VUV-FEL-Statistics: "Reliability Analysis of the VUV- and the X-FEL"

Goal: To analyze and optimize the reliability of the VUV-FEL and to project it to the X-FEL

here: Statistical analysis of VUV-FEL-Downtimes

Goal: Foundation of an objective basis of the actual reliability state of the art of the VUV-FEL

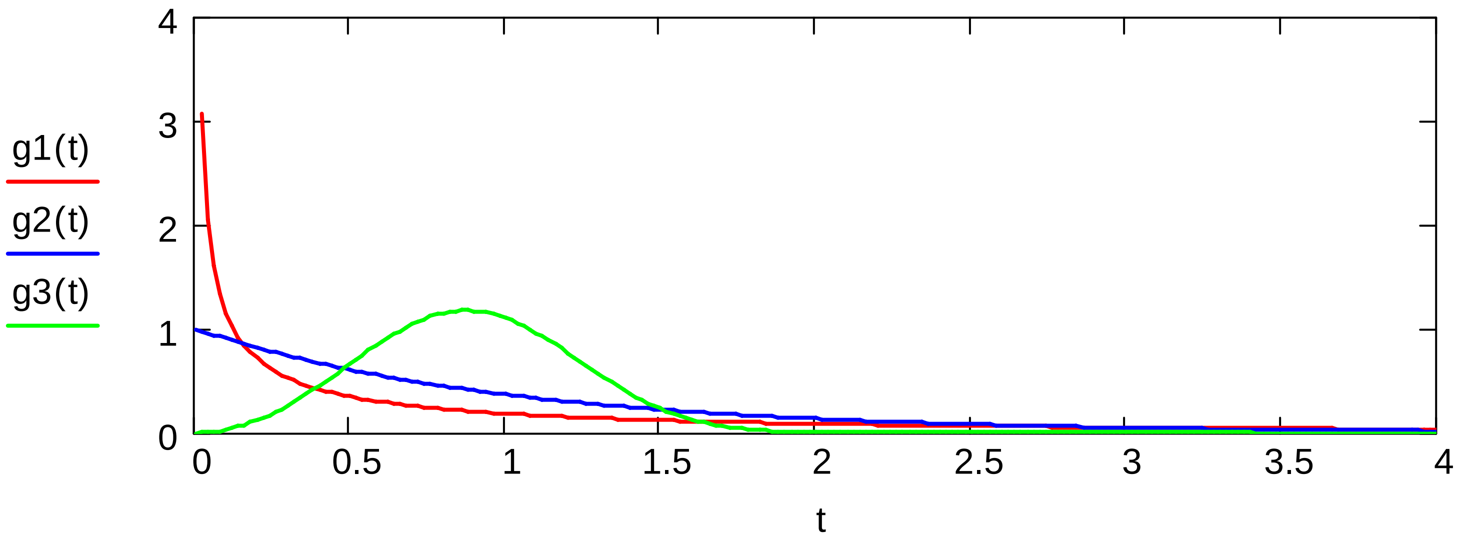
Weibull-Probability-Distribution (WD)

continuous randomvariable	t
"true" Weibull-parameters: characteristic lifetime	T
failurestepness	b
failurefreetime	tO
probabilitydensity (t>0): in 3 parameters:	$g(t) = \left(\frac{t - tO}{T}\right)^{b-1} \cdot \frac{b}{T} \cdot \exp\left[-\left(\frac{t - tO}{T}\right)^{b}\right]$
reduced form in 2 parameters:	$g(t) = \left(\frac{t}{T}\right)^{b-1} \cdot \frac{b}{T} \cdot \exp\left[-\left(\frac{t}{T}\right)^{b}\right]$
reliability = survivalprobability	$R(t) = \exp\left[-\left(\frac{t}{T}\right)^{b}\right]$
failureprobability	$G(t) = 1 - R(t)$
failurerate	$\lambda(t) = \left(\frac{t}{T}\right)^{b-1} \cdot \frac{b}{T}$
mean (also called MTTF, MTBF)	$Et = T \cdot \Gamma\left(\frac{1}{b} + 1\right)$

$t := 0, 0.02 .. 4 \qquad T := 1 \qquad b1 := 0.5 \qquad b2 := 1 \qquad b3 := 3$

$$g1(t) := \left(\frac{t}{T}\right)^{b1-1} \cdot \frac{b1}{T} \cdot \exp\left[-\left(\frac{t}{T}\right)^{b1}\right] \qquad g2(t) := \left(\frac{t}{T}\right)^{b2-1} \cdot \frac{b2}{T} \cdot \exp\left[-\left(\frac{t}{T}\right)^{b2}\right]$$

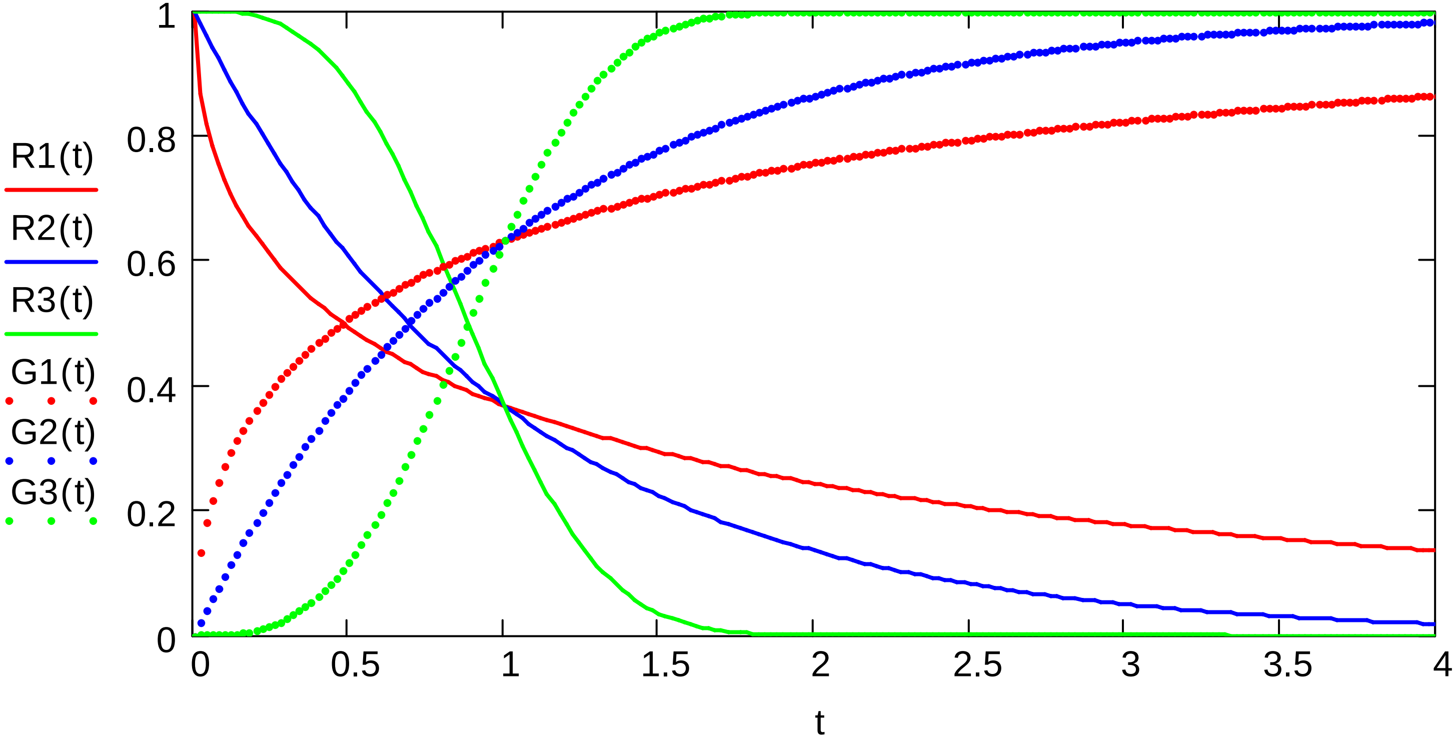
$$g3(t) := \left(\frac{t}{T}\right)^{b3-1} \cdot \frac{b3}{T} \cdot \exp\left[-\left(\frac{t}{T}\right)^{b3}\right]$$



$t := 0, 0.02 .. 4 \qquad T := 1 \qquad b1 := 0.5 \qquad b2 := 1 \qquad b3 := 3$

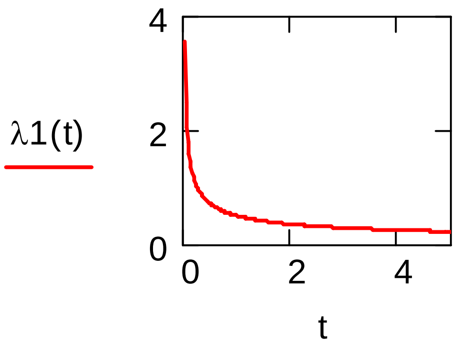
$$R1(t) := \exp\left[-\left(\frac{t}{T}\right)^{b1}\right] \qquad R2(t) := \exp\left[-\left(\frac{t}{T}\right)^{b2}\right] \qquad R3(t) := \exp\left[-\left(\frac{t}{T}\right)^{b3}\right]$$

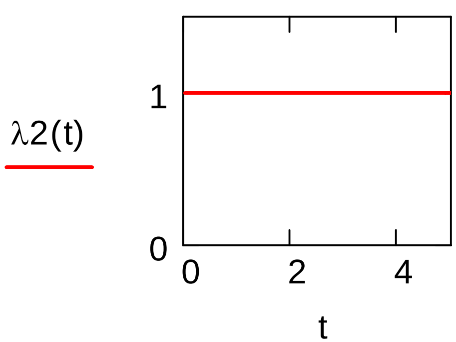
$$G1(t) := 1 - R1(t) \qquad G2(t) := 1 - R2(t) \qquad G3(t) := 1 - R3(t)$$

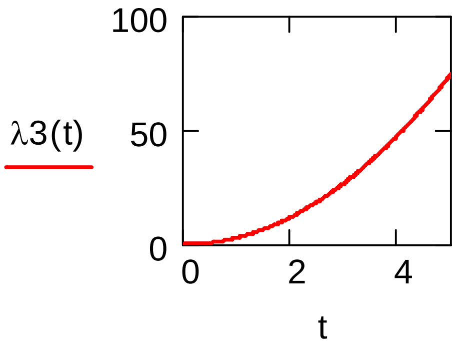


$t := 0, 0.02 .. 5 \qquad T := 1 \qquad b1 := 0.5 \qquad b2 := 1 \qquad b3 := 3$

$$\lambda 1(t) := \left(\frac{t}{T}\right)^{b1-1} \cdot \frac{b1}{T} \qquad \lambda 2(t) := \left(\frac{t}{T}\right)^{b2-1} \cdot \frac{b2}{T} \qquad \lambda 3(t) := \left(\frac{t}{T}\right)^{b3-1} \cdot \frac{b3}{T}$$







early failures

random failures

weareout failures

Statistical Estimates of Weibull-Parameters

Since the parameters are unknown, they are "statistically estimated" basing on n measured lifetimes, constituting a "complete statistical sample" of "size n".

Here, this statistical treatment is performed with the "Maximum-Likely-Method":

$$\ln(L) = \ln\left[\prod_{i=1}^{n} \left(\frac{t_i - tO}{T}\right)^{b-1} \cdot \frac{b}{T} \cdot \exp\left[-\left(\frac{t_i - tO}{T}\right)^{b}\right]\right]$$

The solution of the equation-system, unsolvable in closed form,

$$\frac{\delta}{\delta T}\ln(L) = 0$$

$$\frac{\delta}{\delta b}\ln(L) = 0$$

$$\frac{\delta}{\delta tO}\ln(L) = 0$$

delivers the "statistical estimates" TML, bML, tOML for the unknown Weibull-parameters T, b, tO.

Confidence Ranges of Weibull-Parameters based on Statistical Estimates

At a selectable "confidence" probability P, the "true", principally unknown parameter is in the "confidence range" between an upper limit, indexd "o" ("oben"), and a lower limit, indexed "u" ("unten"), with a symmetrically partitioned error-probability (1-P)/2 above and below these limits.

$$TMLo = \frac{2 \cdot n \cdot TML}{qchisq\left[\frac{(1-P)}{2}, 2 \cdot n\right]} + \Omega To \qquad TMLu = \frac{2 \cdot n \cdot TML}{qchisq\left[\left[1 - \frac{(1-P)}{2}\right], 2 \cdot n\right]} + \Omega Tu$$

$$bMLo = \frac{2 \cdot n \cdot bML}{qchisq\left[\frac{(1-P)}{2}, 2 \cdot n\right]} + \Omega bo \qquad bMLu = \frac{2 \cdot n \cdot bML}{qchisq\left[\left[1 - \frac{(1-P)}{2}\right], 2 \cdot n\right]} + \Omega bu$$

$$tOMLo = \frac{2 \cdot n \cdot tOML}{qchisq\left[\frac{(1-P)}{2}, 2 \cdot n\right]} + \Omega tOo \qquad tOMLu = \frac{2 \cdot n \cdot tOML}{qchisq\left[\left[1 - \frac{(1-P)}{2}\right], 2 \cdot n\right]} + \Omega tOu$$

Ω=Ω(TML,bML,tOML)=higher order terms qchisq= chisquare-distribution-quantile

For each of the thirteen downtime-categories, documented in the TTF-Logbook from 6.1.2005 to 26.3.2006, these 95%-confidence ranges of T,b,tO for "Betrieb"(down=0) and "Stillstand"(down>0) are filed in a seperate document. Excerpts shown during presentation.

Proposed to be considered for amelioration of reliability - projection

1.) clock-time-mark for failure-start and for failure-end by hand or by system (preferred)

2.) individualizing optimally each present downtime-category into their subparts