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Beam Loading Generated by the LOLA-IV Structure in TTF-II*

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I. INTRODUCTION AND CONCLUSION

The LOLA-IV transverse deflecting cavity [1] is used in the Sub-Picosecond Photon Source (SPPS) as a diagnostic for measuring the length of very short bunches (with rms length on the order of tens of microns). It is envisioned to use the same structure for the same purpose in the Tesla Test Facility (TTF)-II. However, unlike the SPPS, the TTF-II may also run in multi-bunch mode, and the question arises, How serious is the beam loading that would be induced?

In this report we address this question and find that, for LOLA-IV in TTF-II, the variation in beam-loading induced energy is confined to the first ~ 80 bunches, and that the total spread in induced energy—the difference in energy between the bunch in the train with the highest energy and the one with the lowest energy—is very small, $\sim 0.03\%$.

II. BEAM LOADING

Consider a train of N evenly-spaced bunches traversing an empty rf cavity. The energy changed induced by the n^{th} bunch is given by

$$\Delta E_n = -2eq_b\kappa_0 L\left(\frac{1}{2}+\sum_{n'=1}^{n-1}\cos k_0(s_n-s_{n'})\,e^{-\frac{k_0(s_n-s_{n'})}{2Q_0}}\right)\ , \tag{1}$$

where q_b is the charge per bunch, L is the structure length, and s_n is the longitudinal position of the n^{th} bunch; where k_0, κ_0, Q_0, are, respectively, the wave number ($= 2\pi f_0/c$ with f_0 the frequency and c the speed of light), the loss factor, and the quality factor of the fundamental mode of the cavity. Here we ignore the effect of higher cavity modes. We can rewrite $(s_n - s_{n'}) = 2\pi\nu(n-n')/k_0$, where the tune $\nu \equiv f_0/f_b = ck_0/(2\pi f_b)$, with f_b the bunch frequency. Therefore,

$$\begin{aligned}\Delta E_n &= -2eq_b\kappa_0 L\left(\frac{1}{2}+\mathcal{R}\mathrm{e}\left[\sum_{n'=1}^{n-1}e^{-\alpha n'}\right]\right)\\ &= -2eq_b\kappa_0 L\left(\frac{1}{2}+\mathcal{R}\mathrm{e}\left[e^{-\alpha}\frac{1-e^{-(n-1)\alpha}}{1-e^{-\alpha}}\right]\right)\ ,\end{aligned} \tag{2}$$

where $\mathcal{R}\mathrm{e}$ means to take the real part and $\alpha = (\frac{1}{Q_0}-2i)\pi\nu$. We see that the result of Eq. 2 remains unchanged if we replace the imaginary part of α ($= -2\pi i\nu$) by $-2\pi i\nu_{frac}$, where ν_{frac} is the fractional (non-integer) part of the tune. Note that beam loading is more sensitive to the fractional part of the tune than to the integer part, and that there is symmetry about the half integer tune [the result for tune ν_{frac} is (nearly) the same as for tune $(1-\nu_{frac})$]. From Eq. 2 we note also that we reach near steady-state at bunch n when the quantity $e^{-n\pi\nu/Q_0}$ becomes small. Finally note that at the integer tune the steady-state energy change $\Delta E \approx -2eq_b\kappa_0 L Q_0/(\nu\pi)$.

For multi-bunch operation of TTF-II we have hundreds of bunches, each with charge $q_b = 3$ nC, bunch frequency $f_b = 1.3$ GHz/144, and energy $E = 600$ MeV. For the LOLA-IV cavity, the deflection mode frequency $f_d = 2.856$ GHz and structure length $L = 3.64$ m; the fundamental mode has properties: $f_0 \approx 2.1$ GHz, $\kappa_0 = 9$ V/pC/m, and $Q_0 = 12000$. Note that since the cavity operates in the dipole mode, the fundamental mode is cut off, and the total Q is the same as the cavity Q. Note also that $\nu = f_0/f_b \approx 233.$, so that if we want to know the fractional part of ν to better than 0.1, we need to know f_0 to better than 4.3×10^{-4} (or 0.9 MHz) accuracy.

In Fig. 1 we show the energy change of the first 80 bunches in the train as given by Eq. 2, for cases with the fractional part of $\nu = 0.03$, 0.10, and 0.30, if the LOLA structure is used at TTF-II (for ease of visualization, the calculated points are connected by straight lines). Note that the single bunch energy loss (the energy loss of the first bunch) is 0.016%. Also, note that the quantity $e^{-n\pi\nu/Q_0} = 7 \times 10^{-3}$ for $n = 80$, which we can see from the figure is near steady-state.

In Fig. 2 we plot the total spread of energy in the bunch train—the difference between the bunch with the highest energy and the one with the lowest energy—*vs.* fractional part of tune. Note that the curve can be divided roughly into three regions: for ν_{frac} in ranges (i) [0, 0.035], (ii) [0.035, 0.25], and (iii) [0.25, 0.5] (remember the mirror symmetry for values beyond $\nu_{frac} = 0.5$). A beam loading example for each of the 3 regions is given in Fig. 1. Beginning with Region (iii), note that, in this region, the energy change as function of bunch number can be roughly described as a damped oscillation that is limited in amplitude to $\epsilon = eq_b\kappa_0 L$, the single bunch energy loss. Therefore, the total spread in energy in the train is $\approx 2\epsilon$, which for our parameters is equivalent to $\Delta E/E = 0.032\%$.

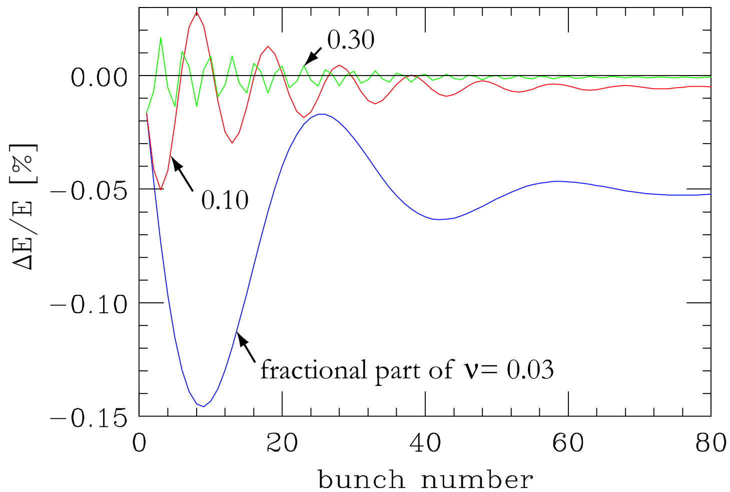

FIG. 1: Energy change of the first 80 bunches in the bunch train, assuming the fractional part of ν is 0.05, 0.10, and 0.20. Note that the results for the 80 bunches are connected by straight lines.

In Region (ii) the oscillation amplitude becomes larger than ϵ; as ν_{frac} decreases, the energy spread in the train gradually increases (here $\sim 1/\nu_{frac}$). Finally, in Region (i), near the integer, all bunches in the train lose energy, and the energy variation increases steeply as ν_{frac} decreases. The boundary to Region (ii), $\nu_{frac} \approx 1.82\nu/Q_0$ (which here $= 0.035$), is the tune for which the maximum energy in the train $\Delta E_{max} = 0$. At the integer tune, we obtain as minimum energy in the train (also the steady-state energy) $\Delta E_{min} = -2\epsilon Q_0/(\pi\nu)$, which here is equivalent to $\Delta E/E = -0.54\%$. To minimize beam loading due to the LOLA-IV cavity we want to operate in Region (iii), and at all cost avoid Region (i).

III. MODE CALCULATIONS

To find the strength of beam loading in the LOLA-IV structure, we need to obtain, to high accuracy, the fundamental mode frequency in the structure. The structure length is 3.64 m. The geometry of one cell of the structure (all cells, except the coupling cells, are identical) is indicated in Fig. 3. Fig. 3a displays the shape of one cell, showing the iris and the mode alignment holes; Figs. 3b,c give a negative image, showing the shape of the enclosed volume in a cell. In Table I we reproduce, from Ref. [1], geometric parameters that

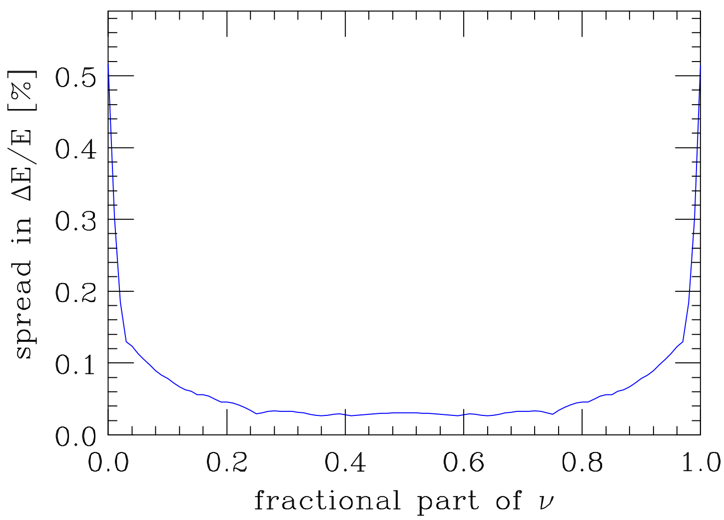

FIG. 2: Total spread in energy in the bunch train *vs.* fractional part of tune.

we use in our calculations.

TABLE I: LOLA-IV Geometric parameters, in inches [1]. Note that the period length $d = 3.5$ cm.

Cavity ID	$2b$	4.5805
Iris aperture diameter	$2a$	1.7670
Disk thickness	t	0.230
Iris tip radius	ρ	0.1215
Iris tip flatness	s	0.031
Alignment holes diameter	$2p$	0.750
Alignment holes offset	c	1.457

As a check on our calculations we also compute the dipole mode frequency, which was measured in the real structure. This frequency was measured in air with 40% humidity, at a temperature of 75°F, and was found to be 2.8562 GHz. The operating temperature, however, is meant to be 113°F with the the cavity under vacuum. It turns out that increasing the temperature reduces the frequency and evacuating the cavity increases the frequency in a such a way that, under operating conditions, the frequency is again 2.856 GHz [2]. Note that increasing the temperature by the 38°F alone yields a relative change in dimension of 3.7×10^{-4}, and a relative change in frequency of -3.7×10^{-4}; evacuating the cavity at room

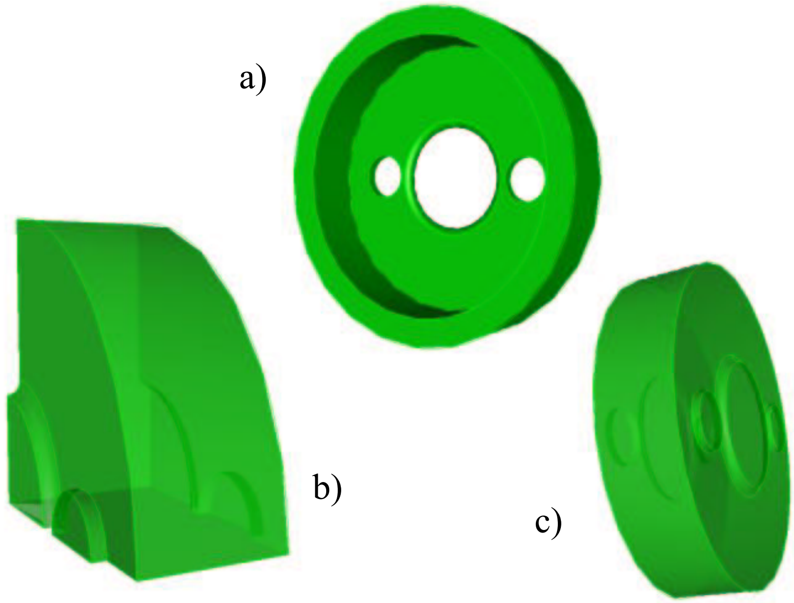

FIG. 3: The LOLA-IV cell geometry. Shown are the shape of one cell (a), and the shape of the enclosed volume of one quarter of a cell (used for calculations) (b), and the enclosed volume in a cell (c).

temperature will yield the negative of this frequency change. Our calculations will use the (room temperature) dimensions of Table I and will assume vacuum. Under these conditions we expect a frequency increase of 1 MHz, or a dipole mode frequency of 2.8572 GHz [6].

We begin by finding the modes in a 2 dimensional (2D) approximation of the LOLA structure using the frequency domain program OMEGA2 [3]. We approximate the iris tip as being perfectly rounded (a half circle in longitudinal cut). We obtain $f_0 = 2.1127$ GHz (near $\pi/2$ per cell phase advance), corresponding to a tune $\nu = 234.02$ and a dipole mode frequency $f_d = 2.8725$ GHz ($2\pi/3$ phase advance). Note that $f_d/f_0 = 1.36$. To obtain a more accurate result, we next perform a 3 dimensional (3D) calculation for the structure of Fig. 3, including the alignment holes, using OMEGA3P [4]. Again we approximate the iris tip as being perfectly round. This time we obtain $f_0 = 2.1078$ GHz ($\nu = 233.48$), and $f_d = 2.858$ GHz. Note that f_d/f_0 is nearly the same as for the cylindrically symmetric model. Also note that the dipole mode with the other polarization has a frequency that is 25 MHz higher.

To study the effect of the more complicated iris tip, we have repeated the 2D calculation, but now using the actual tip geometry. The shape of the iris tip can be described (in longitudinal cut) by two, symmetric circular arcs of radius ρ that, on one end, meet the iris tangentially, and, on the other (the tip end), meet a flat line of length s at iris radius a (see Table I). We find that the dipole mode frequency decreases due to this modification by a small amount, 0.2 MHz. Finally, our best result for the dipole mode frequency at room temperature is 2.8578 GHz, which is 0.6 MHz higher than the expected 2.8572 GHz. This discrepancy is larger than our expected accuracy, ~ 0.2 MHz, and is not understood. Nevertheless, the reasonable agreement gives us some confidence in our numerically obtained fundamental frequency.

Our (room temperature) fundamental mode frequency $f_0 = 2.1078$ GHz becomes, at operating temperatures, $f_0 = 2.1070$ MHz ($\nu = 233.39$). Note that even if this result were uncertain by as much as ± 1 MHz the tune would be in the range [233.28, 233.50], still safely away from the integer resonance. Thus, we conclude that the total spread in bunch energy due to beam loading for the LOLA-IV structure in TTF-II, to good confidence, will be $\sim 0.03\%$.

TABLE II: Calculation results assuming perfectly rounded iris tips.

Model	f_d [GHz]	f_0 [GHz]	ν
2D	2.8725	2.1127	234.02
3D	2.8580	2.1078	233.48

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[6] To allow for manufacturing errors, the real structure was built with $2b$ oversized by 0.5 mil (13 μm) and then tuned by dimpling from the outside (as was done in the manufacture of the SLAC linac rf structures) [5]. A 0.5 mil change in $2b$ changes the dipole mode frequency by 0.3 MHz, suggesting that the dimensions of Table I are believed to yield the correct frequency to within $\lesssim 0.1$ MHz.