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Cavity Control System – Optimization Methods For Single Cavity Driving and Envelope Detection.

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ABSTRACT

The paper is an introduction to the optimization methods of the linear accelerator cavity control system. Three distinct time periods of cavity operation are considered; filling with the EM field energy, field stabilization for the accelerated beam, and field decay. These periods are considered as different states of the cavity corresponding to adequate solutions of differential equation modeling the cavity behavior. The cavity could be operated by several different methods in each work phase:
During the filling – feedback and feed-forward alone, feedback and feed-forward together, self-tuning;
During the flattop – feed-forward and feedback alone or together,
During the decay – detuning and quality factor may be measured.
The optimization is understood as a choice of the most efficient way of the cavity control during each period. The control may be done in terms of minimum power consumption from the klystron during whole work cycle and efficient field stabilization in the cavity, during flattop period. The introductory analysis of the cavity operational modes in three mentioned periods is presented in this paper.
Additionally, the alternative, more precise algorithm of the cavity field detection is proposed. The cavity field is estimated as a result of intermediate frequency signal envelope decoding.
All considerations in this paper deal with a single cavity system in noiseless and deterministic conditions.

Keywords: Superconducting cavity control, linear accelerators, control theory, free electron laser

1. INTRODUCTION

The LLRF (Low Level Radio Frequency) cavity control system for the TESLA - TeV–Energy Superconducting Linear Accelerator project has been developed to stabilize the accelerating fields of the resonators. The control section, powered by one klystron, consists of four cryomodules containing eight cavities each. The control feedback system regulates the vector sum of the pulsed accelerating fields in multiple cavities. The fast and precise amplitude and phase control of the cavity field is accomplished by modulation of the signal driving the klystron. The cavity RF signal is down-converted to an intermediate frequency of 250 KHz preserving the amplitude and phase information. The ADC and DAC converters link analog and digital part of the system. The digital signal processing is realized for the field vector detection and filtering. The digital controller stabilizes the detected real (in-phase) and imaginary (quadrature) components of the incident wave according to the desired set point. Additionally the adaptive feed-forward is applied to improve compensation of repetitive perturbations induced by the beam loading and by the dynamic Lorentz force detuning (figure 1).

The digital LLRF controller has been implemented applying the very fast FPGA technology. The single cavity SIMULINK model has been developed to test the hardware controller in feedback and feed-forward operation.
Optimal cavity control depends on proper set point for feedback mode supported by adequate feed-forward signal. The main methods of cavity operation during *filling* and *flattop* time are shortly reviewed below.

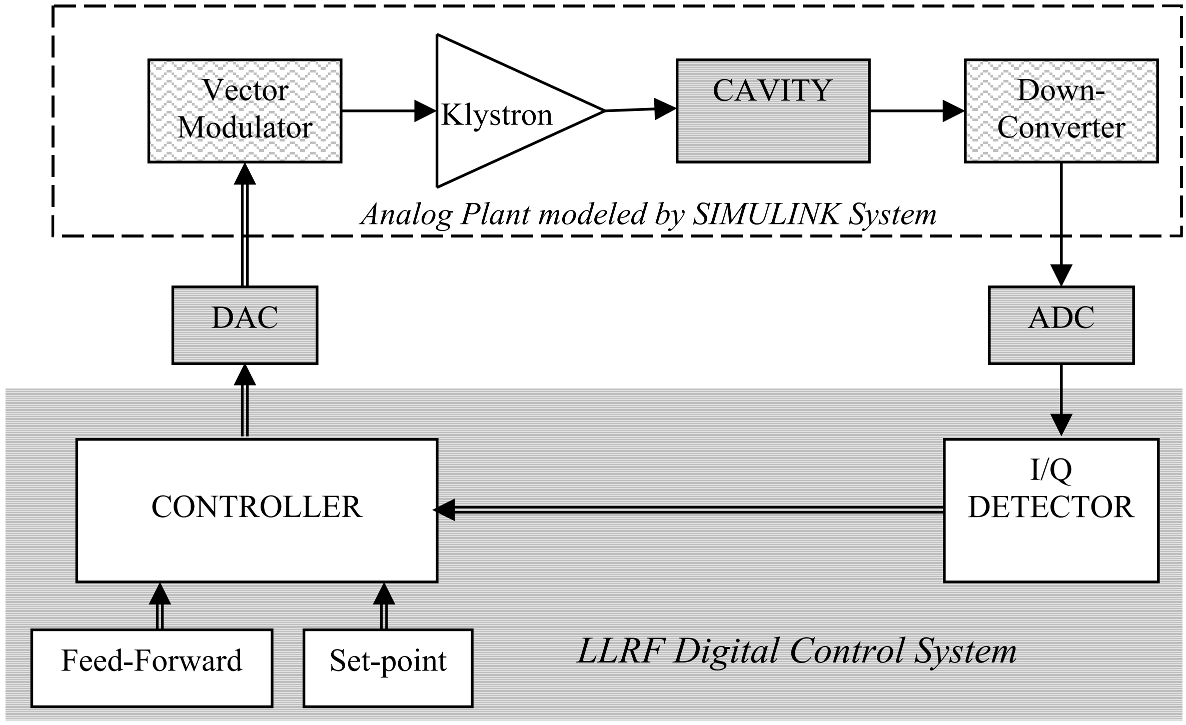

Figure 1. Simplified block diagram of cavity control system.

The cavity electrical model is based on the state space equation for the envelope of the cavity voltage $\mathbf{V}(t)$ driven by the resultant current $\mathbf{I}(t)$ caused by the generator and beam loading as follows:

$$d\mathbf{V}(t)/dt = \mathbf{A}\cdot\mathbf{V}(t) + \omega_{1/2}\cdot R_L\cdot\mathbf{I}(t)$$

where phasor $\mathbf{A} = (-\omega_{1/2}, \Delta\omega)$ for complex representation, or matrix $\mathbf{A} = [-\omega_{1/2}, -\Delta\omega; \Delta\omega, -\omega_{1/2}]$ for vector representation, R_L – cavity shunt impedance, $\omega_{1/2}$ – cavity half-bandwidth, $\Delta\omega$ – cavity detuning.

This state space equation is applied for the initial analysis and simulation of the cavity behavior. The extended model as an electromechanical one performs more real features of the system concerning to the cavity Lorentz force detuning. The mechanical part of the cavity model describes the dynamic Lorentz force detuning $\Delta\omega(t)$, which is the function of the square of the time varying gradient. All results presented below are based on the electromechanical cavity model.

2. CAVITY VOLTAGE ENVELOPE DETECTION.

The cavity voltage envelope is the fundamental data desired for the digital controller. The effective algorithm for envelope detection is an important contribution to the optimization of the cavity control system.

The cavity RF signal is down-converted to an intermediate frequency f_i preserving amplitude $A(t)$ and phase $\varphi(t)$ information as follows:

$$x(t) = A(t)\cdot\cos(2\pi f_i\cdot t + \varphi(t)).$$

Let's assume the representation of the cavity voltage as an *analytical signal* as follows:

$$\mathbf{U}(t) = A(t)e^{i\varphi(t)}\cdot\exp(\boldsymbol{i}\cdot 2\pi f_i\cdot t) = \mathbf{V}(t)\cdot\exp(\boldsymbol{i}\cdot 2\pi f_i\cdot t)$$

where the complex envelope $\mathbf{V}(t) = A(t)e^{i\varphi(t)}$ with the real part called *in-phase* component (I), and the imaginary part called *quadrature* component (Q).

The digital signal processing is performed applying the signal of intermediate frequency as a real part of an *analytical signal* with sampling period $T = 1/4f_i$.

Therefore the *analytical signal* $\mathbf{U}(t) = \mathbf{U}(kT) = \mathbf{U}_k$ for successive samples k is as follows

$$\mathbf{U}_k = \mathbf{V}(kT)\cdot\exp(\boldsymbol{i}\cdot 2\pi f_i\cdot kT) = \mathbf{V}_k\cdot(\boldsymbol{i})^k = x_k + \boldsymbol{i}\cdot y_k$$

where sample of the real signal equals x_k and the imaginary sample equals y_k.

Zero Order Envelope Detection.

Let's assume the **stable** value of an amplitude and phase of the cavity voltage for successive steps k, so that $\mathbf{V}_{k-1} = \mathbf{V}_k =$ const.

Two consecutive samples of the real signal x_{k-1}, x_k are required for *zero order* envelope detection according to relations:

$$\mathbf{V}_{k-1}\cdot(\boldsymbol{i})^{k-1} = x_{k-1} + \boldsymbol{i}\cdot y_{k-1} \quad \text{and} \quad \mathbf{V}_k\cdot(\boldsymbol{i})^k = x_k + \boldsymbol{i}\cdot y_k.$$

Applying upper equations the complex envelope is determined by digital demodulation as follows:

$$\mathbf{V}_k = (x_k + \boldsymbol{i}\cdot x_{k-1})(-\boldsymbol{i})^k$$

First Order Envelope Detection.

More accurate estimation of (I, Q) components is required for fast varying envelope during cavity *filling* time.

Let's assume **linearly** time varying complex envelope for successive steps k with mutual difference of $\Delta\mathbf{V}$, as follows:

$$\mathbf{V}_{k-1} = \mathbf{V}_k - \Delta\mathbf{V},\ \ \mathbf{V}_{k+1} = \mathbf{V}_k + \Delta\mathbf{V}.$$

Three sequential samples of the real signal x_{k-1}, x_k, x_{k+1} are required for *first order* envelope detection according to the relations:

$$(\mathbf{V}_k - \Delta\mathbf{V})\cdot(\boldsymbol{i})^{k-1} = x_{k-1} + \boldsymbol{i}\cdot y_{k-1} \qquad \mathbf{V}_k\cdot(\boldsymbol{i})^k = x_k + \boldsymbol{i}\cdot y_k \qquad (\mathbf{V}_k + \Delta\mathbf{V})\cdot(\boldsymbol{i})^{k+1} = x_{k+1} + \boldsymbol{i}\cdot y_{k+1}.$$

Applying upper equations the complex envelope is determined by digital demodulation as follows:

$$\mathbf{V}_k = (x_k + \boldsymbol{i}\cdot(x_{k-1} - x_{k+1})/2)\cdot(-\boldsymbol{i})^k\ .$$

The *first order* envelope detection algorithm may possibly engage more samples covering the period of the intermediate frequency signal for an advance improvement.

Five sequential samples of the real signal x_{k-2}, x_{k-1}, x_k, x_{k+1} x_{k+2}, are required for modified algorithm of the *first order* envelope detection, removing the *offset* of the intermediate frequency signal according to the relations:

$$\mathbf{V}_k = (\mathbf{V}_{k-1} + \mathbf{V}_{k+1}\ )/2 = ((x_k - x_{k-2})/4 + (x_k - x_{k+2})/4 + \boldsymbol{i}\cdot(x_{k-1} - x_{k+1})/2)\cdot(-\boldsymbol{i})^k\ .$$

Therefore the averaging process is involved causing better envelope estimation and efficient *offset* removing for intermediate frequency signal.

Furthermore the fitting calibration factor corrects the resultant cavity voltage envelope and compensates the measurement channel attenuation and phase shifting for an individual cavity.

Both algorithms for the cavity voltage envelope detection are implemented in the MATLAB controller coupled to the SIMULINK cavity. The simulation results are presented in figure 2 and comparisons of errors are presented in figure 3. The results for the real *off-line* signal envelope detection are presented in figure 4 for both algorithms.

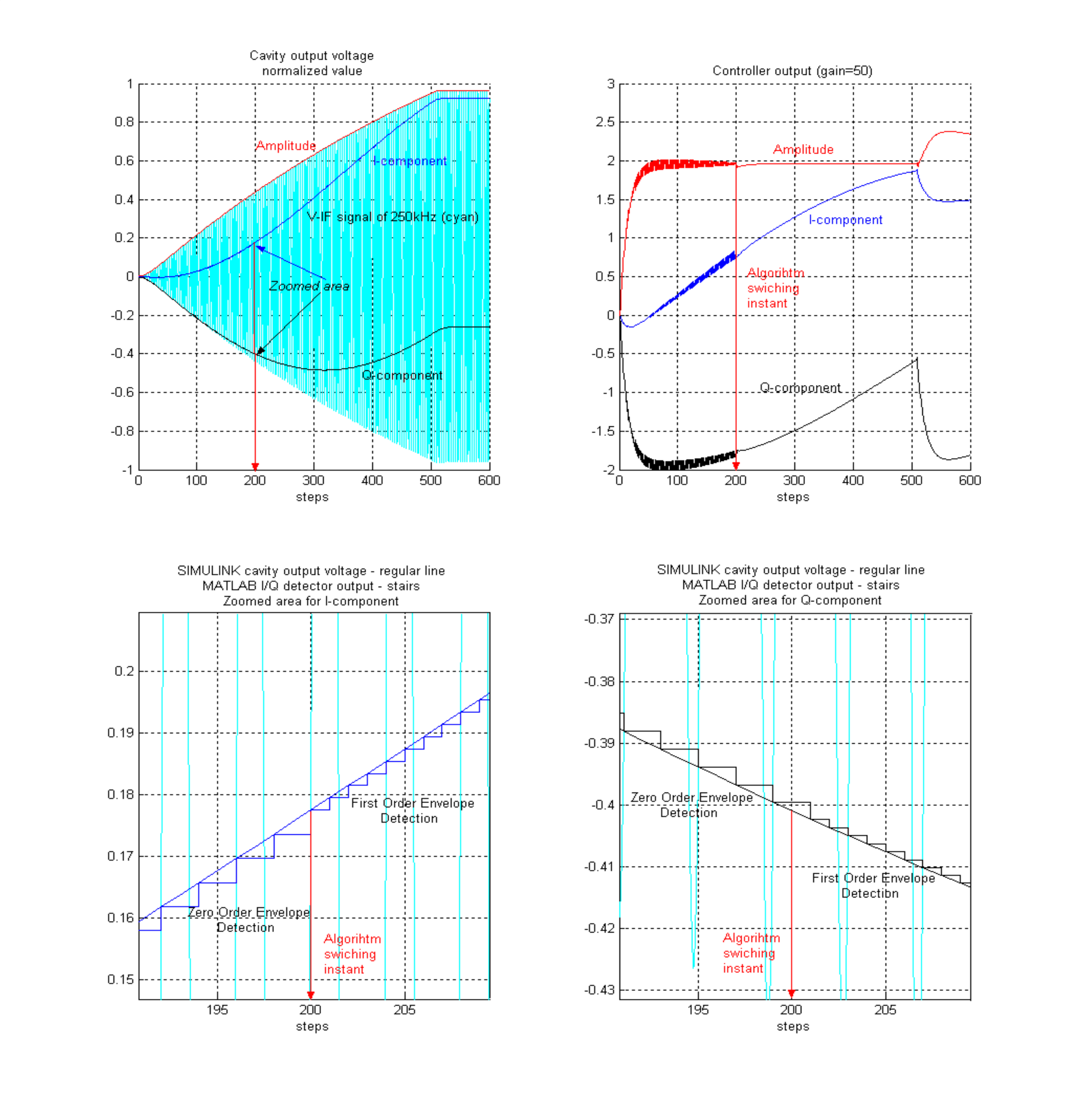

Figure 2. SIMULINK cavity output and MATLAB output for I/Q detector and controller in feedback operation. Two envelope algorithms are applied for different periods of cavity *filling* time.

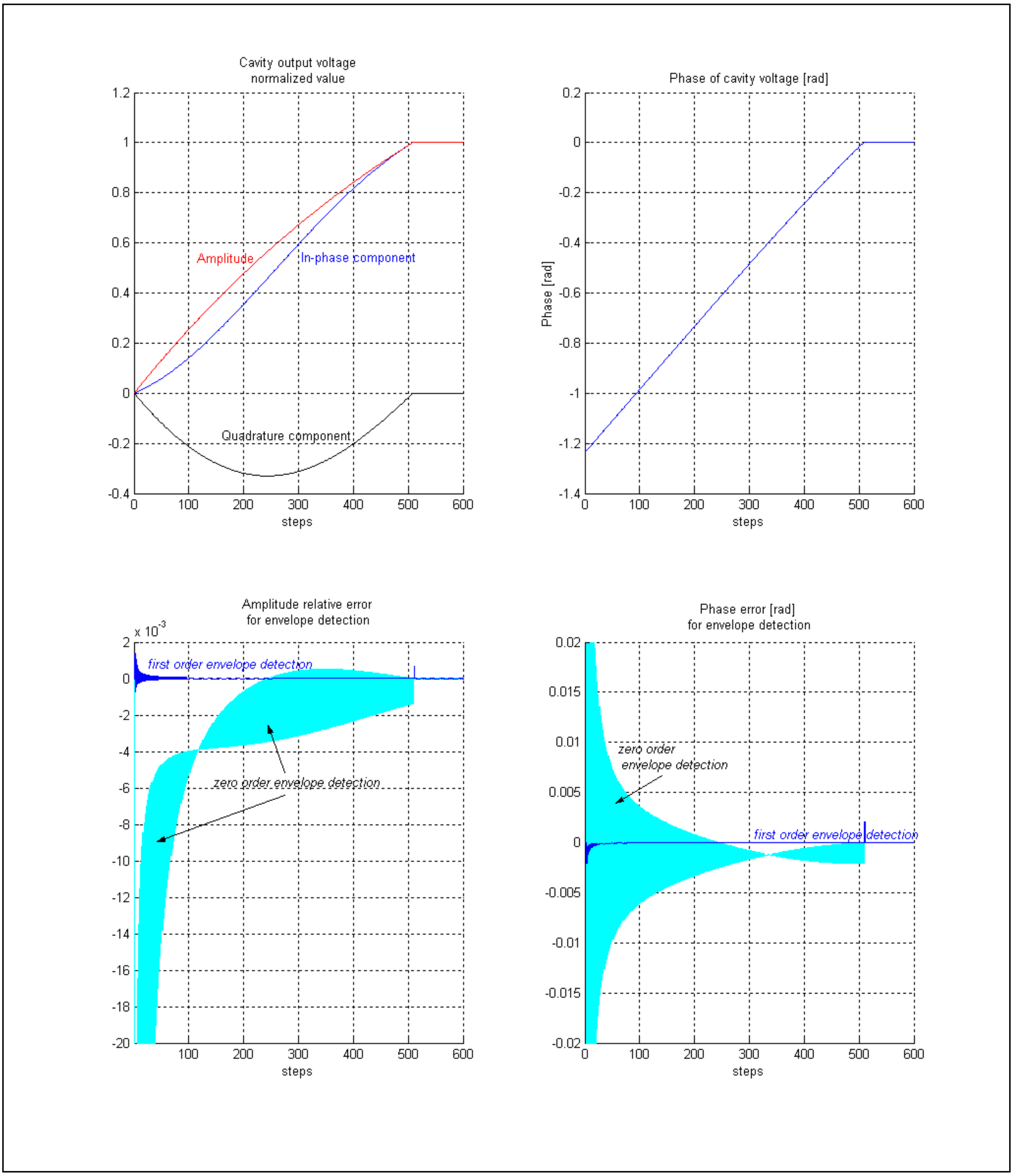

Figure 3. Errors comparison for two algorithms of envelope detection during ideal feed-forward cavity driving.

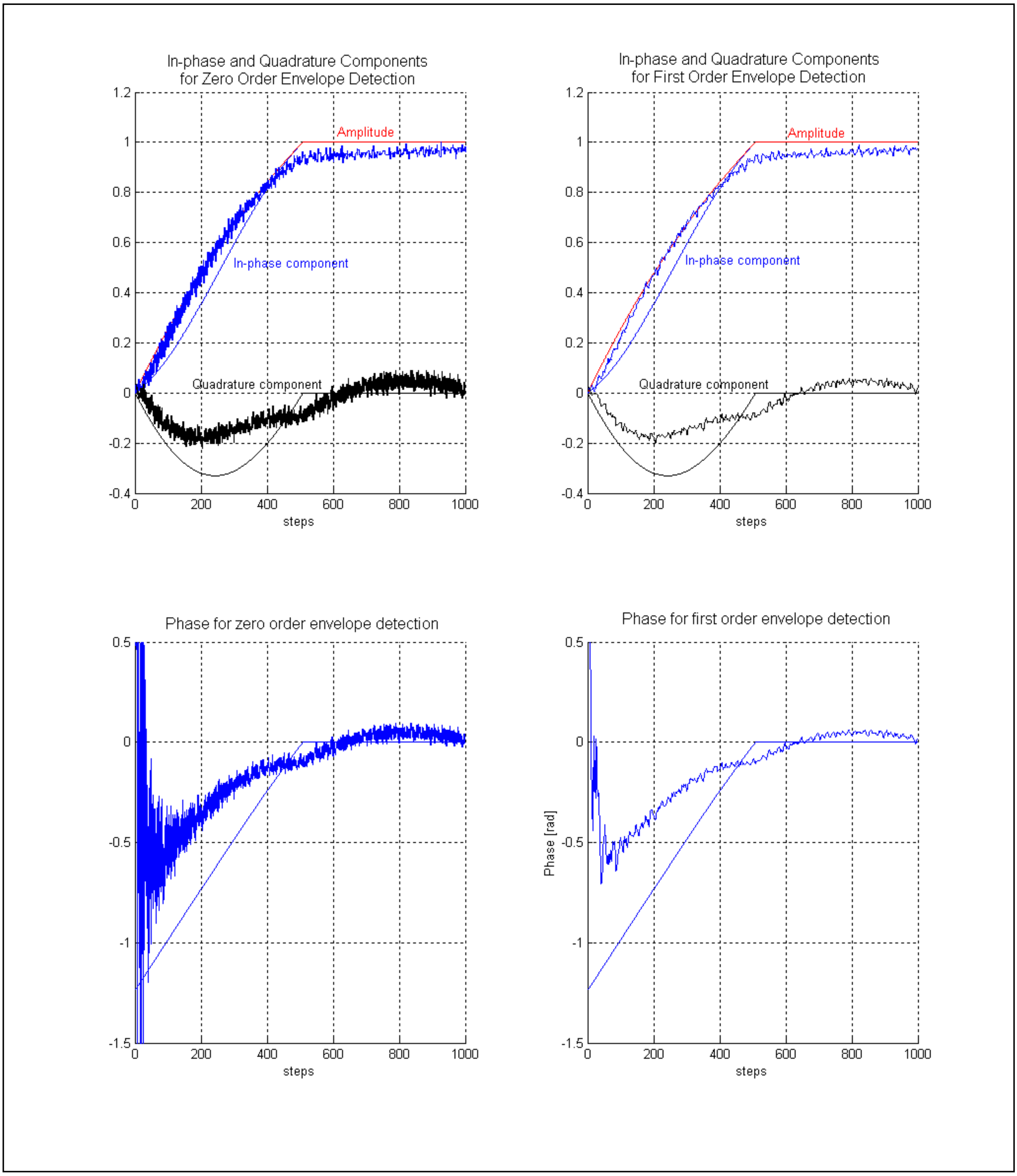

Figure 4. Envelope of ideal and real *off-line* cavity voltage for two detection algorithms.

3. CAVITY *FILLING* OPERATION

The main purpose of the cavity filling is to achieve proper operating conditions for the beam loading. The cavity is filled with the preset, constant, forward power, during the first stage of operation. This results in an exponential increase of the envelope of RF electromagnetic field according to its natural behavior in the resonance condition. But the Lorentz force decreases the cavity resonance frequency with the rising field gradient.

Lets assume a phase modulated current step as an envelope of cavity input, in the complex domain of (I, Q)

$$\mathbf{I}(t) = I_o \cdot \exp(\boldsymbol{i}\Phi(t))$$

The appropriate phase modulation $\Phi(t)$ of the incident wave compensates the time varying cavity detuning $\Delta\omega(t)$, if

$$d\Phi/dt = \Delta\omega(t).$$

Thus, the RF signal follows the cavity resonance frequency.

The envelope of the cavity response in the complex domain of (I, Q), as a solution of the state space equation, is as follows:

$$\mathbf{V}(t) = I_o \cdot R_L \cdot (1 - \exp(-\omega_{1/2} \cdot t)) \cdot \exp(\boldsymbol{i}\Phi(t)),$$

where, $\omega_{1/2}$ = cavity half-bandwidth and R_L = cavity load resistance.

Therefore, the exponential relation between the cavity input, with stable magnitude, and the cavity output is valid in the resonance condition, as follows:

$$\mathbf{V}(t)/\mathbf{I}(t) = R_L \cdot (1 - \exp(-\omega_{1/2} \cdot t))$$

Thus, the zero input-output phase difference is enforced.

The required phase $\Phi(t)$ is the integral of the detuning, for the given function $\Delta\omega(t)$. The amplitude of the cavity voltage attains half of the asymptotic value (for ideal case) at the beam injection instant $T_f = \ln 2/\omega_{1/2}$, and is set stable as the desired value $|\mathbf{V}|$ at the *flattop* level. The proper initial condition $\Phi_0 = \Phi(0)$ adjusts the required final *flattop* value $\Phi(T_f) = \varphi$ for *filling* time T_f.

Thus, the cavity can be driven in the resonance condition during the *filling* time, for the given cavity detuning $\Delta\omega(t)$:

- in **feed-forward** mode by the proper phase modulated step of the incident wave:

$$\mathbf{i}(t) = 2|\mathbf{V}| \cdot \exp(\boldsymbol{i}\Phi(t))/R_L$$

- in **feedback** mode by the phase modulated exponentially rising set-point:

$$\mathbf{v}(t) = 2|\mathbf{V}| \cdot (1 - \exp(-\omega_{1/2} \cdot t)) \cdot \exp(\boldsymbol{i}\Phi(t)).$$

On the other hand, the cavity can drive itself in the resonance without external detuning information in so-called ***self-tuning*** mode. It is a kind of a *self-exciting* mode, where the controller forces the cavity input initial condition. The cavity is driven by constant current equal $\mathbf{I_o} = 2|\mathbf{V}| \cdot \exp(\boldsymbol{i}\Phi_0)/R_L$ during the initial step corresponding to the sampling period T. The resultant cavity output ΔV can be estimated as $\Delta V/|\mathbf{V}| \approx 2\omega_{1/2} \cdot T$ relatively to the final value $|\mathbf{V}|$ desired at the *flattop* level.
The cavity output takes driving itself, for any step number $k \geq 1$, according to the resonance relation:

$$\mathbf{I}(k \cdot T) = \mathbf{V}(k \cdot T)/(R_L \cdot (1 - \exp(-\omega_{1/2} \cdot k \cdot T))).$$

Therefore, the cavity forces itself to track its resonance frequency. The digital controller applies the table with exponentially changing values and equalizes the cavity output and input phases.

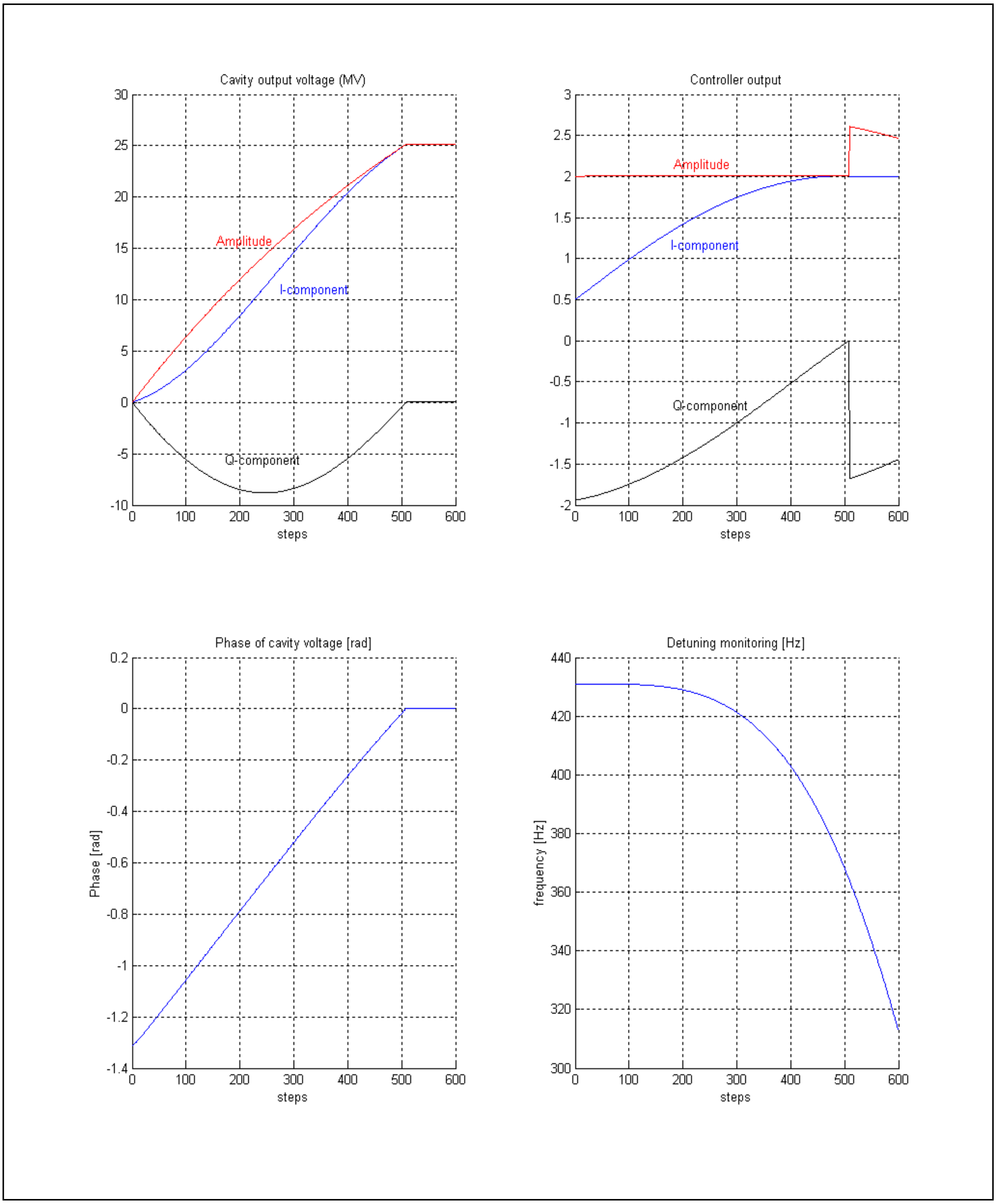

Figure 5. Cavity and controller output, operated in self-tuning mode, during *filling* time.

In any case, the magnitude of the current $|\mathbf{I}_o| = 2|\mathbf{V}|/R_L$ and the initial phase Φ_0 should be correctly established, so that to accomplish the desired steady state at the *flattop* level.

The cavity control system has been modeled applying the Matlab code. Three modes of the cavity *filling* operation have been implemented for sampling time T=1µs.

The auto calibration algorithm has been developed for initial phase Φ_0, from pulse to pulse, in the Matlab system.

The simulation results for the *self-tuning* mode of the cavity operation, for the Matlab system, are presented in figure 5. The results agree with the feed-forward operation for the given cavity detuning $\Delta\omega(t)$ during *filling* time.

4. CAVITY *FLATTOP* OPERATION

The envelope of the cavity voltage is stable during the *flattop* operation at the expense of an additional power reflected, due to the mismatch caused by the cavity detuning. The solution of the state space equation, for the steady state, in complex domain of (I, Q), is as follows:

$$\mathbf{V} = \mathbf{I}\cdot R_L/(1-\boldsymbol{i}\Delta\omega/\omega_{1/2}) = \mathbf{I}\cdot R_L\cos\psi\cdot e^{i\psi}$$

where, voltage phasor **V -** desired envelope of the cavity output value, resultant current phasor **I** - required envelope of the cavity input value and tuning angle $\psi = \tan^{-1}(\Delta\omega/\omega_{1/2})$.

The driving feed-forward current $\mathbf{I}_f$ can be estimated, for the stable value $\mathbf{V}=|\mathbf{V}|e^{i\varphi}$ desired at the *flattop* level, with the beam loading $\mathbf{I}_b$, and the detuned cavity, as follows:

$$\mathbf{I}_f = |\mathbf{V}|e^{i\varphi}\cdot(1-\boldsymbol{i}\Delta\omega/\omega_{1/2})/R_L + \mathbf{I}_b, \qquad \text{where} \quad \mathbf{I}_f - \mathbf{I}_b = \mathbf{I}.$$

Thus, the predicted current component, equal to $\mathbf{I}_b$, compensates the beam loading current, and detuning dependent factor $\Delta\omega/\omega_{1/2}$, compensates the cavity impedance variation. The required input current can be established, for the given cavity detuning $\Delta\omega(t)$, and results in the desired cavity output.

The simulation results of the feed-forward operation for the Matlab system are presented in figure 6.

The required cavity voltage can be as well accomplished through the feedback operation with accuracy limited by the controller gain with set-point value $\mathbf{V}=|\mathbf{V}|e^{i\varphi}$ without cavity detuning information.

The simulation results of feedback operation are presented in figure 7 for the Matlab system.

To minimize the peak power the proper cavity pre-detuning $\Delta\omega_0$ shifts the time varying detuning, so that required forward power is symmetrically extended along the *flattop* range. The cavity pre-detuning $\Delta\omega_0$ and initial phase Φ_0 are adequately set up for all changes of the cavity operating parameters.

On the other hand, the results achieved by the feedback mode, carry information about the condition of the cavity operation during one pulse. The controller output for feedback mode is approximated reproduction of the ideal feed-forward signal (compare figure 6 and 7). Therefore, the cavity detuning during the pulse can be estimated applying (I, Q) relation for the steady state as follows:

$$\Delta\omega(t) \approx -\omega_{1/2}\cdot R_L/|\mathbf{V}|\cdot Im(\mathbf{I}\cdot e^{-i\varphi}),$$

The result of detuning estimation compare to the cavity model detuning monitoring for feedback mode is presented in figure 7.

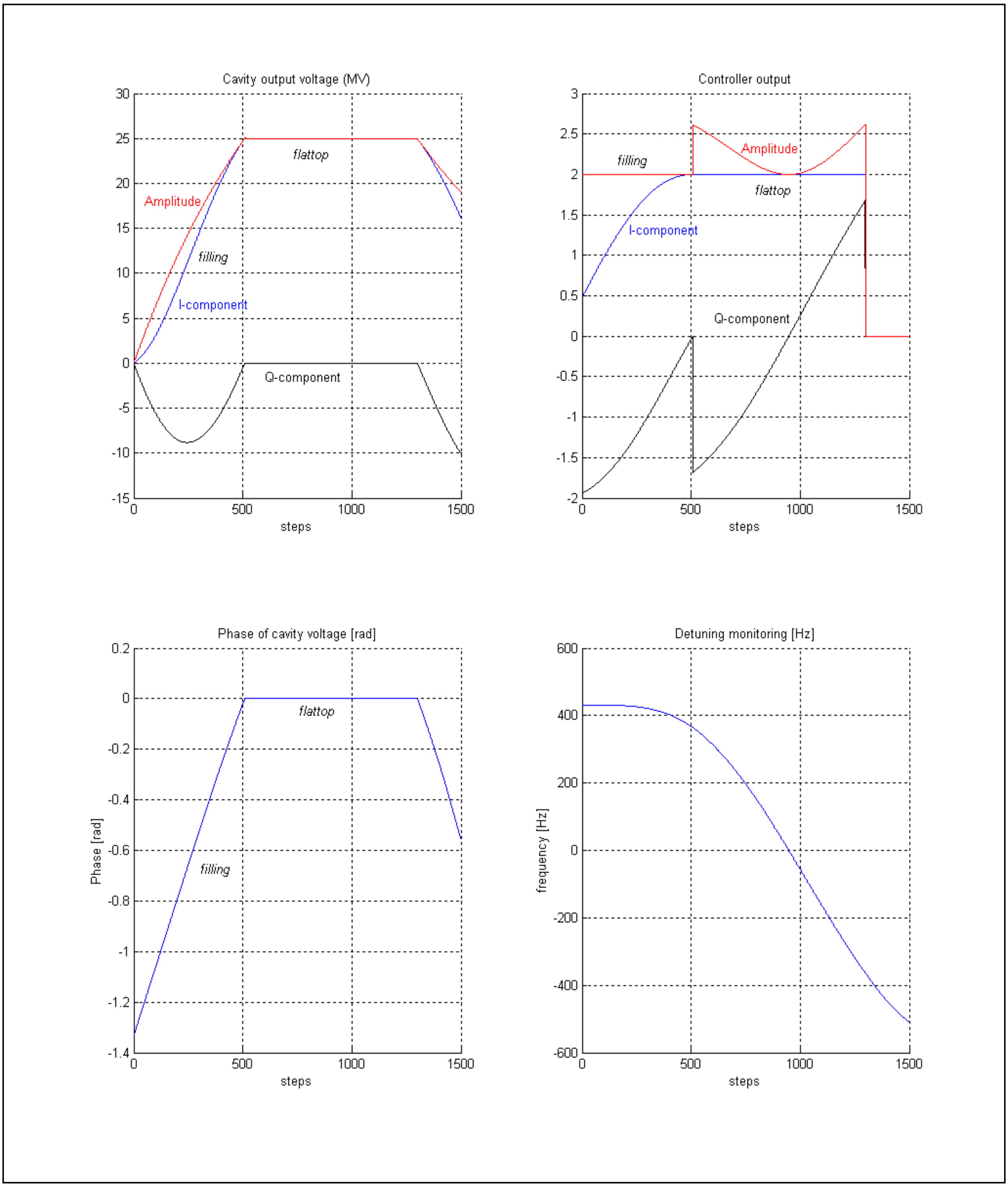

Figure 6. Cavity and controller output, operated by ideal feed-forward, during *filling* and *flattop* time.

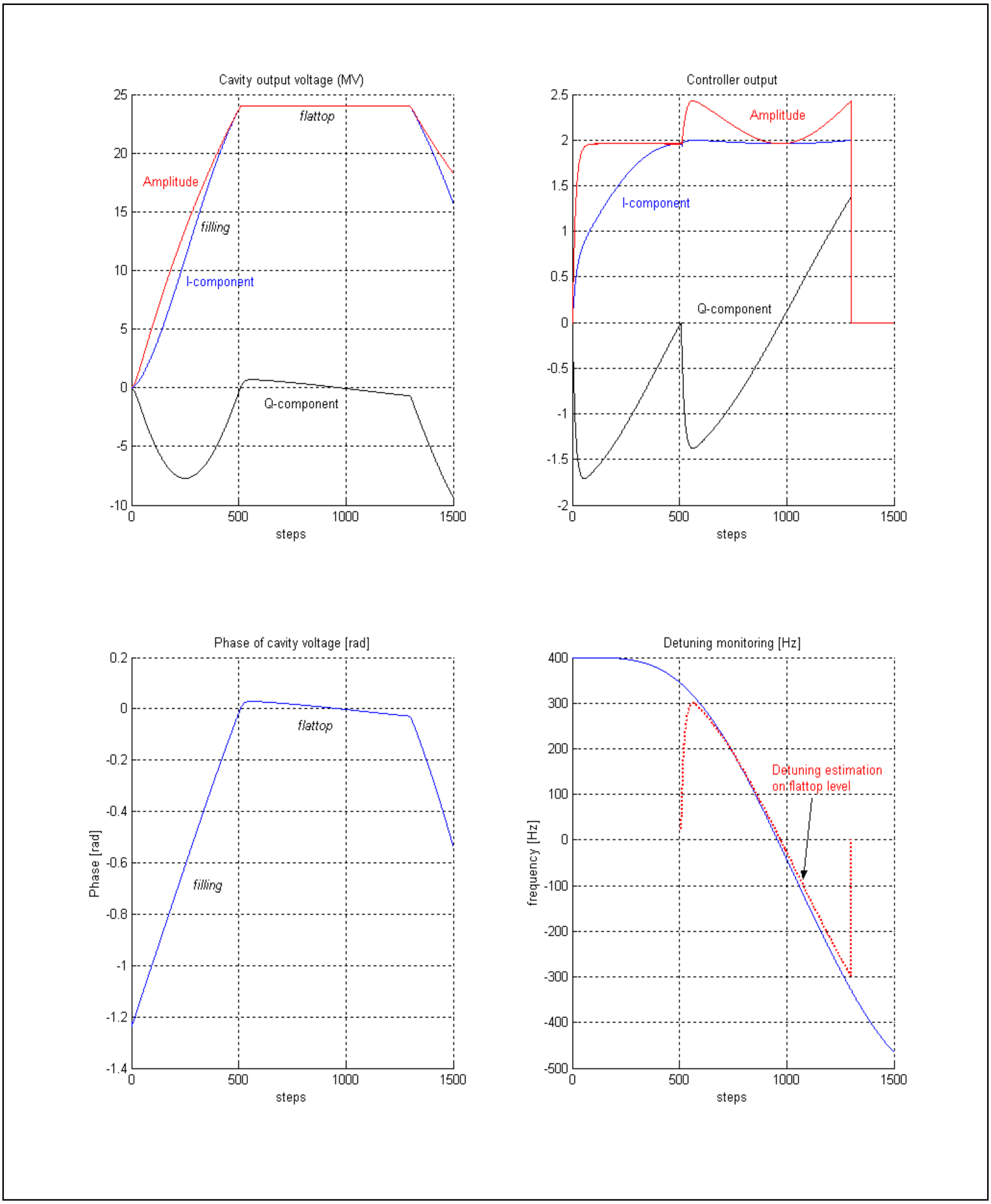

Figure 7. Cavity and controller output, operated by feedback, during *filling* and *flattop* time (gain=50).

5. CAVITY *DECAY* OPERATION

The envelope of the cavity voltage yields an exponential *decay*, after turning off both the generator and the beam, according to the homogeneous solution of the state space equation in complex domain of (I, Q) as follows:

$$\mathbf{V}(t) = |\mathbf{V}|e^{i\varphi}\cdot\exp(-\omega_{1/2}+\boldsymbol{i}\Delta\omega)t = v(t)\cdot\exp(-\boldsymbol{i}\Phi(t)).$$

where, time dependent amplitude $v(t) = |\mathbf{V}|\cdot\exp(-\omega_{1/2}\cdot t)$, and phase $\Phi(t) = \Delta\omega\cdot t + \varphi$.
Therefore, the cavity parameters can be determined according to the equations, in turning off instant:

cavity half-bandwidth $\quad \omega_{1/2} = -\, dv/dt\,/v(t)$

cavity detuning $\quad \Delta\omega = d\Phi/dt$

6. CONCLUSIONS

The algorithm of the *first order* envelope detection has been verified as much better estimation of (I, Q) components comparing to *zero order* one for fast varying envelope during cavity *filling* time. The comprehensive reduction of the systematic error (*offset*) is the substantial feature of that algorithm.

The efficient feedback or feed-forward mode can be applied only on condition that we possess information about the time varying cavity detuning during *filling* time. Thus, the *self-tuning* method is the only solution for optimal cavity driving during *filling* time requiring no information of the cavity detuning.

The efficient *flattop* feed-forward mode can be applied only on condition that we possess information about the time varying cavity detuning. The desired cavity voltage can be accomplished through the *flattop* feedback operation with accuracy limited by the controller gain without cavity detuning information. The results achieved by the feedback mode, provide some information about the cavity operation during one pulse. For example time varying cavity detuning can be estimated. It is to consider how to apply this information for successive pulses in repetitive conditions.

The feedback/feed-forward and self-tuning methods seem at first sight apparently exclusive. It turns out, however, that in real conditions, they may be efficiently supplemented. This results in a comparatively new method of weighted cavity control incurring the usage of self-tuning and feedback/feed-forward methods together.

REFERENCES

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