Linear Tuning and Estimates for the Initial Beam Based Alignment Requirements for the TESLA Beam Delivery System (BDS)

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1. Introduction

The Beam Delivery System (BDS) of the proposed TESLA linear collider contains some of the tightest magnet alignment tolerances of the entire machine. In [1], luminosity stability issues were covered in some detail, with specific emphasis on ground motion and magnet vibration. It was shown that ground motion effects and fast magnet vibration leads to relatively high values of both linear and second-order dispersion at the interaction point (IP). In this note, we estimate the related effects of the residual dispersion left after performing beam-based alignment of the magnets. Specifically, we want to estimate an RMS alignment goal which will need to be attained with beam-based techniques in order to achieve the required 5 nm vertical beam size at the IP.

The exact method of beam-based alignment used is not the subject if this report. Instead, we assume that the method allows us to perform the following two steps:

1. accurate determination of the magnetic centre of the magnets with respect to the local BPM (*i.e.* determination of BPM offsets), and
2. adjustment of the transverse position of each magnet, such that they have some small remnant RMS alignment error with respect to the design beamline axis.

Random magnet alignment errors due to the initial optical survey of the magnets are typically of the order of 100 μm RMS. It is the goal of beam-based alignment to significantly reduce the alignment error to a number less than ~10 μm RMS. Furthermore, it is assumed that the "orbit" will then be steered with a one-to-one or similar algorithm, such that the beam is "zeroed" at each down stream BPM. The position of the beam with respect to the magnet centres is then given by the accuracy to which the BPM offset was determined (point 1 above.)

After beam-based alignment and orbit correction the remaining aberrations will perturb the vertical beam size at the IP. The most significant aberrations are the linear terms, such as vertical dispersion, x-y coupling and waist shift. We expect, however, to be able to tune these linear aberrations out using orthogonal knobs based on the beam offset in the strong sextupoles of the horizontal and vertical chromatic correction sections (HCCS and VCCS respectively, see section 2.) If we assume that the linear terms can be removed, then the remaining non-linear terms will dominate: of these, the most important term is second-order dispersion.

In this report we will estimate a beam-based alignment tolerance based on the maximum allowed residual second-order dispersion generated at the IP, and assuming the linear terms are completely corrected. We begin by first defining our linear orthogonal tuning knobs (section 2). In section 3, we estimate the second-order dispersion at the IP arising from the combination of residual random alignment errors and a simple one-to-one steering algorithm.

2. Linear Tuning Knobs

As of writing, we expect to use the strong CCS sextupole pairs to produce orthogonal waist, coupling and dispersion knobs for both planes. Conceptually, the sextupoles are misplaced with respect to the beam, effectively generating a quadrupole (horizontal motion) or a skew-quadrupole (vertical motion) at that point. Such techniques were first successfully demonstrated at the SLC final focus [2] and later at the FFTB [3]. In the latter case, the sextupoles were placed on translatable stages which allowed them to be physically moved. In principle, the same effect can be achieved by steering the beam in the sextupole. A third possibility is to add pairs of quadrupoles or skew-quadrupoles close to the sextupoles to effectively perform the same job. In this section, we estimate the beam motion relative to the sextupole centre that is necessary to achieve the required tuning knobs.

By design the two sets of strong sextupole pairs are positioned $-I$ apart, and are $(n+½)\pi$ in both x- and y-phase away from the IP. In both the HCCS and VCCS, the dispersion function is symmetric at each sextupole in the pair. The net effect is for the unwanted geometric terms to cancel, while the required chromatic terms add. Such an arrangement allows us to use combinations of symmetric or anti-symmetric motions at the sextupole pairs to generate the required orthogonal linear tuning knobs. Table 1 summarises the five linear aberrations we need to correct, and the associated pair motion and coefficients.

In the current design of the TESLA BDS, a single sextupole from a pair is actually split into two sextupoles, separated by a quadrupole. Each sextupole has a length of 2 m. We assume for this study that only the upstream sextupole is used for the tuning knob. Table 2 shows the numerical coefficients for each aberration "knob". The coefficients are per unit sextupole pair motion (symmetric or anti-symmetric): for example, a 1 μm symmetric horizontal motion of the HCCS sextupoles would generate a 383 μm and 1.6 μm x- and y-waist shift at the IP respectively.

Table 1: Combinations of sextupole pair motions to produce the required linear tuning. Here K_2 is the integrated sextupole strength (m^{-2}), R_{12} and R_{34} are the linear Greens functions from the sextupole to the IP (m), and D_x the design linear dispersion at the magnet.

aberration		**pair motion**	**coefficient / unit offset**
waist	x	horizontal symmetric	$2K_2R_{12}^2$
	y		$2K_2R_{34}^2$
dispersion	x	horizontal anti-symmetric	$2K_2D_xR_{12}$
	y	vertical anti-symmetric	$2K_2D_xR_{34}$
coupling		vertical symmetric	$2K_2R_{12}R_{34}$

Table 2: Coefficients for the various sextupole mover combination.

		coefficients per unit offset	
		X	**Y**
HCCS	waist	382.8	1.634
	dispersion	3.243	0.2118
	coupling	25.01	
VCCS	waist	47.89	158.2
	dispersion	0.8438	1.534
	coupling	87.04	

From the coefficients in Table 2, we can now estimate the required mover ranges (or beam motion) for a typical IP scan. We assume a scan range which gives us an effective increase in the IP β-function (σ^2) by a factor of three. Table 3 lists the required sextupole pair motions to produce a 3β change at the IP.

Table 3: Required sextupole mover motion for a 3β scan of the IP beam. The 2% limit is defined as that motion which would increase the beam size by 2%.

Aberration		**Sextupole Pair**	**Motion Type**	**3β Scan Range (μm)**	**2% limit (μm)**
waist	*x*	HCCS	horizontal symmetric	52	7
		VCCS		413	58
	y	HCCS		346	48
		VCCS		3.6	0.5
dispersion	*x*	HCCS	horizontal anti-symmetric	121	18
		VCCS		465	70
	y	HCCS	vertical anti-symmetric	19	2.6
		VCCS		2.6	0.4
coupling		HCCS	vertical symmetric	7.4	1
		VCCS		2.1	0.3

A typical tuning scenario would be to make a ±3β scan of a specific knob, measuring the beam size at each step. The minimum of the resulting parabola would then be determined and the sextupole movers adjusted accordingly. If we set ourselves a tuning goal of 2% from the design beam size, then this effectively defines the required mover resolution. The 2% limit for motions are also given in Table 3. Magnet movers of the type similar to those used at the FFTB [4] have a range ±1 mm and a typical resolution (step size) of ~1 μm, which would be too large for three of the entries in Table 3 (the VCCS knobs for coupling, *y*-dispersion and *y*-waist.) Clearly in these cases, the alternative HCCS knobs could be used.

3. Estimates of the alignment tolerance goal for beam-based alignment

A realistic machine will clearly have magnets with non-zero alignment errors. The initial "tuning" that is performed will be to steer the orbit to zero at all the BPMs: the resulting steered orbit will go off-centre through all the magnets by an amount equal to the BPM offsets, assuming that each magnet has its own dedicated BPM. These magnet offsets, together with the dispersive kicks from the correctors, generate to first-order those aberrations listed in Table 1, namely dispersion, waist shifts and coupling, the latter two coming only from offsets in the sextupoles. We assume that these linear aberrations are then tuned out using the sextupole knobs described in section 2. Now only non-linear aberrations remain, and of those, the second-order dispersion is likely to be the dominant term.

Assuming a set of magnet alignment errors with a given RMS value, we can estimate using the formalism developed in [1] the second-order dispersion generated at the IP after steering the orbit flat. We can use this value to set a limit on the allowed RMS for the alignment errors.

From [1], the offset of the orbit with respect to the magnet centres (**y**) due to a set of quadrupole offsets (**Y**) is given by

$$\mathbf{y} = -\mathbf{Q} \cdot \mathbf{Y}, \tag{1}$$

where **Q** is the linear response matrix. The *measured* orbit ($\tilde{\mathbf{y}}$) is given by

$$\tilde{\mathbf{y}} = -\mathbf{Q} \cdot \mathbf{Y} + \mathbf{b}, \tag{2}$$

where **b** are the BPM offsets. Assuming that every quadrupole has a corrector coil associated with it, we can now steer the BPM readings to zero (*i.e.* $\tilde{\mathbf{y}} = \mathbf{0}$.) The linear dispersion resulting from the corrector kicks in this case is just given by $\tilde{\mathbf{y}}$. After steering, the beam offset in each of the magnets is only the BPM offsets **b**. In [1], the linear dispersion response matrix (Δ) was used to estimate the dispersion at each magnet generated from a set of beam offsets in the upstream magnets:

$$\mathbf{D}_{\mathrm{y}} = \boldsymbol{\Delta} \cdot \mathbf{y}\,. \tag{3}$$

Hence before tuning but after steering, we can write down following expression for the linear dispersion along the beamline:

$$\begin{aligned}\mathbf{D_y} &= \mathbf{\Delta}\cdot\mathbf{b}+\tilde{\mathbf{y}} \\ &= (\mathbf{\Delta}+\mathbf{I})\cdot\mathbf{b}-\mathbf{Q}\cdot\mathbf{Y}\end{aligned} \tag{4}$$

The covariance of the dispersion ($\mathbf{V_D}$) is given by

$$\mathbf{V_D} = \sigma_b^2(\mathbf{\Delta}+\mathbf{I})\cdot(\mathbf{\Delta}+\mathbf{I})^{\mathrm{T}} + \sigma_Y^2\mathbf{Q}\cdot\mathbf{Q}^{\mathrm{T}} \tag{5}$$

assuming that the BPM offsets and magnet alignment errors are random and uncorrelated, with the RMS values of σ_b and σ_Y respectively. Figure 1 shows an example of the RMS dispersion generated by random alignment errors of 10 μm RMS, together with BPM offsets of 1 μm RMS (after steering.) The resulting RMS IP dispersion is 21 μm; this value must now be corrected using the sextupole knobs presented in Section 2. From Table 2, we can see that a vertical anti-symmetric motion of the VCCS sextupoles by 21/1.534 ≈ 14 μm would tune the RMS dispersion to zero. However, the sextupoles only generate dispersion at the IP phase, and so they will have almost no impact on the dispersion shown in Figure 1.

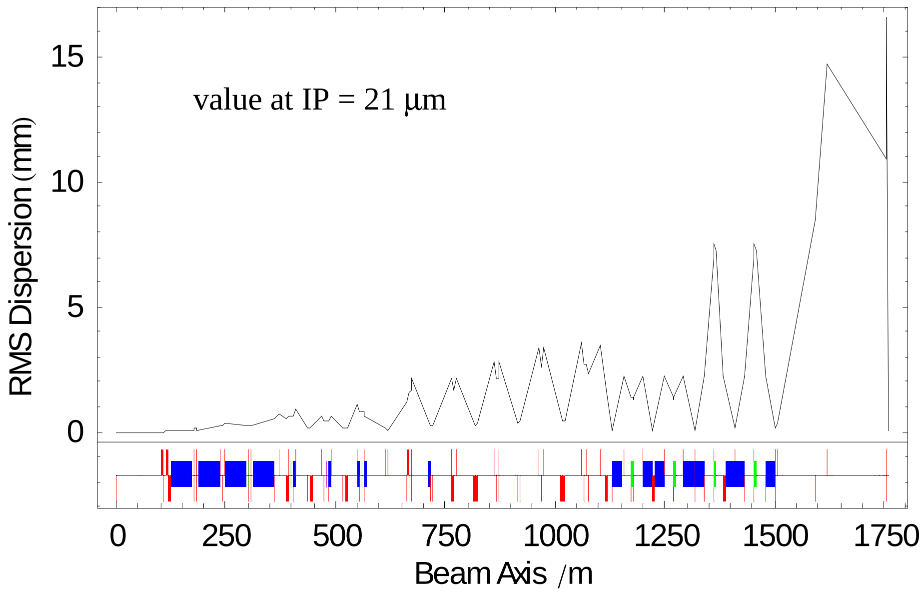

Figure 1: RMS linear dispersion generated after one-to-one steering for an RMS magnet alignment error of 10 μm, together with an RMS BPM offset of 1 μm.

Hence we can assume that the linear dispersion in the FFS itself remains unchanged by the application of the dispersion tuning knob. The second-order dispersion at the magnets (η) is given by

$$\boldsymbol{\eta} = \mathbf{\Delta}\cdot\mathbf{D_y} \tag{6}$$

Substituting (4) into (6), and defining $\mathbf{\Theta} \equiv \mathbf{\Delta} \cdot (\mathbf{\Delta} + \mathbf{I})$ and $\mathbf{\Omega} \equiv \mathbf{\Delta} \cdot \mathbf{Q}$, we can estimate the covariance of the second-order dispersion along the beamline ($\mathbf{V}_\eta$) as

$$\mathbf{V}_{\boldsymbol{\eta}} = \sigma_b^2 \mathbf{\Theta} \cdot \mathbf{\Theta}^{\mathrm{T}} + \sigma_Y^2 \mathbf{\Omega} \cdot \mathbf{\Omega}^{\mathrm{T}} \tag{7}$$

The RMS second-order dispersion for the previous example (σ_b = 1 μm, σ_Y = 10 μm) is shown in Figure 2. From (7), we can estimate the RMS second-order dispersion at the IP[1] for the BDS as

$$\sigma_\eta^2 \approx 626\sigma_b^2 + 609\sigma_Y^2 \tag{8}$$

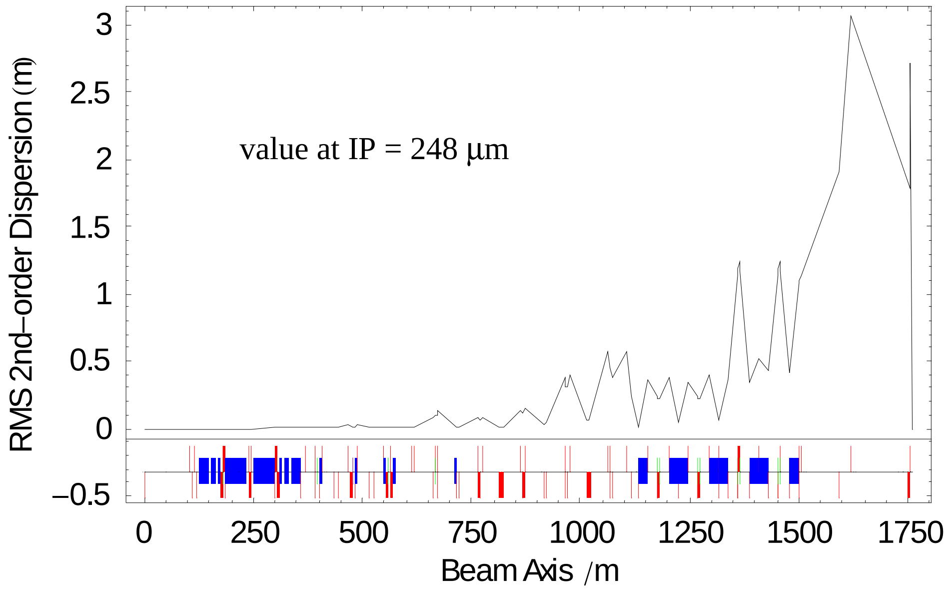

Figure 2: RMS second-order dispersion generated by linear dispersion shown in Figure 1.

As an upper limit on η, we take a 2% increase in vertical beam size at the IP, corresponding to 1 nm added in quadrature to the design 5 nm. Taking the electron energy spread of $\delta \approx 1.8\times10^{-3}$, we have

$$\Delta\sigma_y \approx \sqrt{2}\sigma_\eta \delta^2 \leq 1\,\text{nm} \tag{9}$$

and thus

$$\sigma_\eta \leq 218\,\mu\text{m}\,. \tag{10}$$

This value corresponds to an RMS of ~9 μm for either the BPM offsets or magnet alignment errors, *providing* the effect is entirely due to one or the other. If we assume that beam-based alignment can effectively align the BPM to the magnet centres to within an RMS of (for example) 3 μm, then the RMS magnet alignment will need to be of the order of ~8 μm. However, some overhead is required, since we must expect the

[1] The IP value is given by the last element of η.

RMS magnet alignment to grow over time due to ground motion effects, such as ATL [5]. For ATL-like motion, we estimate the RMS second order dispersion at the IP by

$$\sigma_{\eta,\mathrm{ATL}}[\mu\mathrm{m}] \approx 8\sqrt{T[\mathrm{hours}]} \tag{11}$$

assuming $A = 4\times10^{-6}\ \mu\mathrm{m}^2\mathrm{m}^{-1}\mathrm{s}^{-1}$[6]. If we allow a 2% drop in luminosity due to second-order dispersion before we need to invasively re-tune[2], we have $T \approx 740$ hours, or ~30 days. This assumes that we start with a perfectly aligned machine. Realistically, we must add the ATL contribution (11) in quadrature with the effects of the random magnet and BPM alignment errors (8):

$$\sigma_\eta^2 \approx 626\sigma_b^2 + 609\sigma_Y^2 + 64T \tag{12}$$

where all RMS quantities are now in microns, and *T* is measured in hours as before. Again, assuming $\sigma_b = 3\ \mu\mathrm{m}$, we can estimate what the mean time between invasive tuning (*T*) is as a function of the initial random RMS magnet alignment. The results are shown in Figure 3.

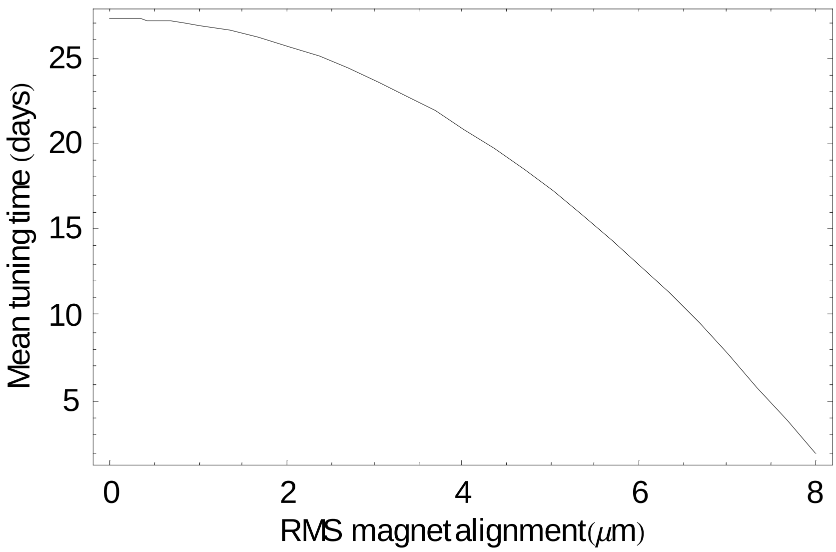

Figure 3: Mean tuning time (in days) as a function of initial RMS magnet misalignment. Calculation is based on a 2% contribution to the vertical beam size by second-order dispersion with $\delta=1.8\times10^{-3}$, *and ATL-like motion* ($A = 4\times10^{-6}\ \mu\mathrm{m}^2\mathrm{m}^{-1}\mathrm{s}^{-1}$).

As a final comment, we should note that the ~8 μm RMS alignment requirement is effectively a short wavelength (magnet to magnet) constraint, where no correlation is assumed. For longer wavelength effects, such as baseline effects arising from optical survey or beam-based alignment techniques, the correlation between the magnets has a significant effect, and larger "amplitudes" of misalignment can be tolerated. Figure 4 shows the maximum allowed RMS amplitude of misalignments as a function of "alignment wavelength". The short wavelength values correspond to the ~8 μm for the random uncorrelated estimate arrived at above. However, at longer wavelengths (>500 m), amplitudes on the order of ~40-100 μm RMS can be tolerated. However, it is still

[2] this may require repeating beam-based alignment, or adjustment of some other tuning knobs

unlikely that such long-baseline tolerances will be met, and more sophisticated alignment procedures, steering algorithms (dispersion free for example) and tuning knobs will probably be required to achieve the tight tolerance set here.

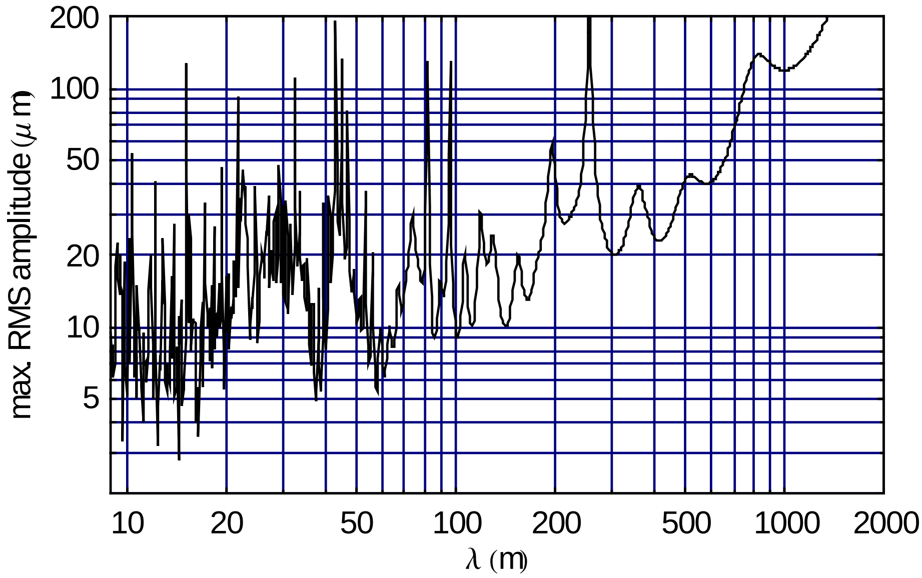

Figure 4: Maximum allowed RMS amplitude of alignment errors as a function of alignment wavelength. Based on a 2% increase in σ_y due to second-order dispersion (after one-to-one steering, with no BPM errors.)

4. Summary

Linear tuning knobs for waist shift, dispersion and coupling can be successfully implemented using the strong sextupole pairs in the CCS sections of the TESLA BDS, placed on mechanical movers with a step size of ~ 1 μm. Using these knobs, the residual linear aberrations at the IP due to random magnet misalignment and BPM offsets, and after one-to-one steering, can be effectively tuned out. The remaining second-order dispersion sets an upper limit on the RMS magnet alignment and BPM offsets. For a 2% increase in vertical beam size due to second-order dispersion, the initial RMS magnet alignment needs to be on the order of ~ 8 μm: such values can only be achieved with beam-based techniques. A value of 6 μm RMS (with an additional 3 μm RMS BPM offset) would allow ~13 days of ATL-like ground motion, before the 2% tolerance is exceeded. These results have been obtained using simplistic analytical models of the BDS: the exact effectiveness of various specific beam-based alignment techniques and subsequent tuning requires more study.

5. References

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