Calculation of Wake Fields in the Bellows of TTF Bunch Compressor

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Abstract

A bunch passing a cavity-like structure leaves electromagnetic fields in this structure. The stored energy will either be carried away by waves propagating along the beam pipe or dissipate heat into the walls of the structure. It is essential to know the amount of power left in such a section to decide whether external cooling is neccessary or not. This paper will present different methods to estimate the power left in the bellows of the middle section (called dipole chamber) of bunch compressor II of the TESLA Test Facility.

1. Introduction

The bunch compressor of the TTF-FEL consists of four dipole magnets which form a dispersive beamline. The middle section is equipped with different flanges for vacuum

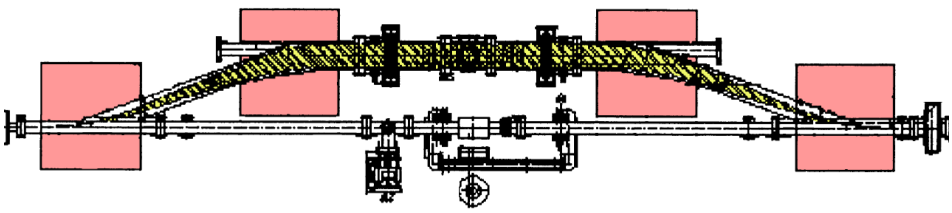

Figure 1. A scetch of the bunch compressor II of the TTF-FEL

pumps and beam monitoring. To avoid mechanical stress on certain components, two bellows are used.

The beam pipe in this middle section (called dipole chamber) consists of a rectangular waveguide with a cross section of 129 mm × 54 mm. The bellows are elliptical with radii

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of $a = 84$ mm, $b = 50$ mm with 20 mm deep folds (see Fig.2); the length of the bellow is 52 mm.

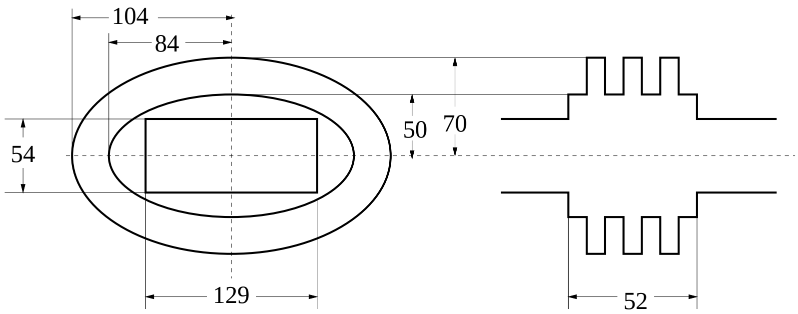


Figure 2. The geometry of the elliptical bellows used in the dipole chamber of BC II in the TTF-FEL

Because of the dispersive arrangement of the dipoles the bunch is transversally spreaded ($\sigma_\perp \sim 4$ mm) in this section. Calculations with bunches on trajectories transversally displaced by 4 mm showed that this fact can be neglected, because there is no essential difference in the wake integrals over these pathes in the bellow section. The bunch length is ~ 1 mm in this section.

2. Methods

Different methods were used to calculate the amount of power left in the bellow section by the bunch train. First an eigenmode calculation was performed to identify trapped modes. Then their loss parameters and the stored energy were calculated. Second the wake integral was calculated directly by 3D time domain calculation. This calculation was

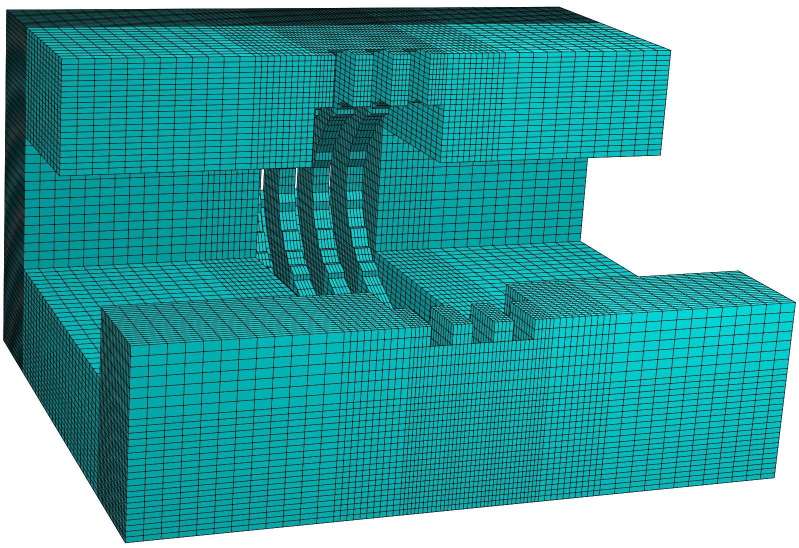


Figure 3. Geometry used to calculate the eigenmodes of the bellow section.

constrained by the computational resources (memory limited to 2 GByte) to a minimal bunch length of 5 mm. To get an estimation of the wake integral for shorter bunches of around 1 mm a 2D calculation was performed using an approximative cylindrical geometry of the bellow section. All field calculations were carried out with MAFIA [1].

3. Eigenmode Calculation

The so-called "trapped modes" oscillate in a cavity-like structure with a frequency below cut-off of the connected waveguide. This means energy cannot be carried away by a travelling wave - the mode is trapped inside the structure.

If the waveguide is long enough the electric and magnetic fields of these modes should not "see" the boundaries at the end of the waveguide. This fact allows to identify these modes in numerical computations because the field distribution and frequency should not depend on these boundary conditions. Therefore two numerical eigenmode calculations

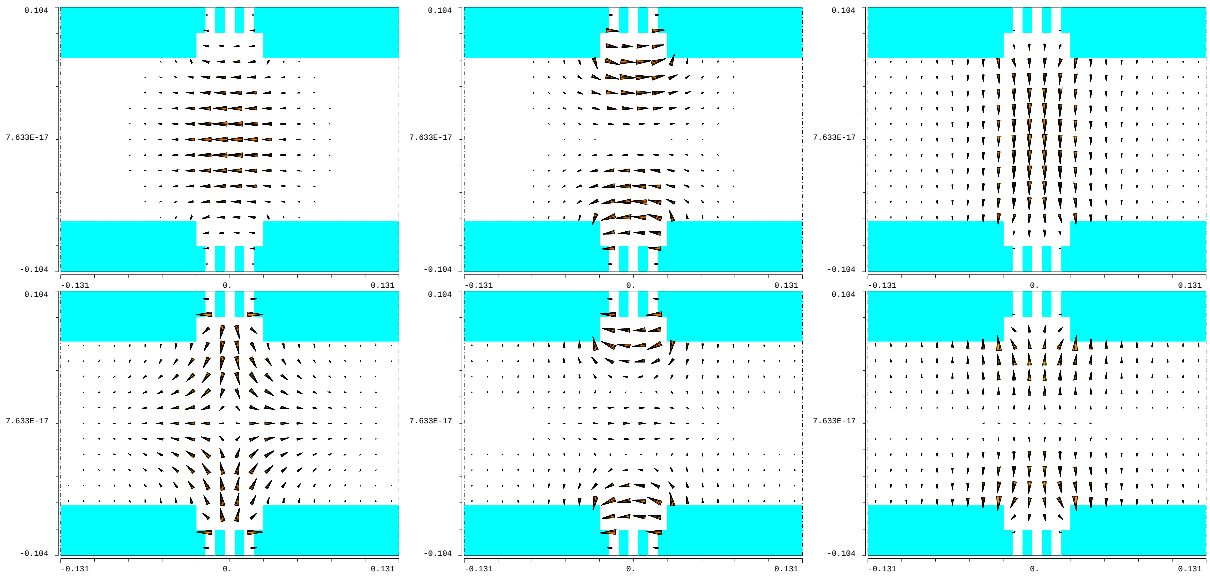


Figure 4. Trapped modes found in the bellow section with $f = 1.744901, 2.324131, 2.462300$ GHz(upper row) and 2.656573, 2.781407, 2.832701 GHz(lower row). A cut in y-z-plane is shown.

with electric and magnetic boundary conditions (resp.) were performed by the MAFIA E-solver using the geometry shown in Fig. 3. The model needs $56 \times 66 \times 84 = 310,464$ mesh points to get an appropriate resolution of the elliptically shaped bellows.

Six localized modes were found in a frequency range of $f = 1.0 \ldots 3.0$ GHz. The corresponding field patterns in y-z-plane are shown in Fig. 4.

The energy lost by one bunch charged with q_b crossing a certain section can be calculated by [2]:

$$\Delta E = k_{loss}\, q_b^2 \tag{1}$$

where k_{loss} is the loss factor which in general can be computed by evaluating:

$$k_{loss} = -W_{||}(\zeta = 0) \tag{2}$$

which is the longitudinal wake potential at the location of the bunch.

For a proposed bunch train of 11,315 bunches with a repetition rate of 10 Hz and a bunch charge of $q_b = 1$ nC the loss factor and dissipated power for these modes is listed in Table 1.

Table 1
Loss factor and dissipated power (calculated for the bunch train of 11315 bunches with a repetition rate of 10 Hz) for the trapped modes in the bellow section of bunch compressor II; the charge of each bunch is 1 nC.

No.	f/GHz	$k_{loss}/\frac{V}{nC}$	P/W
4	1.744901	227.104	0.025
7	2.324131	0.7402893	83.8×10^{-6}
10	2.462300	0.00137	0.155×10^{-6}
14	2.656573	40.35087	4.57×10^{-3}
16	2.781407	13.25481	1.50×10^{-3}
19	2.832701	0.563982	62.8×10^{-6}

The power loss of the bunch by these modes is dominated by the monopole-like mode with a frequency of 1.744901 Hz; the total power stored in the bellow section by all the modes is about 32 mW.

This is of course a kind of worst case calculation because it is supposed that each bunch will deposit the full amount of energy in the section; this neglects the fact that a following bunch may gain energy from the field already stored in the bellow section by a previous one.

4. 3D Calculation of the Wake Integral for a 5 mm-Bunch

MAFIA's time domain solver can directly calculate wake fields of a bunch passing a certain structure. But this calculation requires a geometry with an equidistant mesh in the direction of the bunch motion (z in this case) with a step size at least $\leq \sigma_z/10$. The mesh used above for the eigenmode calculation of the bellow section was modified in this sense for a 5.0 mm-bunch which results in a mesh with $56 \times 66 \times 305 = 1,127,280$ points and a computation time of approximately one day on a 2-GB-Ram-450MHz-workstation.

The resulting wake integral for a bunch charged with 1 nC passing through the center of the bellow is shown in Fig. 5.

The total loss factor of a linear bunch charge distribution $\lambda(s)$ can be computed from

$$k_{tot} = \int ds \lambda(s) W_{||}(s) \tag{3}$$

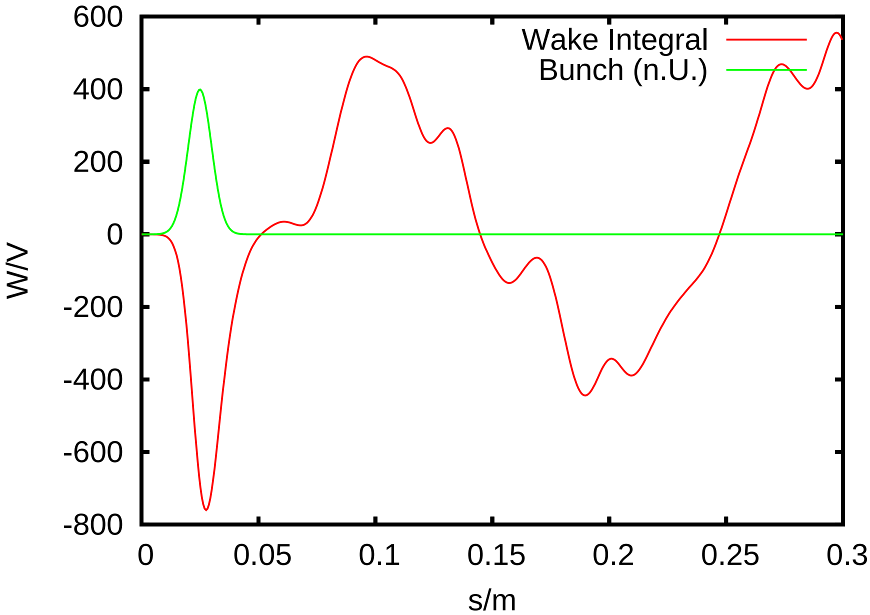


Figure 5. Wake Integral $W_{||}$ of a 5 mm-1 nC-bunch passing through the center of the bellow section. Calculated directly with the MAFIA time domain solver T3.

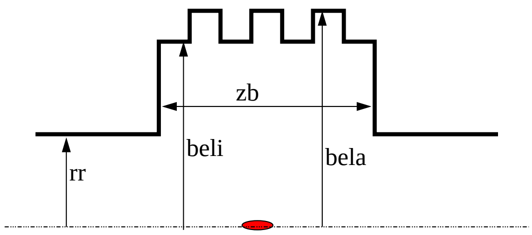
using the wake integral $W_{||}$. This integration is performed numerically using Mathematica [3] and results in

$$k_{tot} \approx 550 \text{ V/nC} \tag{4}$$

for a 5 mm-bunch.

Problem size and computation time rise with $\mathcal{O}(n^4)$ because smaller step size in one direction requires smaller steps in the other directions too and will additionally result in smaller time steps. This is a requirement of the solver to get a reasonable accuracy. Thus an even further reduction of the bunch length and refinement of the step size was not possible.

5. Approximative 2D Calculation of the Wakefield



rr	46 mm
beli	67 mm
bela	87 mm
zb	52 mm
q	1.0 nC

Figure 6. 2D-model of the bellow section used for the wakefield calculation

Due to the restricted computational resources the direct computation of k_{loss} in the bellow section is not possible for bunches far below 5 mm length. To estimate the behaviour of shorter bunches an approximative 2D-model of the bellow section was created (see Fig. 6) which needed only $71 \times 761 = 54,031$ mesh points. Two calculations with bunches of $\sigma_z = 5.0$ mm and 1.0 mm were performed.

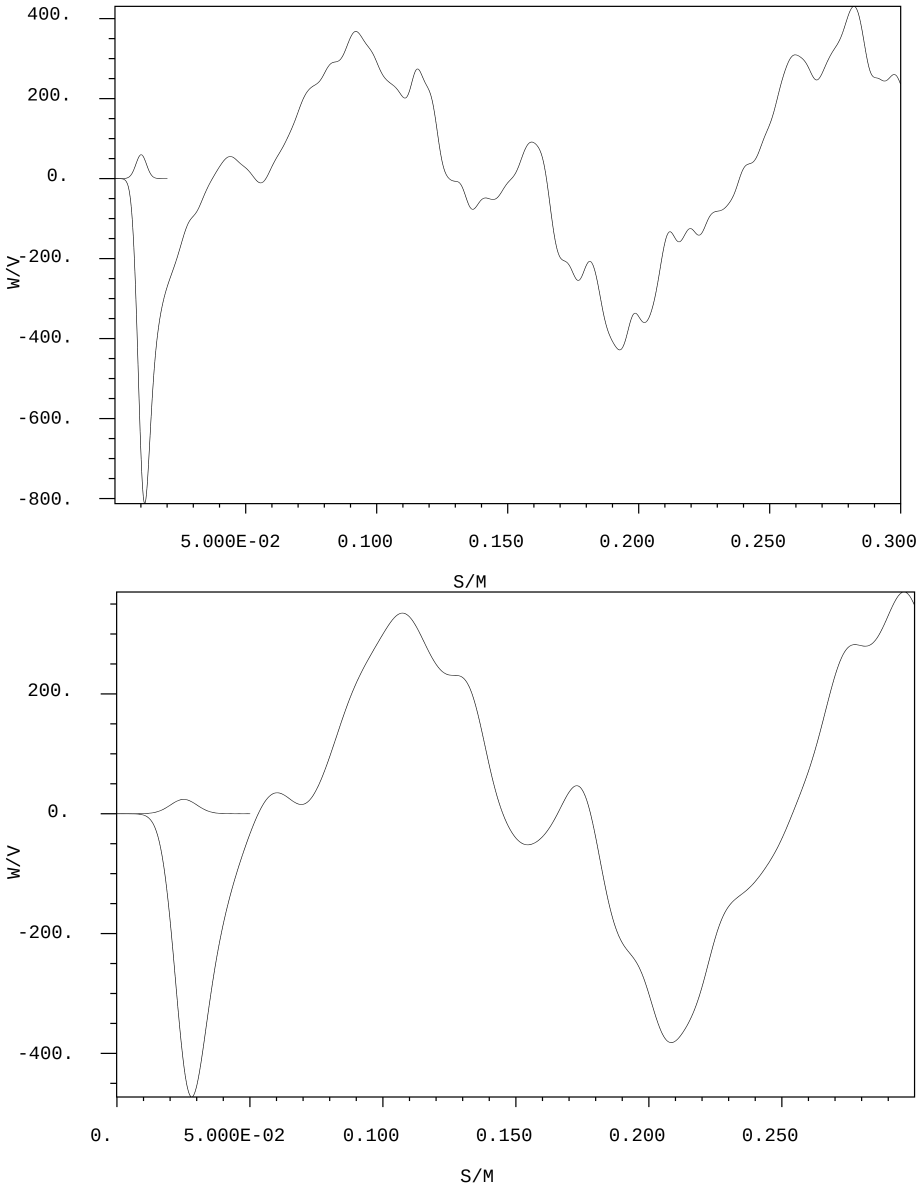


Figure 7. Wake integral $W_{\|}$ calculated for a bunch with $\sigma_z = 1$ mm (upper) and 5 mm (lower) passing a bellow with circular cross-section (modeled in 2D).

In principal the wake integrals of both calculations are similar to the one computed in 3D. This fact should allow to transform the results of the 2D-calculation into 3D.

The 3D and 2D results of $\sigma_z = 5$ mm match quite well applying a factor - which depends on the ellipticity of the 3D-geometry - to get a similar height of the first (negative) peak [2] [4]. So the 2D model gives a sufficient approximation of the 3D geometry and the 2D calculation for $\sigma_z = 1$ mm allows to estimate the loss factor for short bunches by scaling the wake integral and bunch distribution of the 3D-calculation. This results in

$$k_{tot} \approx 1350 \text{ V/nC} \widehat{=} 150 \text{ mW}$$

for a bunch of $\sigma_z = 1$ mm passing the bellow.

6. Conclusion

The power lost by the bunch train passing the bellows of the dipole chamber was calculated by different methods.

The power left by trapped modes which are excited by a passing 1 mm-bunch amounts to ≈ 0.032 W and is dominated by one mode only. Because 3D-wakefield calculations for such short bunches are very expensive in terms of computation time and available storage, a full 3D-computation was only performed for a bunch length of $\sigma_z = 5$ mm. Then 2D calculations were used to estimate the loss factor and power for a 1 mm-bunch. This gives a total power $P_{tot} \approx 0.15$ W left in the bellow section which should not require additional cooling.

All calculations presented have neglected the transversal spreading of the bunch in the dipole chamber of the compressor section. But this should have only minor influence on monopole like modes and rather lead to reduced loss parameters of the dipole like modes.

7. Acknowledgements

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References

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[4] A.Novokhatski: private communication.

[2] Only that part of the wake integral is relevant for loss factor calculation where the bunch charge distribution λ is significantly higher than zero.