Eigenmode Calculation in Long and Complex RF-Structures Using the Coupled S-Parameter Calculation Technique †

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Eigenmode calculations of long and complex structures by 'direct' eigenmode solvers may need an enormous amount of computer resources, especially computation time and memory. This paper presents a technique based on scattering parameters that allows to split the complete geometry into subsections which can be modeled individually. The S-parameter computations of each subsection are significantly smaller problems and can be distributed on different machines. Then, the eigenfrequencies of the complete structure are determined by the combination of the scattering-parameters and the corresponding field distributions are computed in a second step.

I. INTRODUCTION

The analysis of rf components often is based on the knowledge of their eigenmodes. The calculation of the eigenmodes is usually done by 'direct' eigenmode calculations as we will denote the solution of discretized Helmholtz' equation on the complete solution domain at once. Length and complexity of some structures demand even in modern computational environments a significant amount of computational resources to perform these calculations.

This paper presents a method which we call Coupled S-Parameter Calculation (CSC). It allows to split the whole geometry into several subsections. The broadband scattering parameters of each section are calculated seperately with appropriate solution codes (e.g. MAFIA [1], Micro Wave Studio [2], HFSS [3]). CSC calculates the eigenfrequencies of the entire structure using the section's S-parameters. Furthermore, for each resonance it delivers the amplitudes of the waveguide modes in all cuts between the resonator sections. These wave amplitudes are used to specify the wave excitation at the section's ports in further field solver runs which yield the field distribution of every resonance in every section.

The partitioning can be chosen in such a way that geometrical properties like symmetry or repetitions of certain sections or groups of sections can be exploited. Also, very simple subsections would allow for analytical solution.

First, a general procedure to calculate the coupling between the external ports of an arbitrarily structured system of scattering sections is described in section II. There we compare our approach with the commonly used T-matrix representation in order to demonstrate that the application of the latter one is difficult for general geometrical structural topologies.

Resonators, which are entirely closed by definition, are handled as a special case without external coupling. Then, the problem of finding eigenfrequencies is reduced to the repeated solution of low dimensional eigenvalue equations parameterized by frequency. The occurrence of eigenvalue zero indicates resonance frequencies; the according eigenvectors are vectors holding the amplitudes of all the waveguide modes in the system which are needed next to calculate the corresponding field distributions.

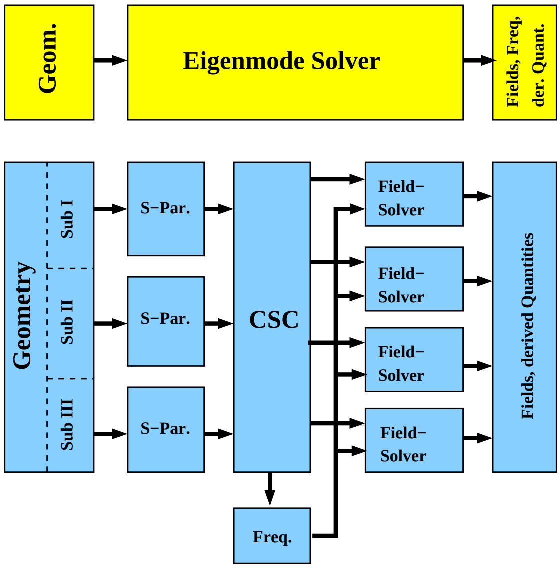

FIG. 1: Calculation of eigenmodes using CSC: The geometry is divided into several subsections. Then their S-parameters are determined. CSC combines them and provides the eigenfrequencies and additional information to compute the field distributions of the eigenmodes in the subsections.

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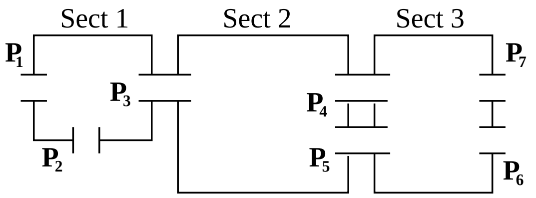

FIG. 2: A chain of three rf-components with external ports (P_1, P_2, P_6, P_7) and internal ports (P_3, P_4, P_5)

A test example is used to demonstrate the method in comparison with direct eigenmode calculation using MAFIA. Then the procedure is applied on the bunch compressor chicane of the TESLA Test Facility [4] in order to identify beam relevant modes in this large, complex shaped beam line device.

II. THEORY

Signal reflection and transmission between the ports of any rf-component can be described by scattering-parameters (briefly: S-parameters). All S-parameters of a linear n-port structure can be represented by a $(n\times n)$-matrix $\mathbf{S}$ where the entry S_{ij} describes the transmission of a signal from port j to port i. Because the S_{ij} contain information about phase and amplitude it is complex functions of frequency. With $\vec{a} = (a_1, \ldots, a_n)^T$ and $\vec{b} = (b_1, \ldots, b_n)^T$ describing the input and output signals, resp., the S-matrix $\mathbf{S}_k$ of the k-th object section forms the relation

$$\vec{b}_k = \mathbf{S}_k(\omega)\ \vec{a}_k = \begin{pmatrix} S_{11}(\omega) & \cdots & S_{1n}(\omega) \\ \vdots & & \vdots \\ S_{n1}(\omega) & \cdots & S_{nn}(\omega) \end{pmatrix}_k \vec{a}_k. \qquad (1)$$

The method is not restricted to single moded waveguide ports. Every mode has its individual scattering properties and therefore enlarges the size of the matrix system; every excited mode can be treated as an individual port of the system in this formalism.

A Combining rf-components using T-matrix representation

The S-parameter formalism allows to connect several structures by concatenating their S-matrices in the following manner (see for example [5]):

$$\mathbf{S}^{(t)} = \mathbf{S}^{(1)} \odot \mathbf{S}^{(2)} \odot \ldots \odot \mathbf{S}^{(\mathbf{N})}. \qquad (2)$$

This special 'product' $\odot$ is defined as

$$\mathbf{S}^{(A)} \odot \mathbf{S}^{(B)} = \begin{pmatrix} \mathbf{S}_{11}^{(A)} + \mathbf{S}_{12}^{(A)} \mathbf{N} \mathbf{S}_{11}^{(B)} \mathbf{S}_{21}^{(A)} & \mathbf{S}_{12}^{(A)} \mathbf{N} \mathbf{S}_{12}^{(B)} \\ \mathbf{S}_{21}^{(B)} \mathbf{N}^T \mathbf{S}_{21}^{(A)} & \mathbf{S}_{22}^{(B)} + \mathbf{S}_{21}^{(B)} \mathbf{N}^T \mathbf{S}_{22}^{(A)} \mathbf{S}_{12}^{(B)} \end{pmatrix} \qquad (3)$$

for two sections A and B with the abbreviation $\mathbf{N} = \left[\mathbf{1} - \mathbf{S}_{11}^{(B)}\, \mathbf{S}_{22}^{(A)}\right]^{-1}$ and $\mathbf{1}$ representing the unity matrix. Equation (3) is based on the transmission-matrix-formulation (e.g. [6, sect. 4.9]): the S-matrices are transformed into T-matrices which are combined by normal matrix multiplication. Afterwards the resulting overall T-matrix is transformed back into an S-matrix again. To calculate the overall S-parameters of a chain of rf-components eq. (3) has to be used iteratively. This goes with the disadvantage of accumulating numerical errors in each calculation step, especially if $\mathbf{N}$ becomes numerically small which may be the case for some frequency points in every calculation step.

To describe a resonator problem all open ports of the chain need to be closed with loss-free reflecting devices. Analytically this is done by setting $a_0 = r_1 b_0$ and $a_N = r_2 b_N$ at the outer most ports; the r_i are commonly called reflection coefficients. Setting $r_1 = r_2 = -1$ for instance will create a loss free electric short cut at both ports. In general r_1 and r_2 will be complex functions of ω of value 1.

Now (1) can be written as

$$\begin{pmatrix} b_0 \\ b_N \end{pmatrix} = \begin{pmatrix} S_{11}^{(t)} & S_{12}^{(t)} \\ S_{21}^{(t)} & S_{22}^{(t)} \end{pmatrix} \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_N \end{pmatrix}. \qquad (4)$$

This can only be fulfilled for discrete frequencies ω_0 which are the resonance frequencies given by the solution of

$$\det(\mathbf{SR} - \mathbf{1}) = 0. \qquad (5)$$

Herein $R = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}$ is set.

The electromagnetic fields corresponding to the eigenfrequencies can be determined by numerically exciting monochromatic waves with the appropriate frequency and amplitude at each port of each section. The amplitude at port $j+1$ can be computed recursively from the in- and outgoing signals at port j again using the T-matrix representation:

$$\begin{pmatrix} a_{j+1} \\ b_{j+1} \end{pmatrix} = \frac{1}{S_{12}^{(j+1)}} \times \begin{pmatrix} S_{12}^{(j+1)} S_{21}^{(j+1)} - S_{11}^{(j+1)} S_{22}^{(j+1)} & S_{22}^{(j+1)} \\ -S_{11}^{(j+1)} & 1 \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix}. \qquad (6)$$

At the starting point - port 0 is chosen for simplicity - the output signal b_0 is given by

$$b_0 = \left(S_{11}^{(t)} + \frac{r_2 S_{12}^{(t)} S_{21}^{(t)}}{1 - r_2 S_{12}^{(t)}} \right) a_0 \qquad (7)$$

where a_0 can be set arbitrarily, for example $a_0 = 1.0$.

If there exists more than one excited waveguide mode at a port Eq. (3) may be generalised: then the entries of the S-matrix will become matrices themselves. Therefore the formalism is applicable to linear chained $2n$-port-structures, which is implied by the definition of the matrix concatenation (3).

The formalism involves immense complications, if there are branches or variations of mode numbers in the system. A further disadvantage of this procedure is its numerical sensitivity given by the iterative use of (3) to compute the overall S-matrix $S^{(t)}$. The same is true for the determination of the wave amplitudes using (6).

B Combining rf-components using CSC-formulation

Here a formulation is given, which avoids the disadvantages mentioned above. We will arrange the S-matrices $\mathbf{S}_k$ for all sections in a block diagonal matrix $\mathbf{S}$. It couples the vector $\vec{a}$ of all incident signals with vector $\vec{b}$ of all scattered signals:

$$\vec{b} = \mathbf{S}\,\vec{a} = \begin{pmatrix} \mathbf{S_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{S_N} \end{pmatrix} \begin{pmatrix} \vec{a_1} \\ \vdots \\ \vec{a_N} \end{pmatrix} = \begin{pmatrix} \vec{b_1} \\ \vdots \\ \vec{b_N} \end{pmatrix} \tag{8}$$

The vector $\vec{a}$ of incident signals is now reordered in a way that all signals coming from outside the system are collected in a vector $\vec{a}_{inc}$ whereas those signals that are incident in one section, but outgoing from a neighbouring one are put together in the coupling vector $\vec{a}_{cop}$. A permutation matrix $\mathbf{P}$ links this reordered vector with $\vec{a}$:

$$\vec{a} = \mathbf{P}\begin{pmatrix} \vec{a}_{cop} \\ \vec{a}_{inc} \end{pmatrix}. \tag{9}$$

In order to illustrate this idea we use the arrangement shown in Fig. 2. For this example the permutation is expressed in a (10×10)-matrix. The exact shape of the matrices of course depends on the chosen numbering of the ports. If the signals are named by their section number (first index) and the port number (second index) $\mathbf{P}$ looks like this:

$$\begin{pmatrix} a_{1,1} \\ a_{1,2} \\ a_{1,3} \\ \hdashline a_{2,3} \\ a_{2,4} \\ a_{2,5} \\ \hdashline a_{3,4} \\ a_{3,5} \\ a_{3,6} \\ a_{3,7} \end{pmatrix} = \left(\begin{array}{cccccc:cccc} & & & & & & 1 & & & \\ & & & & & & & 1 & & \\ 1 & & & & & & & & & \\ \hdashline & 1 & & & & & & & & \\ & & 1 & & & & & & & \\ & & & 1 & & & & & & \\ \hdashline & & & & 1 & & & & & \\ & & & & & 1 & & & & \\ & & & & & & & & 1 & \\ & & & & & & & & & 1 \end{array}\right) \begin{pmatrix} a_{1,3} \\ a_{2,3} \\ a_{2,4} \\ a_{2,5} \\ a_{3,4} \\ a_{3,5} \\ \hdashline a_{1,1} \\ a_{1,2} \\ a_{3,6} \\ a_{3,7} \end{pmatrix} \tag{10}$$

A second (permutation) matrix $\mathbf{F}$ is used to describe the feedback in the system, namely the fact that the outgoing signals of one port are incident at another port. All signals leaving the system are kept untouched and $\mathbf{F}$ reads in our example like

$$\mathbf{F} = \begin{pmatrix} 1 & & & & & & & & & \\ & 1 & & & & & & & & \\ & & & 1 & & & & & & \\ & & 1 & & & & & & & \\ & & & & & & 1 & & & \\ & & & & & & & 1 & & \\ & & & & 1 & & & & & \\ & & & & & 1 & & & & \\ & & & & & & & & 1 & \\ & & & & & & & & & 1 \end{pmatrix} \tag{11}$$

Next, we can order the scattered signals in the same manner like the incoming signals in (9). This can be achieved by applying the inverse permutation leading to

$$\begin{pmatrix} \vec{a}_{cop} \\ \vec{a}_{sct} \end{pmatrix} = \mathbf{P}^{-1}\,\mathbf{F}\,\vec{b}. \tag{12}$$

In our example the product $\mathbf{P}^{-1}\mathbf{F}$ is given by

$$\begin{pmatrix} a_{1,3} \\ a_{2,3} \\ a_{2,4} \\ a_{2,5} \\ a_{3,4} \\ a_{3,5} \\ \hdashline b_{1,1} \\ b_{1,2} \\ b_{3,6} \\ b_{3,7} \end{pmatrix} = \left(\begin{array}{ccc:ccc:cccc} & & & 1 & & & & & & \\ & & 1 & & & & & & & \\ & & & & & & 1 & & & \\ & & & & & & & 1 & & \\ & & & & 1 & & & & & \\ & & & & & 1 & & & & \\ \hdashline 1 & & & & & & & & & \\ & 1 & & & & & & & & \\ & & & & & & & & 1 & \\ & & & & & & & & & 1 \end{array}\right) \begin{pmatrix} b_{1,1} \\ b_{1,2} \\ b_{1,3} \\ \hdashline b_{2,3} \\ b_{2,4} \\ b_{2,5} \\ \hdashline b_{3,4} \\ b_{3,5} \\ b_{3,6} \\ b_{3,7} \end{pmatrix}. \tag{13}$$

Combining (1), (9) and (12) results in the following matrix vector equation:

$$\begin{pmatrix} \vec{a}_{cop} \\ \vec{a}_{sct} \end{pmatrix} = \underbrace{\mathbf{P}^{-1}\,\mathbf{F}\,\mathbf{S}\,\mathbf{P}}_{\mathbf{G}} \begin{pmatrix} \vec{a}_{cop} \\ \vec{a}_{inc} \end{pmatrix} \tag{14}$$

where $\mathbf{G} = \mathbf{P}^{-1}\,\mathbf{F}\,\mathbf{S}\,\mathbf{P}$ describes the structure of the whole system.

The system matrix $\mathbf{G}$ can be split into four block matrices $\mathbf{G}_{ij}$ where the dimensions of the $\mathbf{G}_{ij}$ correspond to the dimensions of $\vec{a}_{cop}$, $\vec{a}_{inc}$ and $\vec{a}_{sct}$. Thus, (14) can be written as the following system of matrix-vector-equations:

$$\vec{a}_{cop} = \mathbf{G}_{11}\,\vec{a}_{cop} + \mathbf{G}_{12}\,\vec{a}_{inc}, \tag{15a}$$

$$\vec{a}_{sct} = \mathbf{G}_{21}\,\vec{a}_{cop} + \mathbf{G}_{22}\,\vec{a}_{inc}. \tag{15b}$$

Herein $\vec{a}_{inc}$ and $\vec{a}_{sct}$ represent the incident and the reflected waves at the external ports. Hence the coupling

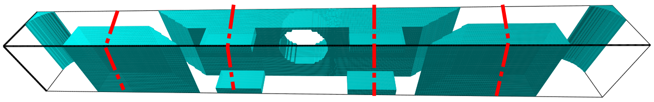

FIG. 3: Geometry used to compare the direct eigenmode calculation with CSC. Additional the cutting planes are shown.

between these signals is given by

$$\vec{a}_{sct} = \left(\mathbf{G}_{21}\ (\mathbf{1} - \mathbf{G}_{11})^{-1}\ \mathbf{G}_{12} + \mathbf{G}_{22}\right) \vec{a}_{inc}. \qquad (16)$$

The overall S-matrix, denoted by $\mathbf{S}^{(T)}$, can be written as

$$\mathbf{S}^{(T)} = \mathbf{G}_{21}\ (\mathbf{1} - \mathbf{G}_{11})^{-1}\ \mathbf{G}_{12} + \mathbf{G}_{22}. \qquad (17)$$

So far our considerations are applicable both for structures with open ports and resonators. We use (17) for the analysis of open structures, like cavities for particle acceleration with attached couplers, which will be described elsewhere [7].

C Resonators

In the case of a resonator problem there are no open (external) ports. Then $dim(\vec{a}_r) = dim(\vec{a}_i) = 0$ holds and only the coupling between the internal ports remains. Simultaneously the block matrices $\mathbf{G}_{12}$, $\mathbf{G}_{21}$ and $\mathbf{G}_{22}$ vanish and (15) reduces to

$$(\mathbf{1} - \mathbf{G}_{11}(\omega_0))\, \vec{a}_{cop} = \mathbf{0} \qquad (18)$$

which has to be fulfilled. This is only valid for discrete frequencies ω_0 which are the resonant frequencies aimed for. In order to find ω_0 the eigenvalues of $(\mathbf{1} - \mathbf{G_{11}}(\omega))$ have to be determined repeatedly. Since the dimension of this matrix is equal to the number of internal signals, which is usually below 10^2, this can be done with standard solvers. A resonant frequency is found if at least one eigenvalue equals zero.

In that case the vector $\vec{a}_{cop}$ contains the amplitudes of the waveguide modes at the location of the internal ports. $\vec{a}_{cop}$ is found as basis of the eigenspace - usually one vector - of the eigenvalue zero which is the kernel of the matrix $(\mathbf{1} - \mathbf{G}_{11}(\omega_0))$.

Afterwards the eigenfields are computed in separate runs for every section, using exciting waves with amplitudes given by $\vec{a}_{cop}$.

III. TEST EXAMPLE

To verify the formulation given above a test geometry was modelled (see Fig. 3) which was split into 5 subsections. The S-parameters of each single subsection were calculated using the MAFIA time domain solver T3 in a frequency range of 1.2 . . . 1.75 GHz.

To monitor the in- and outgoing waves a field decomposition at the particular port planes was performed, using 2D-waveguide-modes that were determined from the port geometry except for an arbitrarily chosen factor. This is a standard procedure, that is prepared in the MAFIA time domain solver T. It has to be guaranteed that corresponding modes of neighbouring sections have the same orientation although they are calculated twice in different runs with probably different meshes. To ensure this, a criterion was implemented that orients all waveguide

CSC	MAFIA E-mod.	$\frac{f_{CSC}-f_{MAFIA}}{f_{MAFIA}}$
1.212265 GHz	1.210309 GHz	1.62×10^{-3}
1.240062 GHz	1.240023 GHz	3.15×10^{-5}
1.348054 GHz	1.347277 GHz	5.77×10^{-4}
1.383414 GHz	1.382202 GHz	8.77×10^{-4}
1.442567 GHz	1.442681 GHz	-7.90×10^{-5}
1.463943 GHz	1.463867 GHz	5.19×10^{-5}
1.503261 GHz	1.502614 GHz	4.30×10^{-4}
1.531568 GHz	1.531387 GHz	1.18×10^{-4}
1.604844 GHz	1.603293 GHz	9.67×10^{-4}
1.653448 GHz	1.652757 GHz	4.18×10^{-4}
1.687855 GHz	1.682436 GHz	3.22×10^{-3}

TABLE I: Comparison of the eigenfrequencies found by the CSC-technique and by MAFIA's E-module (double precision) in the frequency range of 1.2 . . . 1.75 GHz. The same mesh was used inside the resonator section.

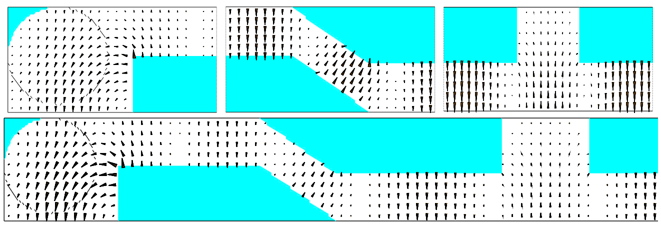

FIG. 4: Electric field of the $f = 1.44258$ GHz-eigenmode computed by CSC (upper plot) and directly with MAFIA's E-module (lower plot).

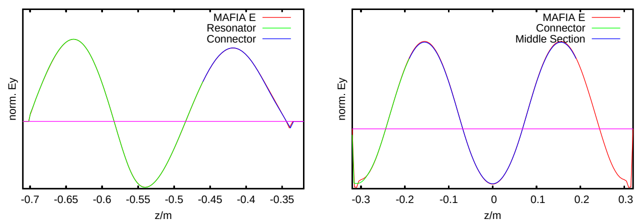

FIG. 5: E_y-component of the $f = 1.44258$ GHz-eigenmode along some path perpendicular to the port plane calculated by CSC and directly with MAFIA's E-module. Partially overlapping curves may be not visible.

modes in a consistent manner.

A Mathematica [8] script was used to compute both the resonant frequencies and the amplitudes at each internal port. The field distributions of the eigenmodes in all subsections were calculated using MAFIA's frequency domain solver W3. The input power and phase of the incident waves are given by CSC and have to be specified for the particular ports of the subsection. During the field calculation the same orientation of all 2D port modes must again be guaranteed like in the S-parameter calculation. This results in the resonant field pattern of the according resonator section.

Alternatively the time domain solver T3 can be used to derive the eigenmode field patterns. In this case monochromatic waves of the given frequency are excited at every port with the appropriate amplitudes and phases. It must be guaranteed that the calculation stabilised to steady state before the resonant fields are evaluated. This can be checked by testing whether all outgoing signals reached a constant amplitude. Nevertheless it may be difficult to decide whether the steady state is reached if resonant substructures with high quality factor are calculated.

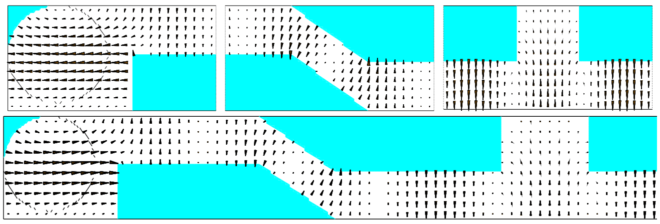

FIG. 6: The electric field of the $f = 1.50325$ GHz-eigenmode computed by the CSC (upper plots) and directly with MAFIA's E-module (lower plot).

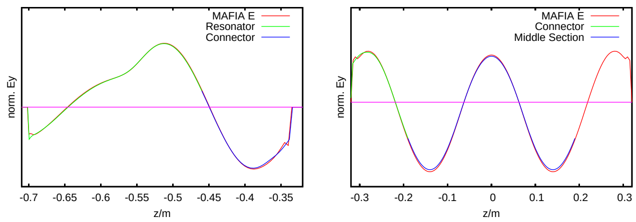

FIG. 7: E_y along some path in z-direction shown for the eigenmode with frequency ($f = 1.50325$ GHz).

For comparison, the eigenmodes of the whole structure were also computed with MAFIA's eigenmode solver E. As shown in Tab. I the frequencies found by the CSC technique match very well those calculated directly.

Fig. 4 and Fig. 6 compare the field distributions of the eigenmodes with frequency $f = 1.44258$ GHz and $f = 1.50325$ GHz calculated with CSC and the direct eigenmode solver. Fig. 5 and Fig. 7 show the y-component of the electric field along some path perpendicular to the port plane and pointing through them. As

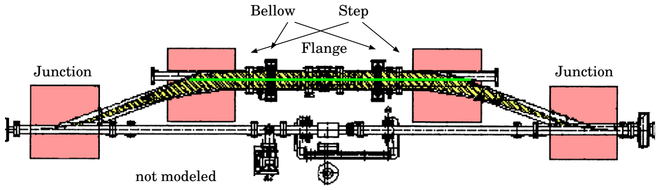

FIG. 8: Geometry of the Bunch Compressor II of the TTF-FEL with the sectioning used for CSC.

can be seen, the field distribution computed by MAFIA's E-module and CSC match extremely well.

IV. APPLICATION

A Resonator

The Bunch Compressor II of the TESLA Test Facility was chosen as a real-life problem. This device is intended to compress electron bunches using a dispersive beam line built by four dipole magnets(a sketch of the geometry is shown in Fig. 8).

The straight part of the chicane is not modelled yet because its cut-off frequency of approx. 3.0 GHz lies well above the examined frequency range of 1.7 . . . 2.2 GHz. In this frequency range only one travelling mode exists and the junction sections act as frequency dependent closings of the structure and only their reflection parameter has to be calculated.

The sectioning was chosen in such a way that the bellow and flange section were symmetric. So only one side of the devices needed to be excited in the T3-runs since their S-matrices reflect this geometrical symmetry in identical reflection parameters. This was not possible for the step section and two runs were needed here to achieve full information.

CSC calculated 23 eigenfrequencies in the range of 1.7 . . . 2.2 GHz. They are shown in Fig. 10, seperated into two groups of symmetric and antisymmetric modes. Fig. 11 shows a sample field distribution, demonstrating the smooth connection between neighbouring sections.

V. CONCLUSION

The presented CSC-technique allows to divide a long and/or complex structure into smaller subsections. Combination of their individual S-parameters yields the S-matrix of the complete structure (for structures with open ports) as well as the eigenfrequencies and the corresponding field distributions in the case of a resonator problem. For a test example it was shown that the results of CSC match very well with those of a well established code for direct eigenmode calculation.

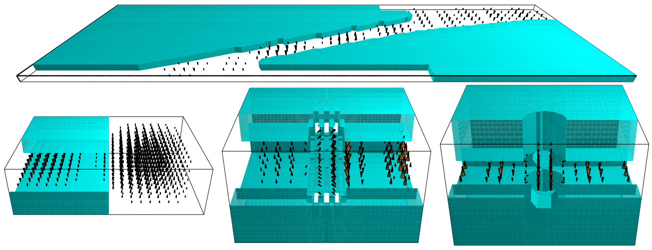

FIG. 9: Geometries of the sections modelled in MAFIA with the electric field of the 1.8558787 GHz-mode. The dimensions of the junction section (upper) are 0.02 m $\times$ 0.421 m $\times$ 1.586 m discretized with 7 $\times$ 103 $\times$ 148 mesh lines. The step section ($25 \times 30 \times 45$ mesh lines) measures 0.054 m $\times$ 0.129 m $\times$ 0.19 m. The bellow section needs $56 \times 66 \times 80$ mesh lines for a sufficient discretization of the folds (0.14 m $\times$ 0.208 m $\times$ 0.24 m)and 32 $\times$ 48 $\times$ 47 mesh lines were used to model the flange geometry (0.27 m $\times$ 0.35 m $\times$ 0.42 m).

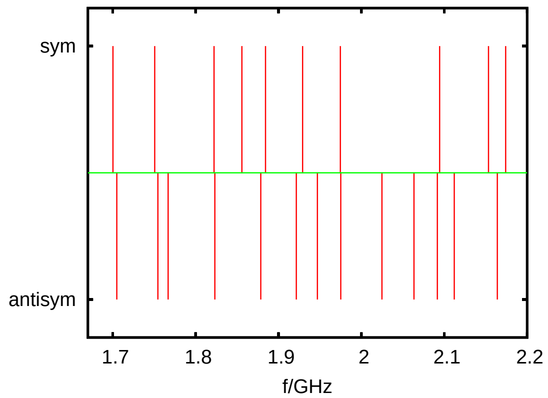

FIG. 10: Eigenmode spectrum of the chicane.

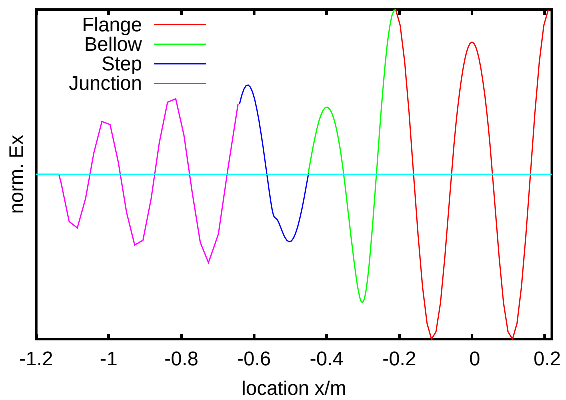

FIG. 11: E_y along a path in z-direction along the centre line of the middle sections shown for the eigenmode with frequency $f = 1.8558787$ GHz.

The advantage of this technique is the possibility to calculate the S-parameters of each subsection on different machines or in separate runs. It also allows to exploit possible symmetries or repetitions of particular subsections groups of sections; some parts of the geometry may be described analytically. Furthermore, it easily allows to specify frequency ranges of the eigenmodes searched for, which is often quite difficult.

If a direct calculation is possible, the overall effort of the CSC-procedure is of course essentially higher. CSC was developed to provide the possibility of eigenmode computation for those structures which *cannot* be handled directly. Additionally it allows to create libraries of S-parameters of sections and makes it possible to optimise small components in a wider context.

A typical example of a structure being far to large for usual calculation is the TTF bunch compressor beam pipe. Here we applied CSC in order to calculate resonant fields. Since they cannot be validated in available computational environment they have to be tested by measurement.

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