

Cavity control system essential modeling for TESLA linear accelerator

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ABSTRACT

The pioneering TESLA linear accelerator and free electron laser project is initially introduced. Elementary analysis of cavity resonator with signal and power considerations is presented. Two alternative simulation models of cavity control system are proposed.

Keywords: TESLA, free electron laser, accelerator, high power microwave cavity, vector and phasor simulation models

1. INTRODUCTION

TESLA stands for TeV–Energy Superconducting Linear Accelerator – a 33 kilometer-long electron-positron collider. This new impressive project is being developed and planned at DESY research center in Hamburg. In order to demonstrate the feasibility of such an immense accelerator, the TESLA Test Facility as a prototype was constructed and is still under intensive improvement during its operation.

Accelerator section uses superconducting niobium structures of so-called cavity resonators. The very low energy spread during acceleration process is necessary to obtain an extremely small particles' interaction point and high beam luminosity. The complex control system for the relativistic beam has been developed to cope with signal disturbances, non-linearity and with time-varying parameters, in order to stabilize accelerating fields of resonators.

A single accelerating module consists of 32 cavities powered by one klystron. The control feedback system regulates the vector sum of pulsed accelerating fields in multiple cavities. The superconducting cavities, due to their narrow bandwidth, are very sensitive to mechanical perturbation caused by microphonics and Lorentz force detuning. In addition to the feedback control loop, which suppresses stochastic errors, the adaptive feed forward is applied to compensate repetitive perturbations induced by the beam loading and by the dynamic Lorentz force detuning.



The TESLA control is a driven feedback system stabilizing the detected real and imaginary parts of the incident wave and thus affecting the amplitude and the phase of a constant frequency signal. In order to analyze and optimally design the system a sophisticated modeling is necessary.

2. PHYSICAL DESCRIPTION OF THE SYSTEM

The TESLA cavity is a 9-cell standing wave structure which is about 1m long and whose fundamental TM mode has a frequency of 1300 MHz and the bandwidth of about 430 Hz (FWHM). A multi-cell structure is applied to maximize the active acceleration space and it is limited by the field's homogeneity requirements and parasitic pass-band modes. The resonator is operated in the π -mode with 180° phase difference between adjacent cells. The RF (radio frequency) oscillating field is synchronized with the motion of a particle moving with the velocity of light across the cavity.

The equivalent representations of the chain of nine cells are resonant LCR circuits which are magnetically coupled. The following considerations are limited to the π -mode of one cavity represented as a single LCR circuit.

The effective accelerating voltage V_{acc} is defined by the energy gain of the unity charge. The cavity voltage V_c is the maximum accelerating voltage acting on the particle. The accelerating voltage of a bunch passing the cavity with a time delay t_b is

$$V_{acc}(t_b) = V_c \cdot \cos(\omega t_b) = V_c \cdot \cos \varphi$$

where $\omega=2\pi f$ is the wave pulsation, $\omega t_b = \varphi$ is the angle phase of the cavity voltage related to the beam current.

According to the energy definition of the voltage, the beam loading can be modeled as a current sink feed by additional power through the cavity electromagnetic field. Bunched beam current has typically ~ 2 ps pulsed structure, 1 MHz rate and an average value of 8 mA.

One 10 MW klystron, through coupled wave-guide with circulator, supplies RF power to 32 cavities which are operated in ~ 1 ms pulsed mode, 10 Hz rate, with an average accelerating gradients of 25 MV/m (Fig. 1).

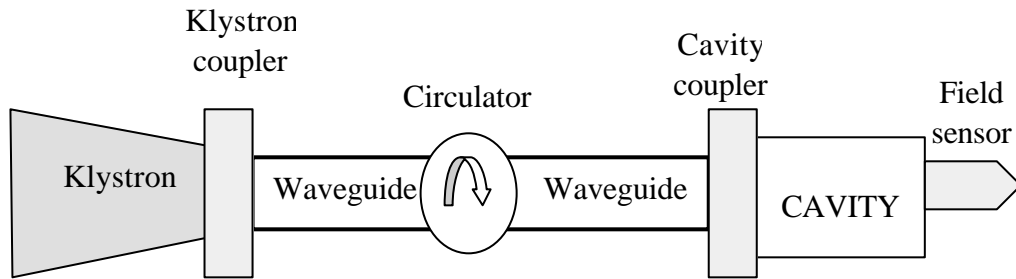


Figure 1. Compact block diagram of the cavity environment.

The fast amplitude and phase control of the cavity field is accomplished by modulation of the signal driving the klystron. The digital controller closes the feedback loop, actuates the vector modulator stabilizing the real (in-phase) and imaginary (quadrature) components of the signal according to set-point input (Fig. 2).

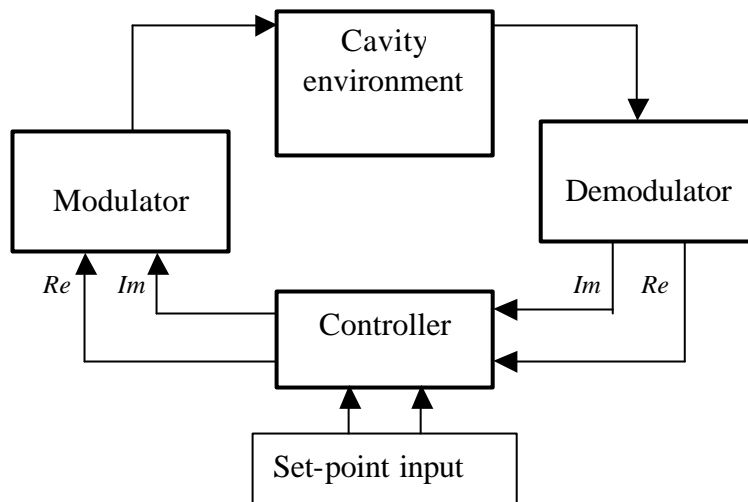


Figure 2. Cavity control system schematic block diagram.

3. CAVITY SIGNAL CONSIDERATIONS

The objective of the control system is to stabilize the cavity voltage which is related to the beam loading current, and to minimize the power consumption supplied by the klystron. Physical model for signal calculations is presented in Figure 3. All the current (J) and voltage (U) quantities are represented in *Laplace transform* space.

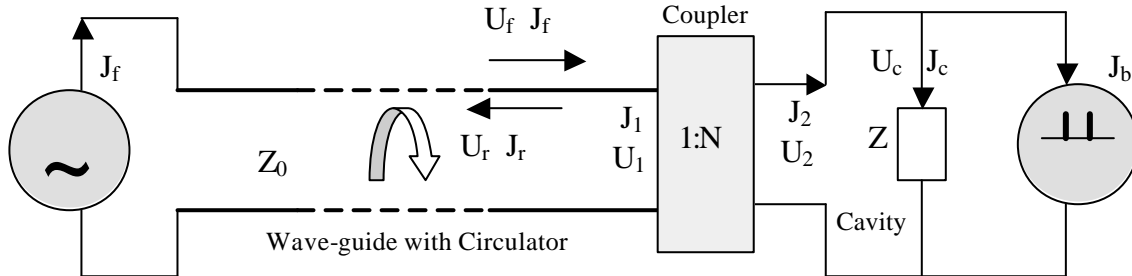


Figure 3. Physical model of cavity environment circuit.

The RF generator is modeled as a current source J_f driving the wave-guide coupled to the cavity. The circulator load matches itself to the wave-guide impedance Z_0 and isolates the generator from the reflected power. Transmission line signals are as follows

$$\begin{aligned} \text{forward current} &= J_f & \text{forward voltage} &= U_f = Z_0 \cdot J_f \\ \text{reflected current} &= J_r & \text{reflected voltage} &= U_r = Z_0 \cdot J_r \end{aligned}$$

Superposition of the forward and reflected waves actuates the coupler, and therefore

$$\text{coupler input current} = J_1 = J_f - J_r \quad \text{coupler input voltage} = U_1 = U_f + U_r = Z_0 \cdot (J_f + J_r)$$

The coupler converts signals according to the transformation ratio 1:N, and therefore

$$\begin{aligned} \text{coupler output current} &= J_2 = J_1/N = (J_f - J_r)/N \\ \text{coupler output voltage} &= U_2 = N \cdot U_1 = NZ_0 \cdot (J_f + J_r) \end{aligned}$$

The beam loading is represented as a current sink J_b with its repetitive pulse-function structure. Superposition of the output coupler current and the beam loading current actuates the cavity, therefore

$$\begin{aligned} \text{cavity current} &= J_c = J_2 - J_b = (J_f - J_r)/N - J_b \\ \text{cavity voltage} &= U_c = Z \cdot J_c = Z \cdot ((J_f - J_r)/N - J_b). \end{aligned}$$

where $Z = (1/R + sC + 1/sL)^{-1}$ is the impedance of the resonant LCR cavity circuit representation.

The output coupler voltage U_2 equals cavity voltage U_c , therefore

$$U_2 = NZ_0 \cdot (J_f + J_r) = U_c = Z \cdot ((J_f - J_r)/N - J_b),$$

Solving upper equations yields

$$U_c = N^2 Z_0 \cdot Z / (N^2 Z_0 + Z) \cdot (2J_f/N - J_b)$$

Applying an equivalent parallel resistance connection $R_L = N^2 Z_0 || R$ and the cavity trans-impedance $Z_L = (1/R_L + sC + 1/sL)^{-1}$ and substituting the current generator $J_g = J_f/N$, results in the cavity voltage

$$U_c = Z_L \cdot (2J_g - J_b) = Z_L \cdot J.$$

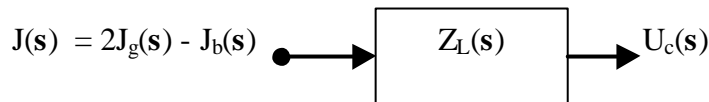


Figure 4. Cavity representation with transfer function $Z_L(s)$.

4. ANALYTICAL SIGNAL MODELING

According to the RF generator constant pulsation ω_g , and due to a narrow resonator bandwidth, the cavity voltage can be modeled in **time domain** as a *generalized oscillation*

$$u_r = A(t) \cdot \cos(\omega_g t + \varphi(t)) \quad \text{with its Hilbert transform} \quad u_i = A(t) \cdot \sin(\omega_g t + \varphi(t)).$$

Both mutually $\pi/2$ shifted oscillations with frequency ω_g are applied for further analysis. This ordered pair of complementary signals called *analytical signal* can be represented as a

$$\text{vector } \mathbf{u} = \begin{bmatrix} u_r \\ u_i \end{bmatrix} \quad \text{or} \quad \text{phasor in complex domain } \mathbf{u} = (u_r, u_i) = u_r + i u_i = A(t) \cdot \exp(i(\omega_g t + \varphi(t))).$$

Because of disturbances, the amplitude A and the phase φ are time-varying components with a relatively narrow spectral range. The cavity control system proceeds with a low level of frequency watching the real signal component (in-phase) = $I = v_r = A \cos \varphi$ and the imaginary signal component (quadrature) = $Q = v_i = A \sin \varphi$.

I, Q components can be detected by complex demodulation,

$$\text{as a phasor} \quad \mathbf{v} = (v_r, v_i) = (A \cos \varphi, A \sin \varphi) = A e^{i\varphi} = \mathbf{u} \cdot \exp(-i\omega_g t) = (u_r, u_i) \cdot \exp(-i\omega_g t)$$

where $\exp(-i\omega_g t)$ stands for complex demodulation operator, or else

$$(v_r, v_i) = (u_r + i u_i) \cdot (\cos(\omega_g t) - i \sin(\omega_g t)) = u_r \cdot \cos(\omega_g t) + u_i \cdot \sin(\omega_g t) + i(-u_r \cdot \sin(\omega_g t) + u_i \cdot \cos(\omega_g t)),$$

therefore demodulation representation for a vector is as follows

$$\begin{bmatrix} v_r \\ v_i \end{bmatrix} = \begin{bmatrix} \cos(\omega_g t) & \sin(\omega_g t) \\ -\sin(\omega_g t) & \cos(\omega_g t) \end{bmatrix} \cdot \begin{bmatrix} u_r \\ u_i \end{bmatrix} = \mathbf{Dem} \cdot \begin{bmatrix} u_r \\ u_i \end{bmatrix}$$

where **Dem** is a demodulation matrix or a vector rotational transformation.

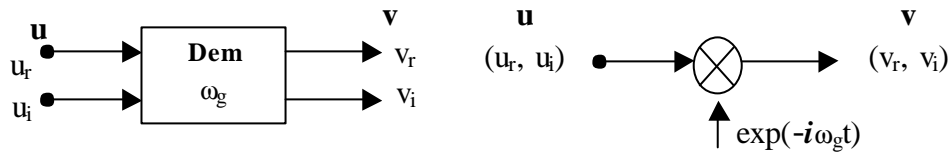


Figure 5. Graphical representation of vector and phasor demodulation.

The complex modulation is the reciprocal operation as follows

$$\text{for a phasor} \quad \mathbf{u} = (u_r, u_i) = (v_r, v_i) \cdot \exp(i\omega_g t) = \mathbf{v} \cdot \exp(i\omega_g t)$$

for a vector

$$\begin{bmatrix} u_r \\ u_i \end{bmatrix} = \begin{bmatrix} \cos(\omega_g t) & -\sin(\omega_g t) \\ \sin(\omega_g t) & \cos(\omega_g t) \end{bmatrix} \cdot \begin{bmatrix} v_r \\ v_i \end{bmatrix} = \mathbf{Mod} \cdot \begin{bmatrix} v_r \\ v_i \end{bmatrix}$$

where modulation matrix = $\mathbf{Mod} = \mathbf{Dem}^{-1}$.

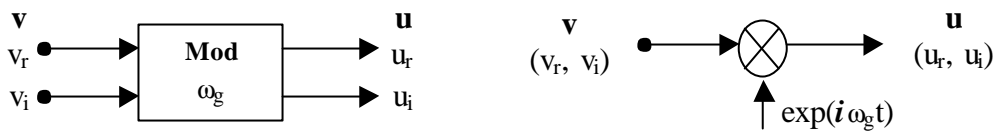


Figure 6. Graphical representation of vector and phasor modulation.

According to the cavity transfer function modeling, the relation between current and voltage *analytical signal* (both mutually $\pi/2$ shifted oscillations) as a phasor or vector representation is given in **Laplace space**:

$$\mathbf{U}(s) = Z_L(s) \cdot (2\mathbf{J}_g(s) - \mathbf{J}_b(s)) = Z_L(s) \cdot \mathbf{J}(s)$$

where the cavity trans-impedance $Z_L(s) = R_L / (1 + R_L \cdot sC + R_L / sL) = R_L / (1 + Q_L \cdot (s/\omega_0 + \omega_0/s))$ with parameters: loaded quality factor = Q_L , resonance frequency = ω_0 , generator frequency = ω_g .

The doubled cavity transfer function representation and the signal conversion modules can be assembled into the complex cavity model according to figure 7.

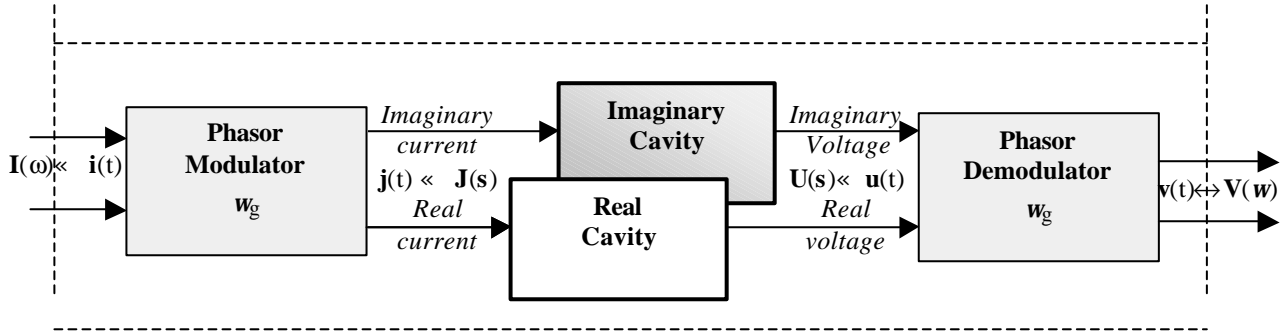


Figure 7. Functional diagram of complex cavity representation.

The complex cavity representation can be effectively simplified for signals with a narrow spectral range relative to the generator frequency ω_g . Modulator input phasor and demodulator output phasor are time dependent functions with adequate **Fourier transform**:

$$\mathbf{i}(t) \ll \mathbf{I}(\omega) \quad \text{and} \quad \mathbf{v}(t) \ll \mathbf{V}(\omega)$$

The complex modulation shifts forward the signal spectrum according to **Fourier transform** relation

$$\mathbf{i}(t) \cdot \exp(i\omega_g t) = \mathbf{j}(t) \ll \mathbf{J}(\omega) = \mathbf{I}(\omega - \omega_g)$$

Cavity output voltage is related to input current according to cavity **Fourier transfer** function

$$\mathbf{U}(\omega) = Z_L(\omega) \cdot \mathbf{J}(\omega) = Z_L(\omega) \cdot \mathbf{I}(\omega - \omega_g)$$

The complex demodulation shifts back the signal spectrum according to **Fourier transform** relation

$$\mathbf{u}(t) \cdot \exp(-i\omega_g t) = \mathbf{v}(t) \ll \mathbf{V}(\omega) = \mathbf{U}(\omega + \omega_g) = Z_L(\omega + \omega_g) \cdot \mathbf{I}(\omega)$$

The cavity **Fourier transfer** function Z_L can be presented for frequency $(\omega + \omega_g)$

$$Z_L(\omega + \omega_g) = R_L / (1 + iQ_L \cdot ((\omega + \omega_g) / \omega_0 - \omega_0 / (\omega + \omega_g))) = R_L / (1 + i(\omega - \Delta\omega) / \omega_{1/2} \cdot (\omega + \omega_0 + \omega_g) / 2(\omega + \omega_g))$$

where: detuning = $\Delta\omega = \omega_0 - \omega_g$, cavity half-bandwidth (HWHM) = $\omega_{1/2} = \omega_0 / 2Q_L$.

Resultant cavity **Fourier transfer** function can be approximated for frequencies $\omega \ll \omega_g \approx \omega_0$

$$Z_L(\omega + \omega_g) \approx R_L / (1 + (i\omega - i\Delta\omega) / \omega_{1/2}) = Z_L(i\omega).$$

Then signals relation in **Fourier space** is as follows

$$\mathbf{V}(i\omega) = Z_L(i\omega) \cdot \mathbf{I}(i\omega).$$

Moving to **Laplace space** with the operator $s = i\omega$ yields

$$\mathbf{V}(s) = Z_L(s) \cdot \mathbf{I}(s).$$

where

$$Z_L(s) = R_L / (1 + (s - i\Delta\omega) / \omega_{1/2}).$$

Therefore signals relation in *Laplace space* can be written for phasor representation

$$(\omega_{1/2} + s - i\Delta\omega) \cdot (V_r, V_i) = \omega_{1/2} \cdot R_L \cdot (I_r, I_i)$$

or for vector representation

$$\begin{vmatrix} \omega_{1/2} + s & \Delta\omega \\ -\Delta\omega & \omega_{1/2} + s \end{vmatrix} \cdot \begin{vmatrix} V_r \\ V_i \end{vmatrix} = \omega_{1/2} \cdot R_L \cdot \begin{vmatrix} I_r \\ I_i \end{vmatrix}$$

Moving to *time domain* yields state space relation

$$d\mathbf{v}/dt = \mathbf{A} \cdot \mathbf{v} + \omega_{1/2} \cdot R_L \cdot \mathbf{i}$$

$$\text{where state matrix } \mathbf{A} = \begin{vmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{vmatrix}$$

5. CAVITY POWER CONSIDERATION

According to the analytical signal model, the cavity voltage and the current can be represented, after complex demodulation, as a relatively stable phasor or vector $\mathbf{v}(t) = \mathbf{V}$ and $\mathbf{i}(t) = \mathbf{I}$, each with its I, Q components.

The real power is given:

in terms of complex voltage and conjugate current for phasor representation

$$P = \frac{1}{2} \text{Re} \{ \mathbf{V} \cdot \mathbf{I}^* \}$$

or as a scalar product for vector representation

$$P = \frac{1}{2} \mathbf{V} \cdot \mathbf{I}$$

The forward power, which is provided by the wave-guide, reflects itself partly due to the mismatched input coupler and dissipates in the circulator load. The residual transmitted power supplies cavity and feeds the beam loading. The objective of the accelerator system is to deliver power to the beam with the best efficiency.

Energy conservation yields

$$P_f = P_r + P_d + dW/dt + P_b$$

where

- forward power $\equiv P_f = \frac{1}{2} \text{Re} \{ \mathbf{V}_f \cdot \mathbf{I}_f^* \} = \frac{1}{2} |\mathbf{I}_f|^2 \cdot Z_0 \approx \frac{1}{2} |\mathbf{I}_g|^2 \cdot R_L$ ($Q_L \ll Q_0$)
- reflected power $\equiv P_r = \frac{1}{2} \text{Re} \{ \mathbf{V}_r \cdot \mathbf{I}_r^* \} = \frac{1}{2} |\mathbf{I}_r|^2 \cdot Z_0 \approx |\mathbf{V}_c - R_L \cdot \mathbf{I}_g|^2 / 2R_L$ ($Q_L \ll Q_0$)
- dissipated power $\equiv P_d = \frac{1}{2} \text{Re} \{ \mathbf{V}_c \cdot \mathbf{I}_c^* \} = |\mathbf{V}_c|^2 / 2R \approx 0$
- electromagnetic energy stored in cavity $\equiv W = |\mathbf{V}_c|^2 / 2\omega_0 = 73\text{J}$ – for 25 MV cavity voltage
- electromagnetic power transferred to cavity in transient states $\equiv dW/dt$
- beam loading power $\equiv P_b = \frac{1}{2} \text{Re} \{ \mathbf{V}_c \cdot \mathbf{I}_b^* \}$

The main cavity parameters and the typical value for power considerations are as follows

- resonance pulsation $\equiv \omega_0 = (LC)^{-1/2} = 2\pi \cdot 1300$ [rad/s]
- characteristic resistance $\equiv ? = (L/C)^{1/2} = 520 \Omega$
- resonant impedance = $R = Z(\omega_0) = \sim 5 \text{ T}\Omega$
- quality factor (unloaded) $Q_0 \equiv ? = \omega_0 W / P_d = R / \rho = \sim 10^{10}$
- fictitious loading as a parallel connection $R_L \equiv N^2 Z_0 || R = \sim 1.5 \text{ G}\Omega$
- loaded quality factor $\equiv Q_L = R_L / \rho = 3 \cdot 10^6$
- half- bandwidth (HWHM) $\equiv \omega_{1/2} = \omega_0 / 2Q_L = 2\pi \cdot 215$ [rad/s]

In a steady state ($dW/dt = 0$) without beam loading ($\mathbf{I}_b = 0$) on resonance condition ($\omega = \omega_0$) calculations for forward, dissipated and reflected power yields

- $P_f \approx |\mathbf{V}_c|^2 / 8R_L \approx 50 \text{ kW}$ – for 25 MV cavity voltage ($Q_L \ll Q_0$)

- $P_d = 4Q_L/Q_0 \cdot (1 - Q_L/Q_0) \cdot P_f \approx 0$
 - $P_f = (1 - 2Q_L/Q_0)^2 \cdot P_f \approx 50 \text{ kW}$
- Effective quality factor $Q_f \equiv \omega_0 W/P_f = 4Q_L \cdot (1 - Q_L/Q_0) \approx 4Q_L$ ($Q_L \ll Q_0$).

In a steady state with the beam loading and the detuned cavity ($\omega \neq \omega_0$) the main signals and parameters are as follows

- beam loading average current = $I_{b0} = 8 \text{ mA}$ – typical value
- RF component beam loading current $I_b = |I_b| = 2I_{b0}$ - determined as a reference phasor
- cavity required voltage $V_c = |V_c| \cdot e^{i\phi} - |V_c|$, ϕ – determined, stabilized amplitude and phase
- generator current $I_g = |I_g| \cdot e^{i\theta} - |I_g|$, θ – amplitude and phase actuated by controller
- complex trans-impedance $Z_L = Z_L(i\omega) = R_L / (1 - iQ_L \cdot (\omega_0/\omega - \omega/\omega_0)) = R_L \cdot \cos\psi \cdot e^{i\psi}$
- tuning angle $\equiv \psi$, $\tan\psi = Q_L \cdot (\omega_0/\omega - \omega/\omega_0) \sim \Delta\omega/\omega_{1/2}$ (detuning = $\Delta\omega = \omega_0 - \omega$)
- cavity actual voltage $V_c = Z_L(i\omega) \cdot (2I_g - I_b) \approx 2R_L / (1 - i\Delta\omega/\omega_{1/2}) \cdot (I_g - I_{b0})$.

Equating the required and the actual cavity voltage V_c , yields the stabilization equation. Solving this equation, the required forward power P_f is obtained, which is dependent on the beam loading and the cavity detuning ($Q_0 \gg Q_L$):

$$P_f = ((1 + 2R_L \cdot I_{b0} \cdot \cos\phi / |V_c|)^2 + ((\Delta\omega/\omega_{1/2})^2 + 2R_L \cdot I_{b0} \cdot \sin\phi / |V_c|)^2) \cdot |V_c|^2 / 8R_L$$

The beam loading required power = $P_b = \frac{1}{2} \text{Re}\{V_c \cdot I_b^*\} = |V_c| \cdot I_{b0} \cdot \cos\phi$.

6. COMPLEX CAVITY SYSTEM MODELING

The cavity control system using complex cavity - modeling is composed according to figure 8.

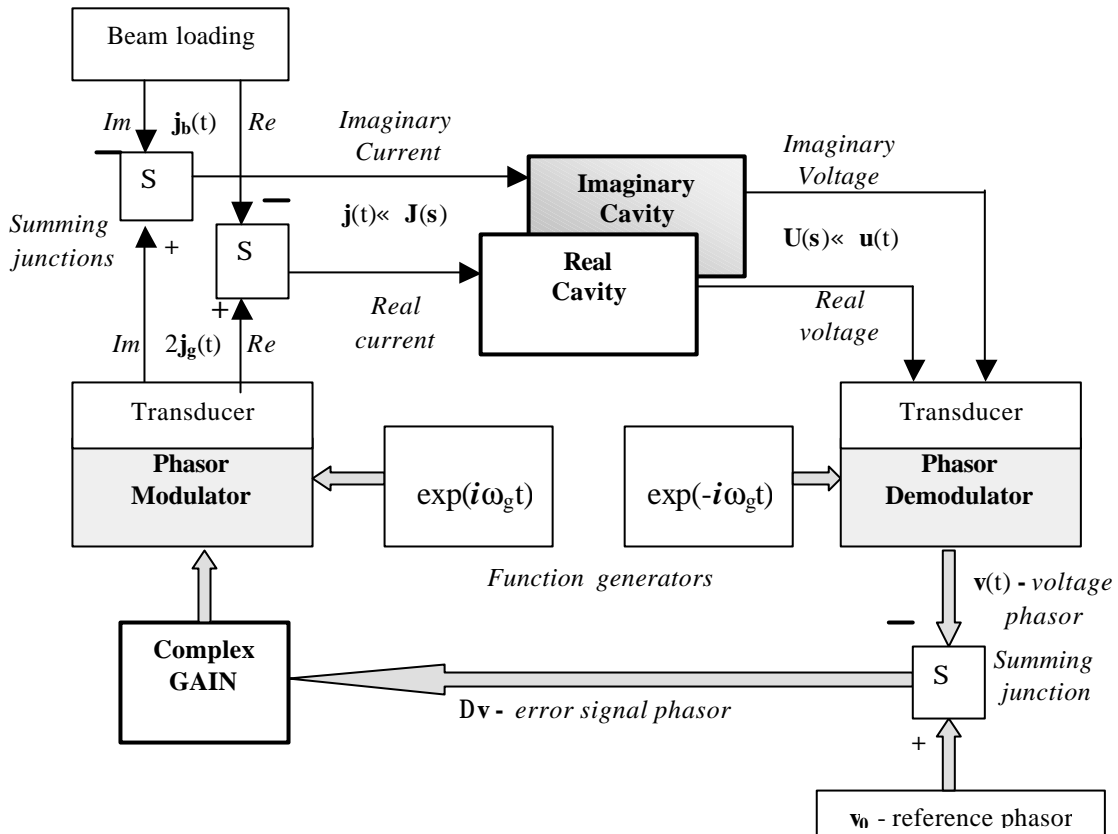


Figure 8. Functional diagram of complex cavity control system.

The complex cavity plant is represented as a transfer function by the double cavity trans-impedance

$$Z_L(s) = R_L / (1 + R_L \cdot sC + R_L/sL) = R_L / (1 + Q_L \cdot (s/\omega_0 + \omega_0/s))$$

The input current vector $\mathbf{J}(s)$ is *Laplace transform* of superposition $2\mathbf{j}_g(t) - \mathbf{j}_b(t)$, each one with its mutually $\pi/2$ shifted components (*Re, Im*). The cavity transfer function $Z_L(s)$ transforms the input current vector $\mathbf{J}(s)$ to the output voltage vector $\mathbf{U}(s) = Z_L(s) \cdot \mathbf{J}(s)$.

The output voltage vector $\mathbf{U}(s)$, retransformed to the *original* $\mathbf{u}(t)$ with its mutually $\pi/2$ shifted components, actuates the transducer and the phasor Demodulator and it is converted to voltage phasor $\mathbf{v}(t)$. The summing junction compares the voltage phasor $\mathbf{v}(t)$ with required voltage \mathbf{v}_0 as a reference phasor and generates the error signal phasor \mathbf{Dv} . The controller, as a Complex Gain, amplifies the error \mathbf{Dv} and closes the feedback system loop. The phasor Modulator and the transducer converts signal to the current vector $2\mathbf{j}_g(t)$ with its mutually $\pi/2$ shifted components oscillating with the frequency ω_g . The Beam Loading current sink subtracts its two mutually $\pi/2$ shifted pulsed structures components and the resultant current vector superposition $\mathbf{j}(t)$ actuates the cavity.

The complex cavity simulation with its doubled $\pi/2$ shifted signal trace performs the features of the fundamental circuit model. Nevertheless due to practical reasons it should be implemented in a reasonably lower level of frequency.

7. CONTROL SYSTEM MODELING FOR STATE SPACE CAVITY REPRESENTATION

The state space cavity representation allows for modeling on a low level frequency range. The dynamics of the closed loop system are dominated by the low frequency poles of the cavity which can be described by the state space relation:

$$d\mathbf{v}/dt = \mathbf{A} \cdot \mathbf{v} + \omega_{1/2} \cdot R_L \cdot (2\mathbf{i}_g - \mathbf{i}_b)$$

where state matrix $\mathbf{A} = \begin{bmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{bmatrix}$

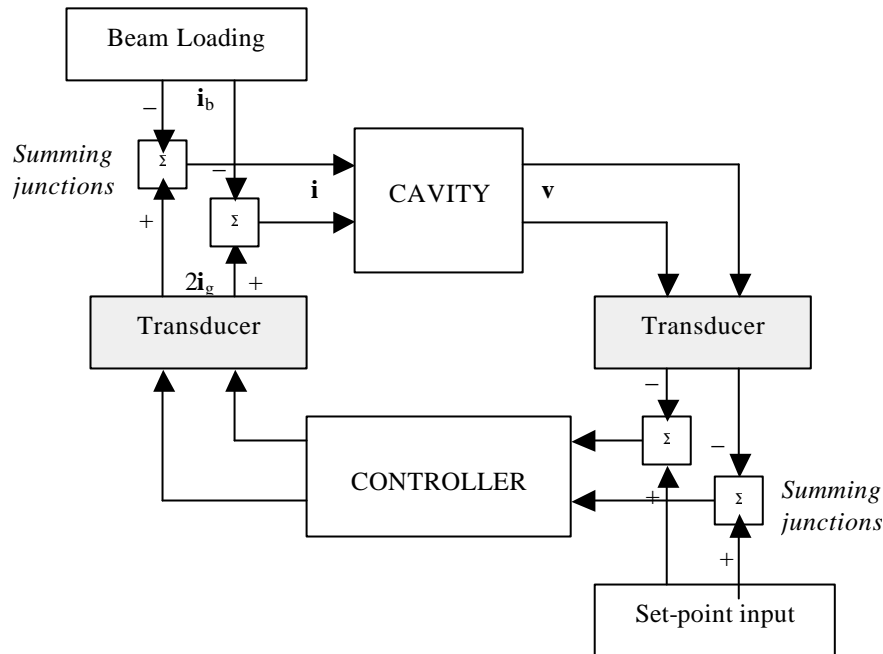


Figure 9. Functional diagram of control system with cavity state space model.

The set-point input delivers the required voltage value which is compared to the actual cavity voltage. The controller amplifies the error signal and closes the feedback system loop. The transducers convert signals to match its value to the cavity environment. The beam loading current is extracted outside the complex cavity model showed in fig. 8. It is represented as a RF component which equals double value of the average current ($i_b = 2i_{b0}$).

8. SUMMARY

The fundamental knowledge about modeling of the cavity resonator for TESLA linear accelerator is presented in this paper. Continuous vectored description of the system applies the linear, time-invariant cavity model. It is useful for the initial analysis and simulations of the cavity behaviors. The complex cavity representation simulates the circuit approximation but has rather academic importance. State space equation constitutes the base for further and advanced design of the control feedback.

The challenging task for the development of the cavity control system is to compensate for the dynamic Lorentz force detuning and to suppress the exciting of parasitic pass-band modes. To obtain the fast and precise stabilization of the cavity field, the digital controller with the efficient algorithm should be carefully designed and implemented.

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