Theoretical Analysis on the Limitation from the Nonlinear Space Charge Forces to TESLA Damping Ring

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[1]

Abstract

By making analogy between space charge and beam-beam effects and by applying the general theoretical method developed in ref. [1] (J. Gao, *Nucl. Instr. and Methods* **A**463 (2001) p. 50), we have explained theoretically the physical cause of the limitation from the space charge effect on TESLA damping ring, which is confirmed by the numerical simulations.

1 Introduction

The problem of nonlinear space charge forces (the forces felt by an individual particle traveling inside and together with the bunched or unbunched beam) in a hadron accelerator has been a subject of research since many decades. In conventional lepton storage rings (from AdA to LEP), however, the nonlinear space charge force effect has never become a critical problem in very contrast to the nonlinear beam-beam forces in circular colliders. In this paper, we

[1]TESLA Report 2003-12

will make a theoretical analysis on the limitation from nonlinear space charge forces to an unconventional storage ring, such as TESLA damping ring [2].

In section 2, we first establish the relationship between the space charge and the beam-beam effects, and then in section 3, by applying the method to treat analytically the beam-beam effects established in ref. [1], we give the analytical formula for the nonlinear space charge determined electron's lifetime. Finally, in section 4, we will show how the nonlinear space charge forces limit the machine performance by taking TESLA damping ring as an example. All our discussions in this paper are restricted to lepton machines.

2 Relation between space charge and beam-beam forces

Considering an electron storage ring, we start with the linear incoherent space charge tune shift of the machine at the center of the bunch [3][4][5] [6][7]:

$$\xi_{sc} = -\frac{r_e N_e \beta_{av,y}}{2\pi\gamma\sigma_y(\sigma_x+\sigma_y)}\left(\frac{L}{\sqrt{2\pi}\beta^2\gamma^2\sigma_z}\right) \tag{1}$$

where r_e is the classical radius of electron, N_e is the particle population inside the bunch, σ_z is the bunch length, $\beta_{av,y}$ is the average over the ring and $\beta_{av,x}$ is assumed to be equal to $\beta_{av,y}$, σ_x and σ_y are the average bunch's transverse dimensions, β and γ are the normalized electron's velocity and the energy, respectively. In fact, one can define the *differential* space charge tune shift from which the space charge tune shift of the ring can be obtained:

$$\xi'_{sc}(s_0) = -\frac{r_e N_e \beta_y(s_0)}{2\pi\gamma\sigma_y(s_0)(\sigma_x(s_0)+\sigma_y(s_0))}\left(\frac{1}{\sqrt{2\pi}\beta^2\gamma^2\sigma_z}\right)ds \tag{2}$$

where s_0 denotes un arbitrary position in the ring. Recalling the expression of the beam-beam tune shift of a storage ring collider, one has

$$\xi_{bb,y}(s_{IP}) = \frac{r_e N_e \beta_{y,IP}}{2\pi\gamma\sigma_y(s_{IP})(\sigma_x(s_{IP})+\sigma_y(s_{IP}))} \tag{3}$$

where s_{IP} denotes the interaction point. Comparing eq. 2 with eq. 3, one finds that the transverse deflecting forces from the differential space charge and the beam-beam interactions have the following relation:

$$f'_{sc}(s) = f_{bb}(s_{IP})G \tag{4}$$

with

$$G = -\left(\frac{1}{\sqrt{2\pi}\beta^2\gamma^2\sigma_z}\right) ds \tag{5}$$

where f'_{sc} and f_{bb} are the total transverse forces including, of course, non-linear parts. We conclude that the differential space charge effect can be made equivalent to the problem of beam-beam interaction in an storage ring collider.

3 Nonlinear space charge forces limited lifetime

It is high time now for us to recall the analytical work on the beam-beam interactions in e^+e^- storage ring colliders [1]. Taking a flat beam ($\sigma_y << \sigma_x$) for example, from eq. 27 of ref. [1] one has the dynamic aperture determined by the nonlinear (octupole is the lowest nonlinear multipole) differential space charge force

$$A_{sc,y}(s) = \frac{\sqrt{\beta_y(s)}}{\beta_y(s_0)}\left(\frac{3\sqrt{2}\gamma\sigma_x(s_0)\sigma_y^3(s_0)}{N_e r_e G}\right)^{1/2} \qquad \text{(FB)} \tag{6}$$

The question which follows immediately is how about the dynamic aperture resulted from the cumulation of the differential space charge effect. To answer this question let's recall the formula given by eq. 53 in ref. [8] and we have

$$A_{total,sc,y}(s) = \frac{1}{\sqrt{\sum_{s_0=0}^{L}\frac{1}{A_{sc,y}(s)^2}}} \tag{7}$$

$$\frac{1}{A^2_{total,sc,y}(s)} = \int_{s_0=0}^{L}\frac{\beta_y(s_0)^2}{\beta_y(s)}\left(\frac{N_e r_e}{6\sqrt{\pi}\beta^2\gamma^3\sigma_x(s_0)\sigma_y(s_0)^3\sigma_z}\right) ds_0 \tag{8}$$

where the differential space charge forces are assumed to be independent (quite similar to our treatment in the estimation of wigglers limited dynamic apertures in a storage ring [9]). After some mathematical simplification and using eq. 1, one gets

$$\mathcal{R}_y^2 = \left(\frac{A_{total,sc,y}(s)}{\sigma_y(s)}\right)^2 = \frac{3}{\sqrt{2}\pi\xi_{sc}} \tag{9}$$

The particle's lifetime due to nonlinear space charge forces can be estimated as [1]:

$$\tau_{sc,y}(\xi_{sc,y}) = \frac{\tau_y}{2}\left(\mathcal{R}_y^2\right)^{-1}\exp\left(\mathcal{R}_y^2\right) = \frac{\tau_y}{2}\left(\frac{3}{\sqrt{2}\pi\xi_{sc,y}}\right)^{-1}\exp\left(\frac{3}{\sqrt{2}\pi\xi_{sc,y}}\right) \quad (10)$$

where τ_y is the damping time in y plane. What should be stressed is that even $\tau_{sc,y}/\tau_y$ is expressed as a function of $\xi_{sc,y}$, the linear space charge tune shift itself, however, is not the physical cause of the limitation.

4 Application to TESLA damping ring

Knowing the particle's lifetime limited by the nonlinear space charge force expressed in eq. 10, one can calculate the relative particle's survival population, $R(\xi_{sc,y})$, at the moment of ejection ($t = \tau_{st}$) by the following formula:

$$R(\xi_{sc}) = \exp\left(-\frac{\tau_{st}}{\tau_{sc,y}(\xi_{sc,y})}\right) \quad (11)$$

Now we apply eq. 11 to TESLA damping ring [7] with $\tau_y = 28$ ms, and storage time $\tau_{st} = 200$ ms, and calculate the relative particle's survival population with respect to the the linear space charge tune shift $\xi_{sc,y}$. Obviously from Fig. 1, one finds that to avoid the particle loss due to nonlinear space charge forces, one has to choose $\xi_{sc,y}$ below 0.07 (less than 1% particles are lost), which coincides with the conclusion from the numerical simulations in ref. [7] which states clearly that the condition $\xi_{sc,y} < 0.1$ should be fulfilled. Taking the TESLA parameters, $E_0 = 5$ GeV, $L = 17$ km, $N_e = 2 \times 10^{10}$, $\sigma_z = 6$ mm, and the normalized transverse emittances, $\epsilon_{x,n} = 9 \times 10^{-6}$ mrad and $\epsilon_{y,n} = 2 \times 10^{-8}$ mrad, one finds $\xi_{sc,y} = 0.248$ and $R(\xi_{sc,y}) = 7.7\%$, which are intolerable. In order to solve this problem, instead of increasing the damping ring's energy, a method has been proposed in ref. [7] to increase the beam dimensions in the long straight sections of the "Dog-Bone" type damping ring by using skrew quadrupoles, which has reduced the space charge tune shift well below the threshold, $\xi_{sc,y} = 0.1$. Finally, it is proposed that experiments could be made on the future TESLA damping ring to verify the validity of the analytical result shown in Fig. 1.

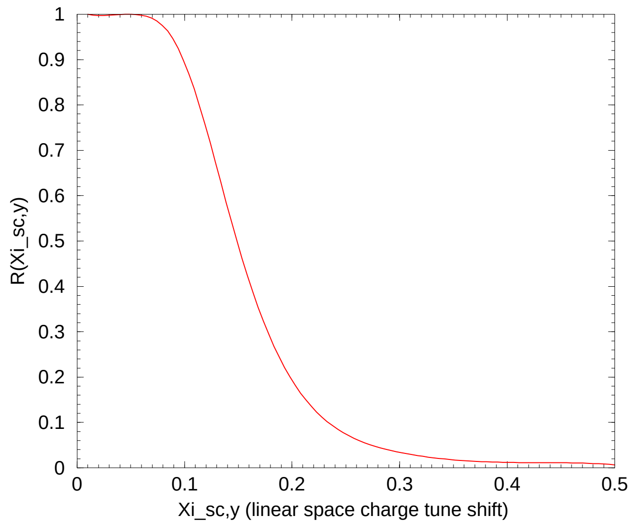

Figure 1: The relative particle's survival population at the moment of ejection, $R(\xi_{sc,y})$, vs the linear space charge tune shift $\xi_{sc,y}$.

5 Conclusion

In this paper, by making analogy between nonlinear space charge forces with that from beam-beam interaction in a storage ring collider, we find the analytical nonlinear space charge force limited particle's lifetime, which confirms the numerical simulation results conducted for TESLA damping ring [7].

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