_{cov}(t)$ and $\vec{x}(t)$ differ from each other. Both are correct, and only refer to different choice of synchronization conventions. However, we criticize standard treatments of the coupling between electromagnetic sources and Maxwell's equations, since the trajectories in the source part of usual Maxwell's equations are always identified with the trajectories obtained in the non covariant (3+1) manner. In other words, $\vec{x}(t)$ is always used, instead of $\vec{x}_{cov}(t)$ as it must be.

usual Maxwell's equations in the lab frame are compatible only with the covariant trajectory $\vec{x}_{cov}(t)$, calculated by using Lorentz coordinates and, therefore, including relativistic kinematics effects

Error in coupling fields and particles

For the first time we showed a difference between conventional and covariant particle tracking results in the lab frame. This essential point has never received attention in the physical community. Only the solution of the dynamics equations in covariant form gives the correct coupling between the usual Maxwell's equations and particle trajectories in the lab frame. We conclude that previous theoretical and experimental results in accelerator and plasma physics should be reexamined in the light of the pointed difference between conventional and covariant particle tracking. In particular, a correction of the conventional synchrotron radiation theory is required

Thank you for your attention.
Any questions?