Spatial resolution of the optical systems for beam profile measurements at TTF

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Abstract

Resolution study for an optical system similar to ones used on TTF for beam size diagnostics was performed, and the the measured resolution is quite sufficient for TTF Phase 2 requirements.

1 Introduction

As far as TTF approaches its Phase 2, requirements for beam profile diagnostics become more demanding. With the beam size decreasing down to 50 μm, the spatial resolution of measuring devices has to be much better than this value.

Even at present, a resolution better than 100 μm is necessary at some places along the linac to perform reliable transverse profile studies of the beam. Certain doubts have arose for the last time whether the present performance of the optics is capable of providing sufficient accuracy for measurements.

It is worth to remind that optical transition radiation (OTR) is widely used at TTF for both beam transport optimization and characterization along with standard fluorescent ceramics [1]. Although different optics is used for the beam imaging, at locations specifically intended for measurements, diagnostic stations are typically equipped with an OTR screen as a light radiator and an achromatic doublet lens as a primary component of the optical system to image the beam profile on a CCD.

As well known, the best resolution attainable in an ideal optical system is determined by diffraction, that transforms the image of a point source of light into the so-called point spread function (PSF). The resolution for the same optics, but in the case of OTR produced by a single particle, has been shown [2, 3, 4] to be about 3 times larger (in terms of the fwhm definition). Real optics may suffer from aberrations, misalignments etc., hence, its resolution is to be measured for every particular situation.

The present note reports results of resolution measurements for a TTF -like optical system in the range of parameters relevant to Phase 2.

2 Experimental setup

Measurements were performed at Frascati Laboratory where an optical system similar to those used at TTF was assembled (Fig. 1). It is, essentially, the simplest configuration based on an achromatic doublet, a combination of positive and negative elements with different refractive indexes to almost cancel the chromatic aberration in a certain wavelength range. Good achromats are usually also optimized to be very nearly free of both spherical aberration and coma. A f/5.9 doublet with a focal length 300 mm was used.

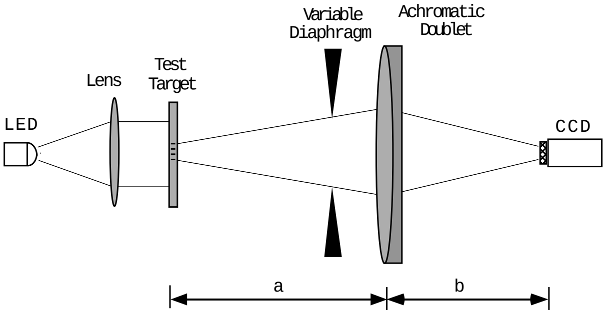

Figure 1: The experimental setup.

Other elements of the setup are the following: a standard LED was used to illuminate an USAF test target at a wavelength of 650 nm. The LED was placed in the back focal plane of a simple lens to create on the test target a nearly uniform and sufficiently large spot of light. The achromatic doublet produced on a CCD the test target image with a given magnification m.

A variable- size circular diaphragm was employed in order to study the field-depth effect as a function of acceptance. The CCD was Philips VCM6250 with the 1/2" sensitive area consisting of 512x582 pixels. The framegrabber resolution was measured by scanning across the CCD by a collimated laser beam to be 8.6 μm in the horizontal and 8.4 μm in the vertical plane, respectively.

The system was tested for two values of the magnification: $m = 1/2$ and $m = 1$.The corresponding distances between the target and the lens a and between the lens and the CCD b are listed in Table 1.

Table 1: Geometric parameters of the setup

Magnification	a(mm)	b(mm)
1/2	900	450
1	600	600

3 Resolution based on the analysis of MTF

The conventional approach used to determine the image-forming quality of an optical device is to measure the modulation transfer function (MTF). MTF describes the ability of a lens or a system of lenses to transfer the object contrast to the image, as a function of spatial frequency.

MTF is best measured using a sine wave chart on which light transmission varies one dimensionally in a precisely sinusoidal fashion. Such charts are difficult to make and they are expensive, so the cheaper solution is the bar charts, one of which, the USAF test target, was used in this measurement.The images of the test target for each of the two values of the magnification are shown in Fig. 2 and Fig. 3.

In the target the number of line pairs per millimeter varies from one element to another in a given order (a "line pair" being a dark bar plus an equally spaced clear bar). Every element consists of two target patterns of three lines each, at right angles to each other.

If the target is viewed against a uniformly illuminated incoherent background, the modulation is defined as

$$M = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{1}$$

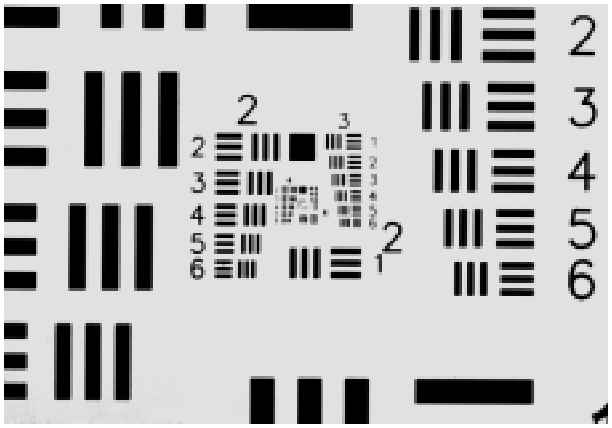

Figure 2: The USAF test target image for $m = 1/2$ magnification.

where I_{max} and I_{min} are the maximum and minimum image intensities in the clear and dark bar regions. MTF of the optical system is the functional dependence of the modulation on the bar spatial frequency f, measured in line pairs per millimeter.

$$M = M(f) \tag{2}$$

We assumed that both x and y plane were equivalent and used only vertically oriented bar elements calculating for each one the modulation depth according to Eq. 1. MTF measured for both magnifications, with the parameters a and b listed in Table 1 and for a set of diaphragm diameters D are shown in Fig. 4 and Fig. 5.

As expected, all MTF in these figures gradually drop from 1 at low spatial frequencies to zero as the spatial frequency increases. The highest frequency above which MTF vanishes is typically called the cutoff frequency. In the case of an ideal optics, the cutoff frequency f_0 is determined only by the diffraction due to the aperture of the iris D. The calculated diffraction limited values for our configurations are given in Table 2. The cutoff frequencies that can be extrapolated from measured MTF in Fig. 4 and 5 are much lower, indicating that in our case the resolution was far from the diffraction limit. To our estimations, this can be mostly attributed to the pixel size of the CCD (~ 9 μm).

Although MTF itself is a very good way to describe the optics quality in general, a guess about the spread function (SF) may be particularly useful to characterize the resolution in the case of a beam diagnostic system. It is worth to remind that the spread function is defined as the two-dimensional intensity distribution in the

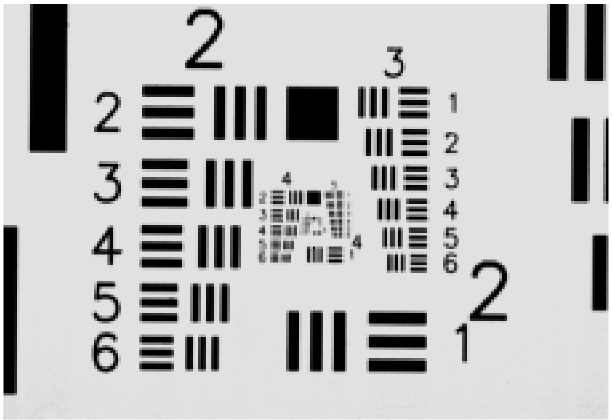

Figure 3: The Usaf test target image for m=1 magnification.

Table 2: Cutoff frequencies for diffraction limited MTF

m	D(mm)	$f_0(pairs/mm)$
1/2	37	59.9
1/2	20	32.4
1/2	15	24.3
1	37	94.7
1	20	51.6
1	15	38.4

image plane of a point light source. However, for a bar test target, a one-dimensional structure, it is more natural to introduce a spread function for a line. In the same way as the rms beam size is a standard parameter specifying the beam dimension, the rms width of the SF can be considered a measure of spatial resolution. This definition is even of more value, as spread functions having the same rms width but different shapes give rise to different behaviour of MTF.

The connection between SF and MTF may be easily established, if SF is an even function $g(-x) = g(x)$, normalized such that

$$\int_{-\infty}^{\infty} g(x)dx = 1\,. \tag{3}$$

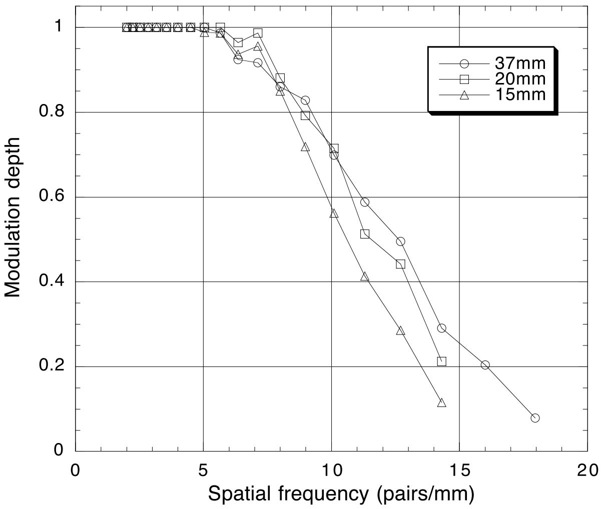

Figure 4: Modulation trasnfer function for M=1/2 magnification.

Then MTF for the infinite one-dimensional bar array reads

$$M(f) = 1 - 4\sum_{n=0}^{\infty}\int_{\frac{n+1/4}{f}}^{\frac{n+3/4}{f}} g(x)dx\,. \tag{4}$$

In Fig. 6, MTF calculated for three SF in the form of a diffration pattern $f_0\left[\sin(\pi f_0 x)/\pi f_0 x\right]^2$, a gaussian and rectangular distributions, all having the same rms width, are shown.

Thus, to find the rms width of SF from MTF one must make a reasonable assumption about its form. Figure 6 indicates that for the measured modulation transfer functions, SF is likely to have a shape between gaussian and rectangular, i.e. gaussian-like edges but flatter top. Flattering of the top could happen as result of certain kinds of aberration and a focus deviation [5].

To this end we tried to approximate our SF by a sum of two equal gaussians with the dispersion σ separated by a distance 2μ. The rms width of a such function is simply $\sqrt{\sigma^2+\mu^2}$. MTF is given by

$$M = 1 - \sum_{n=0}^{N}\left[\Omega\left(\frac{\mu}{\sqrt{2}\sigma}, \frac{3/4+n}{\sqrt{2}\sigma f}\right) - \Omega\left(\frac{\mu}{\sqrt{2}\sigma}, \frac{1/4+n}{\sqrt{2}\sigma f}\right)\right] \tag{5}$$

$$\Omega(x,y) = \left[\mathrm{erf}(y-x) + \mathrm{erf}(y+x)\right] \tag{6}$$

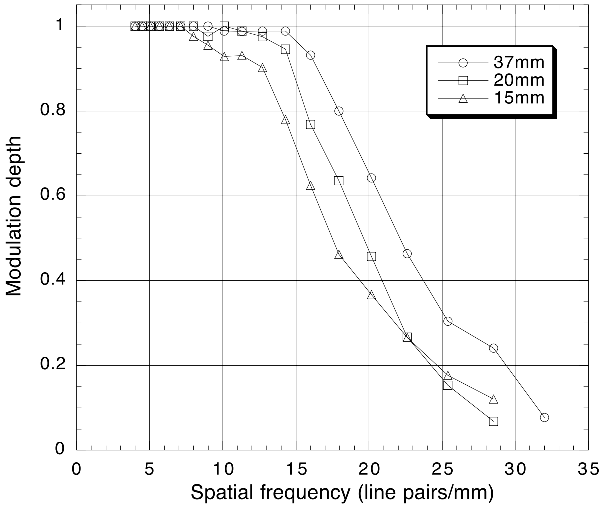

Figure 5: Modulation trasnfer function for M=1 magnification.

where erf(x) is the error function. The fit of Eq. 5 to the measured points gives the values listed in Tabel 1.

Table 3: Fit estimates for double gaussian SF

m	D(mm)	$\mu(\mu\text{m})$	$\sigma(\mu\text{m})$	$\sqrt{\sigma^2+\mu^2}$ (μm)
1/2	37	12.3	12.1	17.3
1/2	20	14.4	10.3	17.7
1/2	15	15.9	11.0	19.3
1	37	7.5	5.21	9.1
1	20	8.7	5.71	10.4
1	15	7.7	8.6	11.5

It is evident, that the rms width of SF increases with the decrease of the iris diameter, but the values, for both magnifications, are very close to the "effective" pixel sizes.

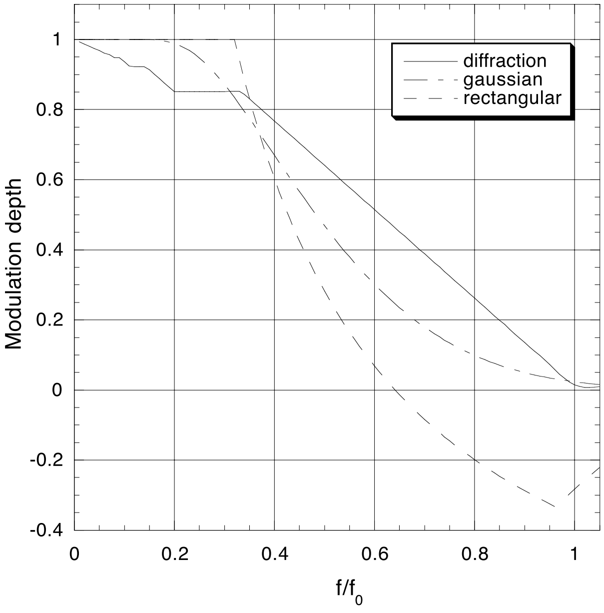

Figure 6: Modulation trasnfer function for diffraction-like, gaussian and rectangular LSF functions having the same rms-width.

4 Smearing of the bar edges in the image

Alternatively, resolution of the system can be estimated from individual bar image profiles, again, by making assumptions about the spread function. This method, though less accurate then the MTF analysis, is simpler and faster, thus allowing to process a large amount of images. It was applied to study the field-depth effect, that is dependence of resolution on a small variation in the test target position. Resolution changes as a result of defocusing and can be evaluated from the analysis of edge smearing in the bar image.

In fact, due to the high quality of the test target, there is sharp change in the light transmittance on the boundary between adjacent clear and dark bars. Smearing of the bar image edges occurs due to the finite optical resolution and the CCD pixel size. Assuming a certain form for SF, an analytical expression can be constructed and compared with the measurement. Since this analysis is sensitive to the SF width, rather than its shape, a simple gaussian is a good approximation to estimate the resolution rms value. In this case, the bar image profile is described by the following formula

$$I(x) = \frac{1}{2}\left[\mathrm{erf}\left(\frac{d/2 - x}{\sqrt{2}\sigma}\right) + \mathrm{erf}\left(\frac{d/2 + x}{\sqrt{2}\sigma}\right)\right] , \tag{7}$$

where the intensity I is normalized to vary between 0 and 1, d is the bar width and σ is the SF width (rms resolution).

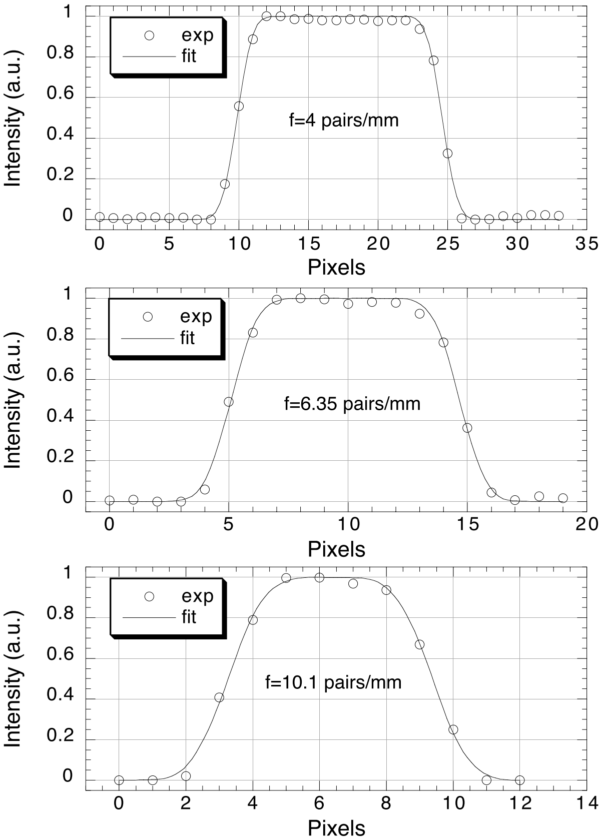

Figure 7: Fit to bar profiles by the gaussian SF.

Resolution measurements were performed for image magnifications 1/2 and 1, each for three different values of the iris diameter. For every magnification the test target position was varied in a given range specified by a visible strong degradation of the image quality on its boundaries. For each image, Eq. 7 was fitted to 4 - 6 bar profiles of different spatial frequencies. An example of the fit to three different bars on an image taken with m=1 is shown in Fig. 7.

The parameter σ was obtained from the fits and its average and standard deviation for every image were calculated. All the results are collected in Figs. 8 and 9, where solid lines are the cubic spline applied to the experimental points.

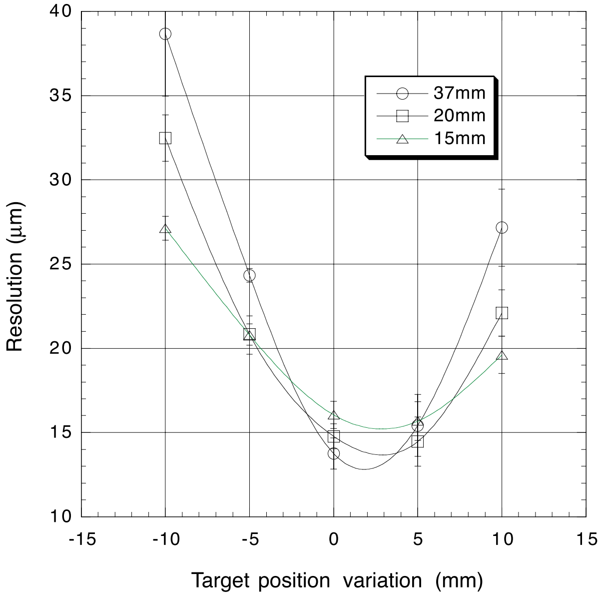

Figure 8: Resolution versus target displacement for m=1/2.

The overall behaviour of the data can be explained by the following effects:

- Resolution becomes worse as the distance from the focus increases. Note, that the target position is relative, defined with respect to the initial point. That is why in Fig. 8 the real focus has a small offset from the zero position.e
- Diffraction limited resolution improves inversely with the diaphragm diameter. This is best observed near the focus.
- Defocusing effect is weaker for a smaller angular acceptance. The effect is linear.
- Resolution due to the CCD pixel size depends on the magnification only. It strongly influences the minimum resolution achieved in the experiment.

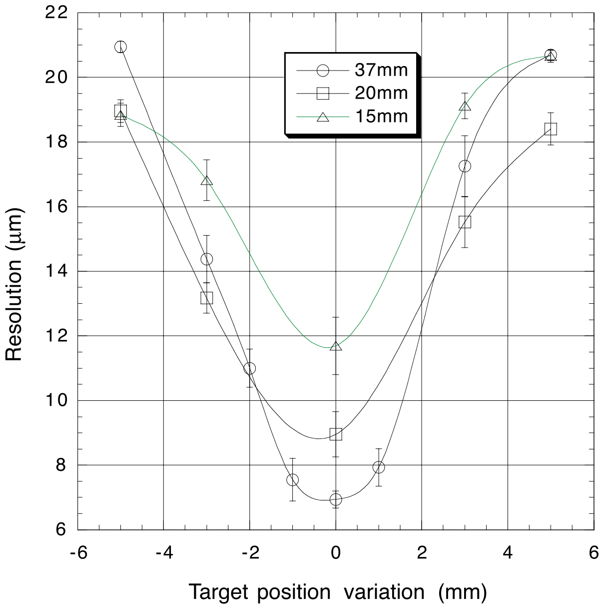

Figure 9: Resolution versus target displacement for m=1.

- Effect of aberrations on resolution is hard to evaluate. It is believed to be small compared to other factors.

It can be seen that resolutions near the minima agree within 25% with those obtained from the MTF analysis reported in the Table 3 and we can still consider it a good consistency between the two different analyses. Compared to the diffraction limits they are just 10% to 60% (depending on the diaphragm diameter) higher and can be improved if a CCD with a smaller pixel size is used. Note, however, that the measured resolution values are quite sufficient for the TTF Phase 2 beam size of 50 μm.

If at focus the resolution is not of concern, the field depth may be. From Figs. 8 and 9 one can find that the inaccuracy in the optics focus alignment should not be larger then few millimeters. Given that the OTR screens are at 45° to the beam axis, for a precise measurement, the beam must be very near to the exact focal point. For this reason, and due to the reduced field of view resulting from the large magnification, certain modification to the current screen design and camera support will be probably necessary.

5 Conclusions

Resolution measurements for a TTF-like optical system in the range of parameters relevant to Phase 2 were performed. The dependence of resolution on magnification, angular acceptance and defocusing was studied in detail. It was found that quantitatively the measured resolution is quite sufficient for Phase 2 being just 10-60 % higher than the corresponding diffraction limits. However, the field depth effect may impose a serious limits on the focus misalignment that should not exceed few millimeters.

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