VUV-FEL-Statistics: "Reliability Analysis of the VUV- and the X-FEL"

Goal: To analyze and optimize the reliability of the VUV-FEL and to project it to the X-FEL

here: Statistical analysis of VUV-FEL-Downtimes

Goal: Foundation of an objective basis of the actual reliability state of the art of the VUV-FEL
**Weibull-Probability-Distribution (WD)**

continuous randomvariable \( t \)

"true" Weibull-parameters: characteristic lifetime \( T \)

failurestepness \( b \)

failurefreetime \( tO \)

probabilitydensity \( g(t) \): in 3 parameters:

\[
g(t) = \left( \frac{t - tO}{T} \right)^{b-1} \cdot \frac{b}{T} \cdot \exp \left[ -\left( \frac{t - tO}{T} \right)^b \right]
\]

reduced form in 2 parameters:

\[
g(t) = \left( \frac{t}{T} \right)^{b-1} \cdot \frac{b}{T} \cdot \exp \left[ -\left( \frac{t}{T} \right)^b \right]
\]

reliability = survivalprobability \( R(t) = \exp \left[ -\left( \frac{t}{T} \right)^b \right] \)

failureprobability \( G(t) = 1 - R(t) \)

failurerate \( \lambda(t) = \left( \frac{t}{T} \right)^{b-1} \cdot \frac{b}{T} \)

mean (also called MTTF, MTBF) \( E_t = T \cdot \Gamma \left( \frac{1}{b} + 1 \right) \)
\[ t := 0, 0.02 .. 4 \quad T := 1 \quad b_1 := 0.5 \quad b_2 := 1 \quad b_3 := 3 \]

\[ g_1(t) := \left( \frac{t}{T} \right)^{b_1-1} \cdot \frac{b_1}{T} \cdot \exp \left[ -\left( \frac{t}{T} \right)^{b_1} \right] \]

\[ g_2(t) := \left( \frac{t}{T} \right)^{b_2-1} \cdot \frac{b_2}{T} \cdot \exp \left[ -\left( \frac{t}{T} \right)^{b_2} \right] \]

\[ g_3(t) := \left( \frac{t}{T} \right)^{b_3-1} \cdot \frac{b_3}{T} \cdot \exp \left[ -\left( \frac{t}{T} \right)^{b_3} \right] \]
\[
\begin{align*}
\text{t} & := 0 \ldots 4, \quad T := 1, \quad b_1 := 0.5, \quad b_2 := 1, \quad b_3 := 3 \\
R_1(t) & := \exp\left[-\left(\frac{t}{T}\right)^{b_1}\right] \\
R_2(t) & := \exp\left[-\left(\frac{t}{T}\right)^{b_2}\right] \\
R_3(t) & := \exp\left[-\left(\frac{t}{T}\right)^{b_3}\right] \\
G_1(t) & := 1 - R_1(t) \\
G_2(t) & := 1 - R_2(t) \\
G_3(t) & := 1 - R_3(t)
\end{align*}
\]
\[ t := 0, 0.02 \ldots 5 \quad T := 1 \quad b_1 := 0.5 \quad b_2 := 1 \quad b_3 := 3 \]

\[ \lambda_1(t) := \left( \frac{t}{T} \right)^{b_1-1} \cdot \frac{b_1}{T} \quad \lambda_2(t) := \left( \frac{t}{T} \right)^{b_2-1} \cdot \frac{b_2}{T} \quad \lambda_3(t) := \left( \frac{t}{T} \right)^{b_3-1} \cdot \frac{b_3}{T} \]

Early failures

Random failures

 Wear-out failures
Statistical Estimates of Weibull-Parameters

Since the parameters are unknown, they are "statistically estimated" basing on n measured lifetimes, constituting a "complete statistical sample" of "size n".

Here, this statistical treatment is performed with the "Maximum-Likely-Method":

\[
\ln(L) = \ln\left[ \prod_{i=1}^{n} \left( \frac{t_i - tO}{T} \right)^{b-1} \cdot \frac{b}{T} \cdot \exp\left[ -\left( \frac{t_i - tO}{T} \right)^b \right] \right]
\]

The solution of the equation-system, unsolvable in closed form,

\[
\frac{\delta}{\delta T} \ln(L) = 0
\]
\[
\frac{\delta}{\delta b} \ln(L) = 0
\]
\[
\frac{\delta}{\delta tO} \ln(L) = 0
\]

delivers the "statistical estimates" TML, bML, tOML for the unknown Weibull-parameters T, b, tO.
Confidence Ranges of Weibull-Parameters based on Statistical Estimates

At a selectable "confidence" probability $P$, the "true", principally unknown parameter is in the "confidence range" between an upper limit, indexed "o" ("oben"), and a lower limit, indexed "u" ("unten"), with a symmetrically partitioned error-probability $(1-P)/2$ above and below these limits.

$$TML_{0} = \frac{2 \cdot n \cdot TML}{\text{qchisq} \left[ \frac{(1 - P)}{2}, 2 \cdot n \right]} + \Omega_{T_{0}}$$

$$TML_{u} = \frac{2 \cdot n \cdot TML}{\text{qchisq} \left[ 1 - \frac{(1 - P)}{2}, 2 \cdot n \right]} + \Omega_{T_{u}}$$

$$bML_{0} = \frac{2 \cdot n \cdot bML}{\text{qchisq} \left[ \frac{(1 - P)}{2}, 2 \cdot n \right]} + \Omega_{b_{0}}$$

$$bML_{u} = \frac{2 \cdot n \cdot bML}{\text{qchisq} \left[ 1 - \frac{(1 - P)}{2}, 2 \cdot n \right]} + \Omega_{b_{u}}$$

$$tOML_{0} = \frac{2 \cdot n \cdot tOML}{\text{qchisq} \left[ \frac{(1 - P)}{2}, 2 \cdot n \right]} + \Omega_{t_{O_{0}}}$$

$$tOML_{u} = \frac{2 \cdot n \cdot tOML}{\text{qchisq} \left[ 1 - \frac{(1 - P)}{2}, 2 \cdot n \right]} + \Omega_{t_{O_{u}}}$$

$\Omega=\Omega(TML,bML,tOML)=$higher order terms

$q\text{chisq}=\text{chisquare-distribution-quantile}$

For each of the thirteen downtime-categories, documented in the TTF-Logbook from 6.1.2005 to 26.3.2006, these 95%-confidence ranges of $T,b,tO$ for "Betrieb"(down=0) and "Stillstand"(down>0) are filed in a separate document. Excerpts shown during presentation.

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Proposed to be considered for amelioration of reliability - projection

1.) clock-time-mark for failure-start and for failure-end by hand or by system (preferred)

2.) individualizing optimally each present downtime-category into their subparts