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Estimation of Relative Field Unflatness and Effective Gradient in TESLA Superstructures

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Abstract

In last years a few proposals for increasing of effective gradient in the TESLA linear collider were appeared. One of them is so called superstructure (SS) [1]. At the present time the superstructure consist of four 7-cells standing wave cavities coupled by tube of big diameter (114mm) and short length ($\lambda/2$). Due to shorter tubes between cavities in comparison with the TTF structure ($\lambda/2$ instead of $3\lambda/2$) the filling factor of the SS is higher than the filling factor of the TTF structure by 16.7%. In addition authors of this proposal are considering that the SS will have unflatness of the accelerating field less than the TTF structure. In this paper it will be shown that accelerating field unflatness of the whole SS is bigger than the unflatness of its sub-cavities and the unflatness of the TTF 9-cells cavity too. The behavior of the cavity chain called the SS is the behavior of a multiperiodic structure and can not be considered separately as four independent cavities. It means also that dynamic adjusting of the SS will be more complicate than that of the TTF structure.

1. Introduction

In order to describe accurately the behavior of the actual cavity chains of the SS a simple mathematical model of the cavity chain can be used. As shown in [2-3] a chain of electrically coupled lumped circuit resonators is the most appropriate for the investigation of properties of electromagnetic modes in accelerating structures. For the considered kind of accelerating structures this model provides accurate dispersion curve and accelerating field distribution of the fundamental mode. Fig. 1 shows schematically the model of the coupled M×N-cells cavities.

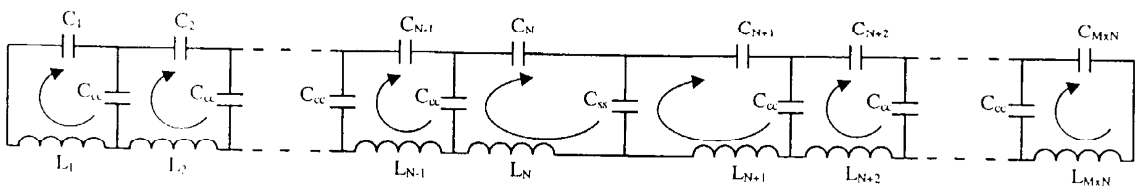

Fig. 1. Coupled lumped circuit resonator model for a multiperiodic chain of cavities

The system of Kirchhoff's equations generated from a simple model of the M×N superstructure is of the form

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$$j_1(i\omega L_1 + 1/i\omega C_1 + 1/i\omega C_{cc}) - j_2/i\omega C_{cc} = 0$$
$$-j_1/i\omega C_{cc} + j_2(i\omega L_2 + 1/i\omega C_2 + 2/i\omega C_{cc}) - j_3/i\omega C_{cc} = 0$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$-j_{N-1}/i\omega C_{cc} + j_N(i\omega L_N + 1/i\omega C_N + 1/i\omega C_{cc} + 1/i\omega C_{ss}) - j_{N+1}/i\omega C_{ss} = 0$$
$$-j_N/i\omega C_{ss} + j_{N+1}(i\omega L_{N+1} + 1/i\omega C_{N+1} + 1/i\omega C_{cc} + 1/i\omega C_{ss}) - j_{N+2}/i\omega C_{ss} = 0$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$-j_{M\cdot N-2}/i\omega C_{cc} + j_{M\cdot N-1}(i\omega L_{M\cdot N} + 1/i\omega C_{M\cdot N} + 2/i\omega C_{cc}) - j_{M\cdot N}/i\omega C_{cc} = 0$$
$$-j_{M\cdot N-1}/i\omega C_{cc} + j_{M\cdot N}(i\omega L_{M\cdot N} + 1/i\omega C_{M\cdot N} + 1/i\omega C_{cc}) = 0$$

where we have neglected losses. This set of equations can be transformed as

$$j_1(1-\omega^2/\omega_1^2 + k_{cc}/2\cdot C_1/C_2) - j_2\cdot k_{cc}/2\cdot C_1/C_2 = 0$$
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$$-j_{N-1}\cdot k_{cc}/2\cdot C_N/C_{N-1} + j_N(1-\omega^2/\omega_N^2 + k_{cc}/2\cdot C_N/C_{N-1} + k_{ss}/2) - j_{N+1}\cdot k_{ss}/2 = 0$$
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$$-j_{M\cdot N-2}\cdot k_{cc}/2 + j_{M\cdot N}(1-\omega^2/\omega_{M\cdot N-1}^2 + k_{cc}) - j_{M\cdot N}\cdot k_{cc}/2 = 0$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$-j_{M\cdot N-1}\cdot k_{cc}/2\cdot C_{M\cdot N}/C_{M\cdot N-1} + j_{M\cdot N}(1-\omega^2/\omega_1^2 + k_{cc}/2\cdot C_{M\cdot N}/C_{M\cdot N-1}) = 0$$

where

$\omega_i = 1/\sqrt{L_i C_i}$ is the eigen frequency of i-th resonator,

j_i is the cells current,

$k_{cc} - 2\cdot C_{m\cdot N+nl}/C_{cc} = 2\cdot C_{in}/C_{cc}$ is the coupling between inner cells ($m\in 0,\ldots,M;\ nl\in 2,\ldots,(N-1)$),

$k_{ss} = 2\cdot C_{ml\cdot N\pm 1}/C_{ss} = 2\cdot C_{se}/C_{ss}$ is the coupling between intermediate side cells ($ml\in 1,\ldots,M-1$).

C_{in}, C_{cc}, C_{se}, C_{ss} are parameters of equivalent circuit for inner resonators, for coupling between resonators into the cavities, for side resonators and for coupling between side resonators correspondingly.

This system may be written in matrix-vector-notation

$$Z\times\vec{j} = 0$$

with 3-diagonal matrix impedance Z expressed as follows

$$z_{11} = z_{M\cdot N,M\cdot N} = 1-\frac{\omega^2}{\omega_i^2}+\frac{k_{cc}}{2}\cdot\frac{C_{ee}}{C_{in}}$$
$$z_{i,i} = 1-\frac{\omega^2}{\omega_i^2}+\frac{k_{cc}}{2}\cdot\frac{C_{se}}{C_{in}}+\frac{k_{ss}}{2} \quad - \text{ if } i = N\pm 1+m\cdot N,\ \ m\in 0,\ldots,M-1$$
$$z_{i,i} = 1-\frac{\omega^2}{\omega_i^2}+k_{cc} \quad - \text{ if } i \neq \pm 1+m\cdot N,\ \ m\in 0,\ldots,M$$

$$z_{i,i+1} = z_{i+1,i} = -\frac{k_{cc}}{2} \cdot \frac{C_{ee}}{C_{in}} \quad - \text{ if } i = 1, M \cdot N - 1$$

$$z_{i-1,i} = z_{i+1,i+2} = -\frac{k_{cc}}{2} \cdot \frac{C_{se}}{C_{in}} \quad - \text{ if } i = m \cdot N, \quad m \in 1,\ldots,M-1$$

$$z_{i,i+1} = z_{i+1,i} = -\frac{k_{ss}}{2} \quad - \text{ if } i = m \cdot N, \quad m \in 1,\ldots,M-1$$

Where k_{cc}, k_{ss}, C_{se}/C_{cc}, C_{ee}/C_{cc} can be calculated numerically by using some electromagnetic code. The eigenvalues of this matrix correspond to the square of the eigenfrequencies, the eigenvectors are proportional to the amplitude of the accelerating field in cells on these modes.

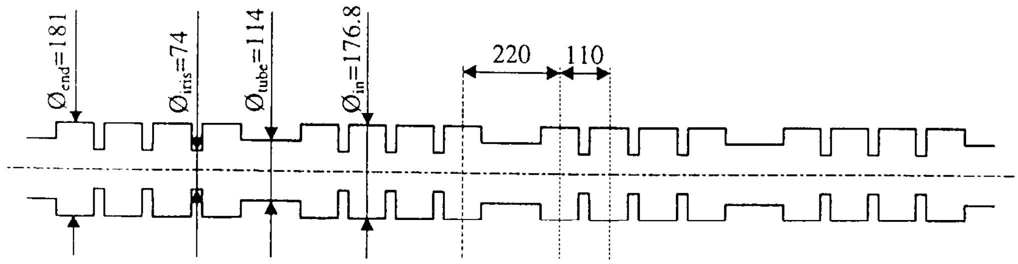


Fig. 2. Geometric parameters of the test 4×4 superstructure.

To test this model a simple 4×4 superstructure (consisting of four 4-cells cavities) will be examined. This 4x4 superstructure (see Fig.2) is close to the investigated SS mentioned above.

The data set for the mathematical model, (ω_i, k_{cc}, k_{ss}, C_{se}/C_{in}, C_{ee}/C_{in}), is calculated by URMEL [4]. In addition, the dispersion curve for the 4×4 superstructure and electric field

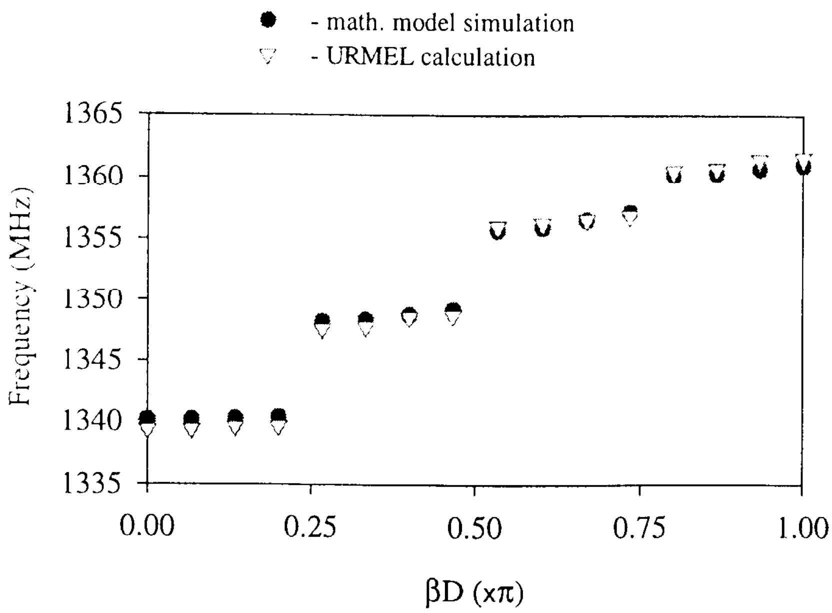
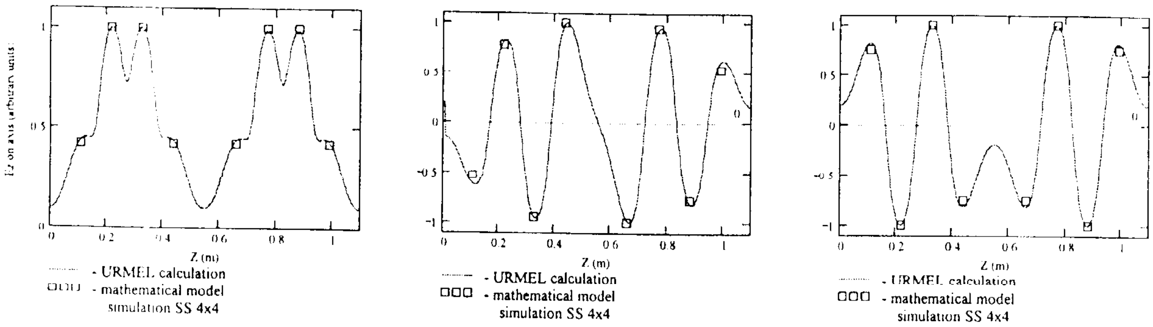


Fig. 3. Dispersion curve of the 4x4 test superstructure.

distribution on axis are calculated by using URMEL. Each of the four cavities has been preliminary tuned individually for a maximal accelerating field flatness. It should be noted that k_{ss} is the coupling between the side cells of the neighboring cavities through the tube.

Fig. 3 shows the dispersion curve obtained by URMEL and by the mathematical model. The accelerating field profiles of three modes 0, π and $13\pi/16$ in comparison with model field amplitudes are shown in Fig.4.



a) 0 mode　　b) π mode　　c) $13\pi/16$ mode (SS mode)

Fig. 4. Comparison between field distribution in URMEL calculation and in mathematical model simulation for test 4x4 superstructure (left half of the SS).

The comparison shows a good agreement between the mathematical model simulations and URMEL direct calculations. Differences between URMEL calculations and the mathematical modeling are less than 0.5% for mode frequencies and ~5% for the field amplitude distribution. This model also illustrates that the superstructure in whole can not be tuned separately by tuning its component parts.

2. The 4×7 superstructure for the TESLA

Properties of the actual 4×7-cells SS (see Fig.5) can be investigated by means of the previous model. The geometry data set of the SS has been obtained from ref. [5]. Calculated by URMEL one finds: f_{in}=1275.65MHz, f_{se}=1287.856MHz, f_{ee}=1286.68MHz, k_{cc}=0.01852

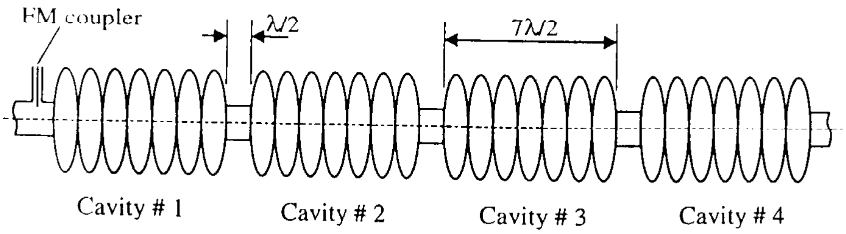


Fig. 5. The 4x7 superstructure for the TESLA.

and k_{ss}=0.0022, where f_{in} is the frequency of the inner cells, f_{se} is the frequency of the side cells of the inner cavities and f_{ee} is the frequency of the side cells of the both sides of the SS.

Fig. 6 represents dispersion curve for the SS. The operation mode of the SS, $25\pi/28$, is marked by a circle there. After adjusting of the SS, the frequencies f_{se} and f_{ee} would be slightly changed, but the shape of the dispersion curve rests almost unchanged. Here, the adjusting of the SS is defined as the finding of cell's frequency set which provides high cells amplitude flatness ($\Delta E_{acc}/E_{acc}<10^{-6}$) for the mathematical model. In the following, to calculate the effects of cavity frequency errors on field levels we assume that each cell has a frequency $f_i+\delta f_i$ where δf_i is a random frequency shift.

For the same comparison, corresponding properties of the TTF 9-cells structure have been obtained by using this model. The simulation of an accelerating standing wave structure shows that the structure field nonuniformity is in direct proportional to the average relative error of cells frequency $\delta f_c/f_c$ and $N^{3/2}$ and in inverse proportion to cell-to-cell coupling k_{cc}

$$\sigma_{\frac{\Delta E}{E}} \sim \sigma_{\frac{\delta f_c}{f_c}} \cdot \frac{N^{\frac{3}{2}}}{k_{cc}} \quad , \text{ where } N \gg 1$$

More accurate relation for field nonuniformity in standing wave structure is produced in ref. [6, 7].

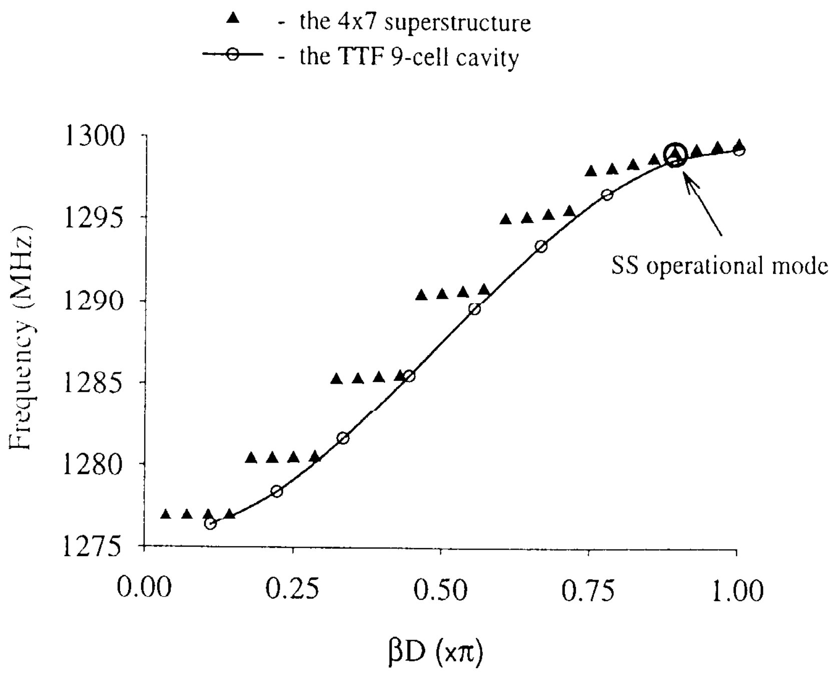


Fig. 6. Dispersion curves of the 4x7 superstructure and the TTF 9-cell cavity

Defining structure field nonuniformity ($\sigma_{\Delta E/E}$) as ratio $(E_{max}-E_{mean})/E_{mean}$, one obtains the average value $\sigma_{\Delta E/E}$ from a big number (~1000) of the simulations with a random set of cells frequency distributions ~$500\cdot\delta f_c/f_c$ (%) for the TTF 9-cells structure and $1830\cdot\delta f_c/f_c$ (%) for the 4×7 SS. Thus for the same tolerance of fabrication and assembling the 4×7 SS has field unflatness 3.6 times higher than the TTF 9-cells structure. Fig. 6 shows that the field unflatness of each subunit of the SS can be less than the $\sigma_{\Delta E/E}$ of the whole SS.

As shown in ref. [8] the effective coupling (k) for a selected mode is in inverse proportion to $(f_i \;\; f_{i+1})$, mode-to-mode distance on a dispersion curve. In addition, for a well adjusted multiperiodic or a biperiodic structure it should be minimized in the operational mode point [6].

Experiments in the present TESLA Test Facility have shown the $\sigma_{\Delta E/E}$ of TTF 9-cells structure ~ 5% [9]. It leads 5% reduction of E_{eff} for this structure. It means also ~18.3% field unflatness for the 4x7 SS with the same errors of fabrication. In addition SLANS [11] calculation has shown a ratio E_{max}/E_{acc} in the side cells is 2.3 (E_{max}/E_{acc}=2.0 for inner cells). Therefore the side cells have the effective accelerating rate 13% lower than inner cells. The total reduction of E_{eff} for the SS is 2/7*13%=3.7%. This factor for the TTF 9-cells structure ($\varnothing_{tube}$=78mm) is negligible ~0.2%. Thus the resulting effective gradient E_{eff} for the 4x7 SS will be 0.1% lower in comparison with the E_{eff} of the TTF 9-cells cavity (see Tab. 1)

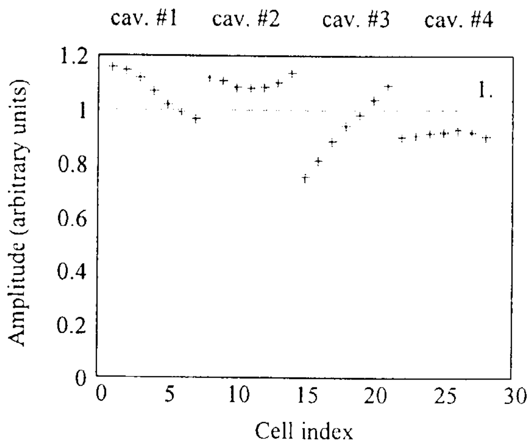


Fig. 7. Computed amplitudes in the model of 4x7 superstructure ($\delta f_c/f_c=10^{-4}$)

To understand more clearly the nature of the high field unflatness of the SS it should be noted that the SS behaves as a multiperiodic structure but not as its subunits. Evidently in this case the cavity-to-cavity field nonuniformity is more higher than cell-to-cell $\sigma_{\Delta E/E}$ into the each cavities of the SS (see Fig. 7). The dispersion curve of a $M\times N$ superstructure has (N-1) stopbands [10]. Like biperiodic [6] or ordinary standing wave structures the multiperiodic structure has the worst point for field flatness π and the best point near $\pi/2$. Pretuning of a multiperiodic structure means the minimization of nearest to work point stopband just as biperiodic structure. The 4x7 SS needs this pretuning too. But for π mode in this 4x7 SS the field unflatness would be ~ $4300 \cdot \delta f_c/f_c$ (%), i.e. ~ 43% for obtained on TTF cells frequency scattering. This value exceeds the unflatness of TTF 28-cells structure ~ 2400 $\delta f_c/f_c$ (%).

The next problem concerning the SS is the complexity of the dynamic structure tuning. For instance one can simulate the effect of cavity tuning by "compression" of one of the cavities. We define a compression of one cavity as an uniform changing of its cells frequencies. Fig. 8 demonstrates the field unflatness of each cavity and of the whole SS vs frequency tuning of

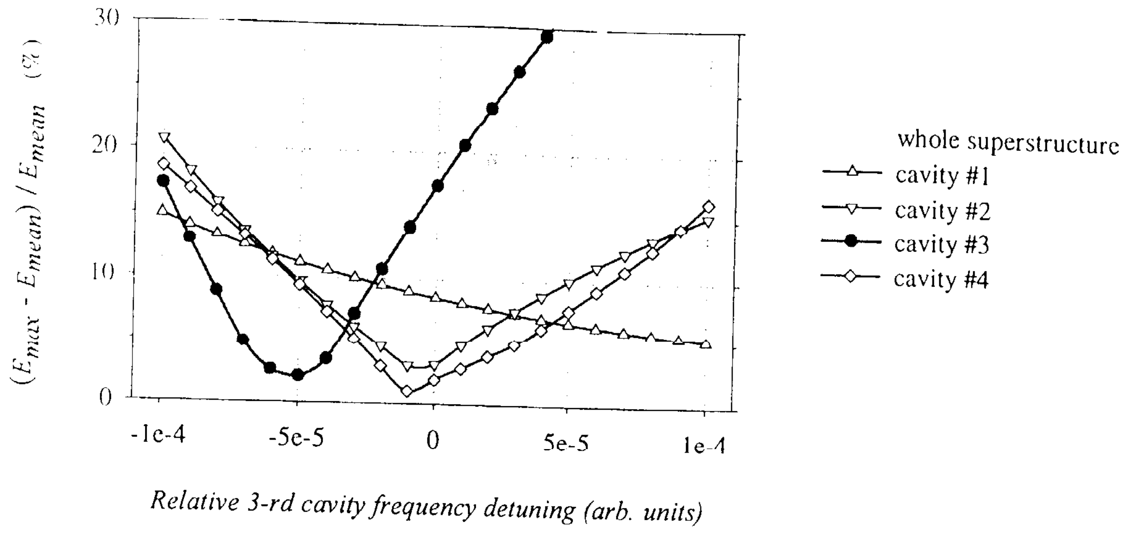


Fig. 8. Field unflatness as a function of frequency detuning of 3-rd cavity in the 4x7 superstructure

third cavity (see Fig. 7). Strong influence of this tuning on the field of other cavities has been shown. It means that an independent cavity tuning is impossible in the SS. The tuning system and operational control for the SS will be more complex than for the TTF structure.

3. Optimization of the 4×7 superstructure.

The field unflatness in the 4×7 SS vs side cell - side cell coupling k_{ss} is shown in Fig. 9. The optimum of $\sigma_{\Delta E/E}$ is situated between the values of k_{ss} 0.03 and 0.04.

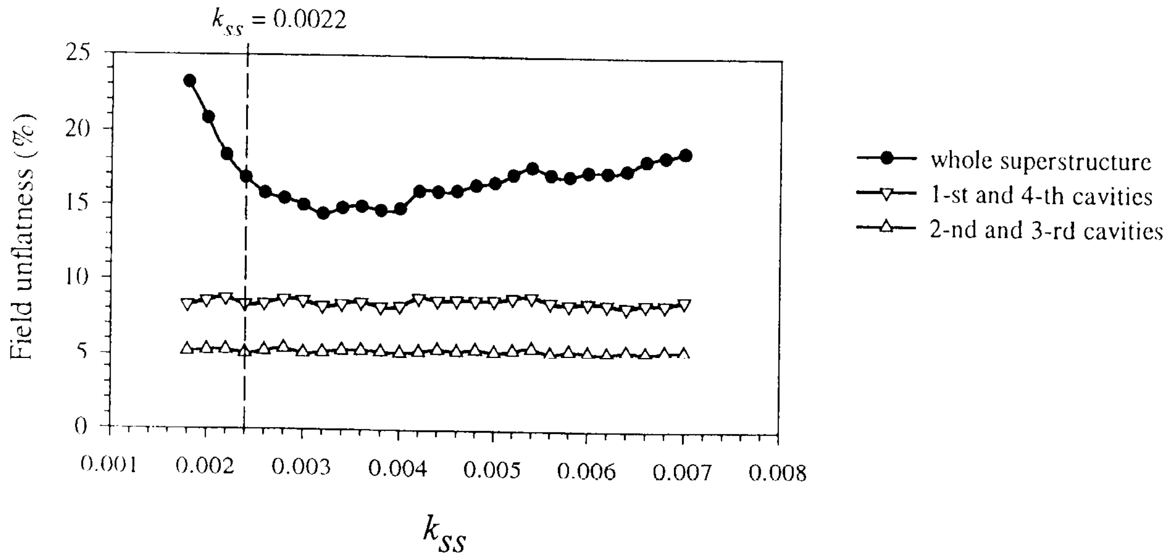


Fig. 9. Relative unflatness in the 4×7 superstructure as function of coupling k_{ss} ($\delta f_c/f_c=10^{-4}$, k_{cc}=0.01852)

As mentioned above the operational mode of the SS has stopbands. According to ref. [2] the field flatness of a multiperiodic structure reach maximum at a minimum width of stopband. In this case the coupling cavity chain of the SS is evidently undercoupled for k_{ss}<0.03 and overcoupled for k_{ss}>0.04. However k_{ss}=0.03 corresponds to diameter of interconnecting tube of about 120mm. This leads to ~14.9% reduction of accelerating rate in side cells. The total gain in E_{eff} expected for the SS with k_{ss}=0.032 is 2.65%.

Parameter	TTF cavity	Superstructure	SS (k_{ss}=0.0032)
Radius of tube (mm)	39	57	60
$N_{cell}/N_{structure}$	9/1	7/4	7/4
Fill factor	0.875	0.75	0.75
k_{cc} (%)	1.852	1.852	1.852
k_{ss} (%)	...	0.22	0.32
Field unflatness for $\delta f_c/f_c=10^{-4}$ (%)	5	18.3	15
E_{max}/E_{acc} in side cells	2.02	2.3	2.351
Total gain in E_{eff} in comparison with TTF (%)	...	-0.1	2.65

Table 1. Parameters of the superstrure and the TTF cavity

4. Conclusion

Examination of the properties of the 4×7 SS shows that there is a strong interference of electric field between the coupled cavities. It doesn't allow tuning the cavities independently. In addition the field unflatness of the 4×7 SS is 3.6 times higher than that of the TTF 9-cells cavity for the same cell frequency scattering $\delta f_c/f_c$. If the reduction of the accelerating rate in the end cells of the cavities is also taken into account, the resulting E_{eff} of the SS is very close to E_{eff} of the present TTF structure. By doing some further optimizations of the end cells coupling k_{ss} the E_{eff} can be slightly increased. But a useful increase of the E_{eff} would need more precise manufacturing or more sophisticated tuning of the cavity chain.

Acknowledgments

The author is indebted to J. Gao and J. Le Duff for their interests in this work and many useful conversations. B. Mouton assisted me in computer utilization.

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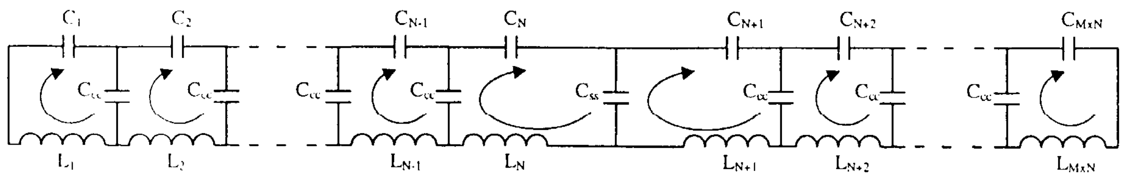

Fig. 1. Coupled lumped circuit resonator model for a multiperiodic chain of cavities

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$$. \quad . \quad . \quad . \quad .$$
$$-j_{N-1}/i\omega C_{cc} + j_N(i\omega L_N + 1/i\omega C_N + 1/i\omega C_{cc} + 1/i\omega C_{ss}) - j_{N+1}/i\omega C_{ss} = 0$$
$$-j_N/i\omega C_{ss} + j_{N+1}(i\omega L_{N+1} + 1/i\omega C_{N+1} + 1/i\omega C_{cc} + 1/i\omega C_{ss}) - j_{N+2}/i\omega C_{ss} = 0$$
$$. \quad . \quad . \quad . \quad .$$
$$-j_{M\cdot N-2}/i\omega C_{cc} + j_{M\cdot N-1}(i\omega L_{M\cdot N} + 1/i\omega C_{M\cdot N} + 2/i\omega C_{cc}) - j_{M\cdot N}/i\omega C_{cc} = 0$$
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where we have neglected losses. This set of equations can be transformed as

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$$-j_{M\cdot N-2}\cdot k_{cc}/2 + j_{M\cdot N}(1-\omega^2/\omega_{M\cdot N-1}^2 + k_{cc}) - j_{M\cdot N}\cdot k_{cc}/2 = 0$$
$$. \quad . \quad . \quad . \quad .$$
$$-j_{M\cdot N-1}\cdot k_{cc}/2\cdot C_{M\cdot N}/C_{M\cdot N-1} + j_{M\cdot N}(1-\omega^2/\omega_1^2 + k_{cc}/2\cdot C_{M\cdot N}/C_{M\cdot N-1}) = 0$$

where

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with 3-diagonal matrix impedance Z expressed as follows

$$z_{11} = z_{M\cdot N,M\cdot N} = 1 - \frac{\omega^2}{\omega_i^2} + \frac{k_{cc}}{2}\cdot\frac{C_{cc}}{C_{in}}$$
$$z_{i,i} = 1 - \frac{\omega^2}{\omega_i^2} + \frac{k_{cc}}{2}\cdot\frac{C_{se}}{C_{in}} + \frac{k_{ss}}{2} \quad - \text{ if } i = N\pm 1 + m\cdot N,\ \ m\in 0,\dots,M-1$$
$$z_{i,i} = 1 - \frac{\omega^2}{\omega_i^2} + k_{cc} \quad - \text{ if } i \neq \pm 1 + m\cdot N,\ \ m\in 0,\dots,M$$

$$z_{i,i+1} = z_{i+1,i} = -\frac{k_{cc}}{2} \cdot \frac{C_{ee}}{C_{in}} \quad - \text{ if } i = 1, M \cdot N - 1$$

$$z_{i-1,i} = z_{i+1,i+2} = -\frac{k_{cc}}{2} \cdot \frac{C_{se}}{C_{in}} \quad - \text{ if } i = m \cdot N, \quad m \in 1,\ldots,M-1$$

$$z_{i,i+1} = z_{i+1,i} = -\frac{k_{ss}}{2} \quad - \text{ if } i = m \cdot N, \quad m \in 1,\ldots,M-1$$

Where k_{cc}, k_{ss}, C_{se}/C_{cc}, C_{ee}/C_{cc} can be calculated numerically by using some electromagnetic code. The eigenvalues of this matrix correspond to the square of the eigenfrequencies, the eigenvectors are proportional to the amplitude of the accelerating field in cells on these modes.

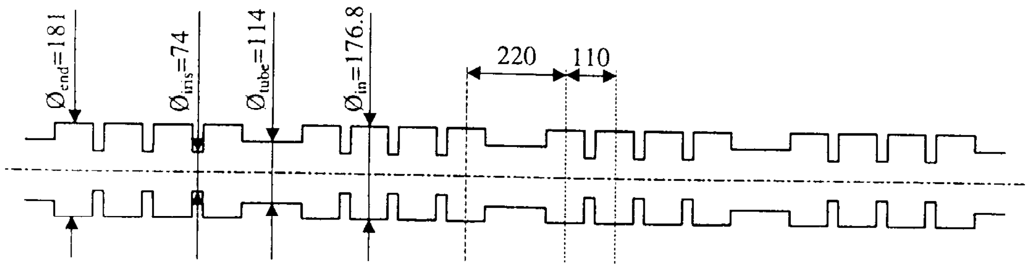


Fig. 2. Geometric parameters of the test 4x4 superstructure.

To test this model a simple 4x4 superstructure (consisting of four 4-cells cavities) will be examined. This 4x4 superstructure (see Fig.2) is close to the investigated SS mentioned above.

The data set for the mathematical model, (ω_i, k_{cc}, k_{ss}, C_{se}/C_{in}, C_{ee}/C_{in}), is calculated by URMEL [4]. In addition, the dispersion curve for the 4×4 superstructure and electric field

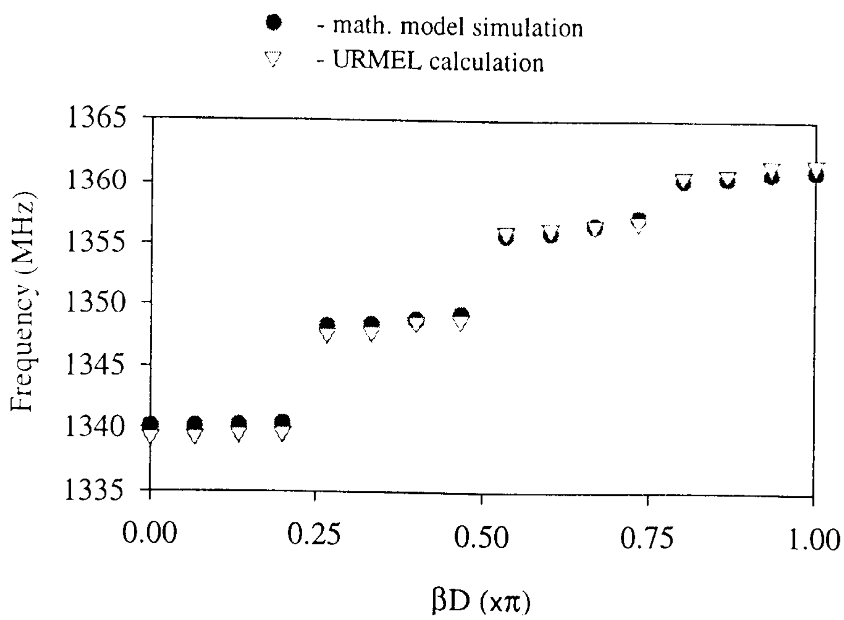
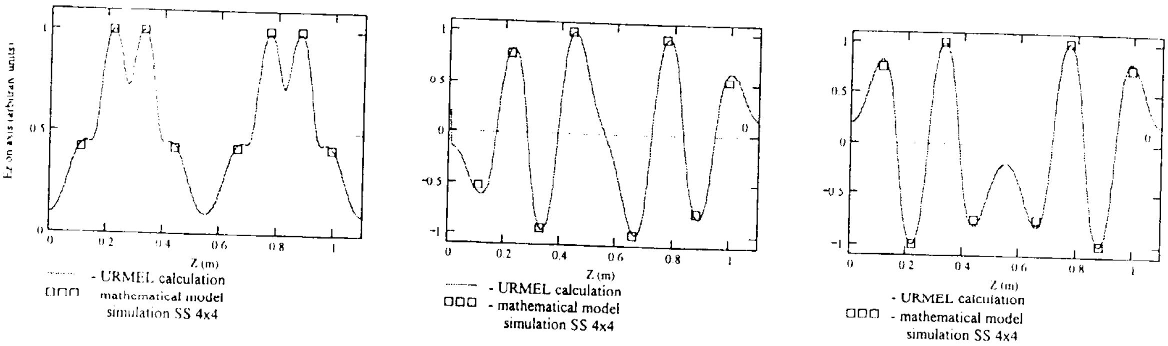


Fig. 3. Dispersion curve of the 4x4 test superstructure.

distribution on axis are calculated by using URMEL. Each of the four cavities has been preliminary tuned individually for a maximal accelerating field flatness. It should be noted that k_{ss} is the coupling between the side cells of the neighboring cavities through the tube.

Fig. 3 shows the dispersion curve obtained by URMEL and by the mathematical model. The accelerating field profiles of three modes 0, π and $13\pi/16$ in comparison with model field amplitudes are shown in Fig.4.



a) 0 mode b) π mode c) $13\pi/16$ mode (SS mode)

Fig. 4. Comparison between field distribution in URMEL calculation and in mathematical model simulation for test 4x4 superstructure (left half of the SS).

The comparison shows a good agreement between the mathematical model simulations and URMEL direct calculations. Differences between URMEL calculations and the mathematical modeling are less than 0.5% for mode frequencies and ~5% for the field amplitude distribution. This model also illustrates that the superstructure in whole can not be tuned separately by tuning its component parts.

2. The 4×7 superstructure for the TESLA

Properties of the actual 4×7-cells SS (see Fig.5) can be investigated by means of the previous model. The geometry data set of the SS has been obtained from ref. [5]. Calculated by URMEL one finds: f_{in}=1275.65MHz, f_{se}=1287.856MHz, f_{ee}=1286.68MHz, k_{cc}=0.01852

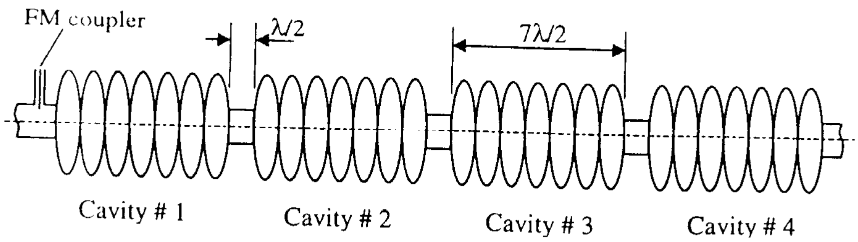


Fig. 5. The 4x7 superstructure for the TESLA.

and k_{ss}=0.0022, where f_{in} is the frequency of the inner cells, f_{sp} is the frequency of the side cells of the inner cavities and f_{ee} is the frequency of the side cells of the both sides of the SS.

Fig. 6 represents dispersion curve for the SS. The operation mode of the SS, 25π/28, is marked by a circle there. After adjusting of the SS, the frequencies f_{sc} and f_{ee} would be slightly changed, but the shape of the dispersion curve rests almost unchanged. Here, the adjusting of the SS is defined as the finding of cell's frequency set which provides high cells amplitude flatness ($\Delta E_{acc}/E_{acc}<10^{-6}$) for the mathematical model. In the following, to calculate the effects of cavity frequency errors on field levels we assume that each cell has a frequency $f_i+\delta f_i$ where δf_i is a random frequency shift.

For the same comparison, corresponding properties of the TTF 9-cells structure have been obtained by using this model. The simulation of an accelerating standing wave structure shows that the structure field nonuniformity is in direct proportional to the average relative error of cells frequency $\delta f_c/f_c$ and $N^{3/2}$ and in inverse proportion to cell-to-cell coupling k_{cc}

$$\sigma_{\frac{\Delta E}{E}} \sim \sigma_{\frac{\delta f_c}{f_c}} \cdot \frac{N^{\frac{3}{2}}}{k_{cc}}, \text{ where } N \gg 1$$

More accurate relation for field nonuniformity in standing wave structure is produced in ref. [6, 7].

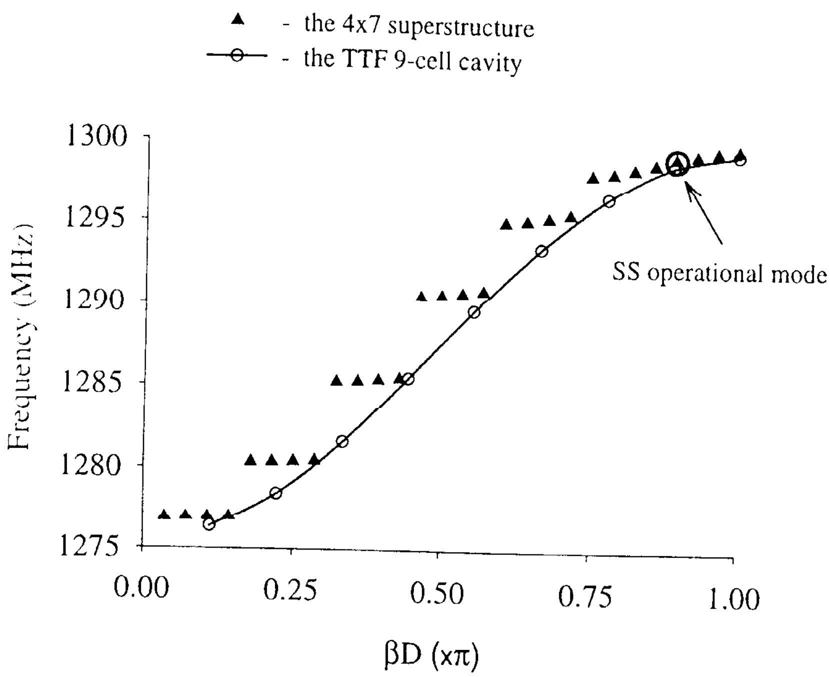


Fig. 6. Dispersion curves of the 4x7 superstructure and the TTF 9-cell cavity

Defining structure field nonuniformity ($\sigma_{\Delta E/E}$) as ratio $(E_{max}-E_{mean})/E_{mean}$, one obtains the average value $\sigma_{\Delta E/E}$ from a big number (~1000) of the simulations with a random set of cells frequency distributions ~$500\cdot\delta f_c/f_c$ (%) for the TTF 9-cells structure and $1830\cdot\delta f_c/f_c$ (%) for the 4×7 SS. Thus for the same tolerance of fabrication and assembling the 4×7 SS has field unflatness 3.6 times higher than the TTF 9-cells structure. Fig. 6 shows that the field unflatness of each subunit of the SS can be less than the $\sigma_{\Delta E/E}$ of the whole SS.

As shown in ref. [8] the effective coupling (k) for a selected mode is in inverse proportion to $(f_i - f_{i+1})$, mode-to-mode distance on a dispersion curve. In addition, for a well adjusted multiperiodic or a biperiodic structure it should be minimized in the operational mode point [6].

Experiments in the present TESLA Test Facility have shown the $\sigma_{\Delta E/E}$ of TTF 9-cells structure ~ 5% [9]. It leads 5% reduction of E_{eff} for this structure. It means also ~18.3% field unflatness for the 4×7 SS with the same errors of fabrication. In addition SLANS [11] calculation has shown a ratio E_{max}/E_{acc} in the side cells is 2.3 (E_{max}/E_{acc}=2.0 for inner cells). Therefore the side cells have the effective accelerating rate 13% lower than inner cells. The total reduction of E_{eff} for the SS is 2/7*13%=3.7%. This factor for the TTF 9-cells structure ($\varnothing_{tube}$=78mm) is negligible ~0.2%. Thus the resulting effective gradient E_{eff} for the 4×7 SS will be 0.1% lower in comparison with the E_{eff} of the TTF 9-cells cavity (see Tab. 1)

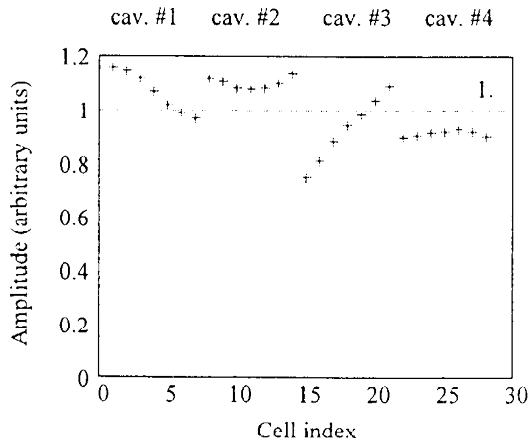


Fig. 7. Computed amplitudes in the model of 4×7 superstructure ($\delta f_c/f_c$=10^{-4})

To understand more clearly the nature of the high field unflatness of the SS it should be noted that the SS behaves as a multiperiodic structure but not as its subunits. Evidently in this case the cavity-to-cavity field nonuniformity is more higher than cell-to-cell $\sigma_{\Delta E/E}$ into the each cavities of the SS (see Fig. 7). The dispersion curve of a M×N superstructure has (N-1) stopbands [10]. Like biperiodic [6] or ordinary standing wave structures the multiperiodic structure has the worst point for field flatness π and the best point near $\pi/2$. Pretuning of a multiperiodic structure means the minimization of nearest to work point stopband just as biperiodic structure. The 4×7 SS needs this pretuning too. But for π mode in this 4×7 SS the field unflatness would be ~ 4300·$\delta f_c/f_c$ (%), i.e. ~ 43% for obtained on TTF cells frequency scattering. This value exceeds the unflatness of TTF 28-cells structure ~ 2400·$\delta f_c/f_c$ (%).

The next problem concerning the SS is the complexity of the dynamic structure tuning. For instance one can simulate the effect of cavity tuning by "compression" of one of the cavities. We define a compression of one cavity as an uniform changing of its cells frequencies. Fig. 8 demonstrates the field unflatness of each cavity and of the whole SS vs frequency tuning of

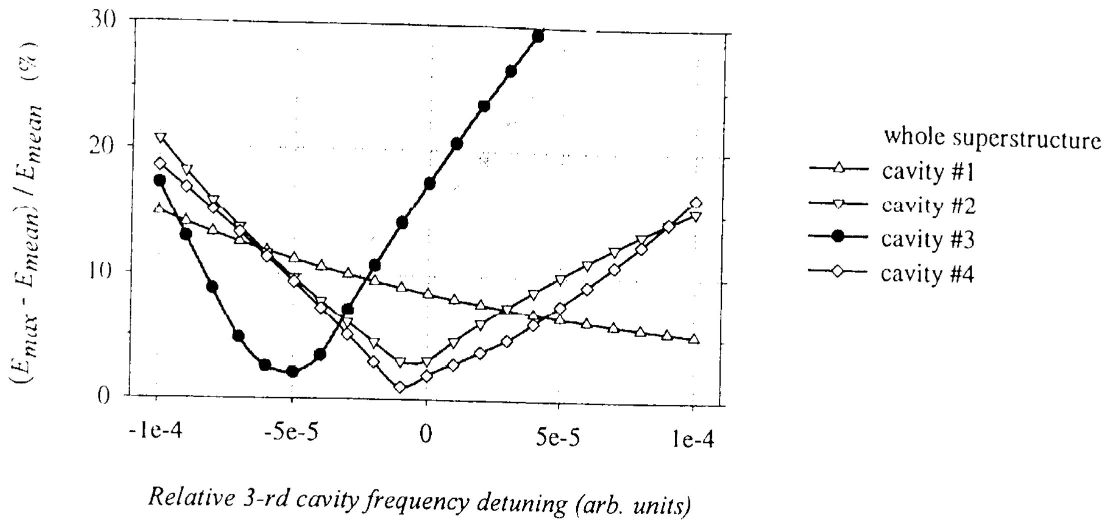


Fig. 8. Field unflatness as a function of frequency detuning of 3-rd cavity in the 4x7 superstructure

third cavity (see Fig. 7). Strong influence of this tuning on the field of other cavities has been shown. It means that an independent cavity tuning is impossible in the SS. The tuning system and operational control for the SS will be more complex than for the TTF structure.

3. Optimization of the 4×7 superstructure.

The field unflatness in the 4×7 SS vs side cell - side cell coupling k_{ss} is shown in Fig. 9. The optimum of $\sigma_{\Delta E/E}$ is situated between the values of k_{ss} 0.03 and 0.04.

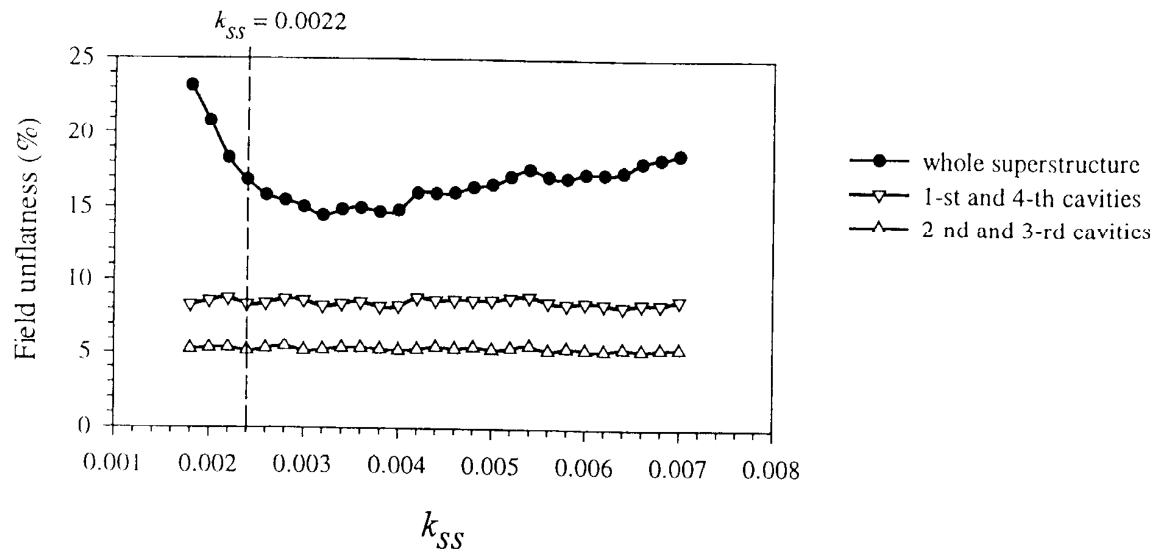


Fig. 9. Relative unflatness in the 4x7 superstructure as function of coupling k_{ss} ($\delta f_c/f_c$=10^{-4}, k_{cc}=0.01852)

As mentioned above the operational mode of the SS has stopbands. According to ref. [2] the field flatness of a multiperiodic structure reach maximum at a minimum width of stopband. In this case the coupling cavity chain of the SS is evidently undercoupled for k_{ss}<0.03 and overcoupled for k_{ss}>0.04. However k_{ss}=0.03 corresponds to diameter of interconnecting tube of about 120mm. This leads to ~14.9% reduction of accelerating rate in side cells. The total gain in E_{eff} expected for the SS with k_{ss}=0.032 is 2.65%.

Parameter	TTF cavity	Superstructure	SS (k_{ss}=0.0032)
Radius of tube (mm)	39	57	60
$N_{cell}/N_{structure}$	9/1	7/4	7/4
Fill factor	0.875	0.75	0.75
k_{cc} (%)	1.852	1.852	1.852
k_{ss} (%)	...	0.22	0.32
Field unflatness for $\delta f_c/f_c=10^{-4}$ (%)	5	18.3	15
E_{max}/E_{acc} in side cells	2.02	2.3	2.351
Total gain in E_{eff} in comparison with TTF (%)	...	-0.1	2.65

Table 1. Parameters of the superstrure and the TTF cavity

4. Conclusion

Examination of the properties of the 4x7 SS shows that there is a strong interference of electric field between the coupled cavities. It doesn't allow tuning the cavities independently. In addition the field unflatness of the 4x7 SS is 3.6 times higher than that of the TTF 9-cells cavity for the same cell frequency scattering $\delta f_c/f_c$. If the reduction of the accelerating rate in the end cells of the cavities is also taken into account, the resulting E_{eff} of the SS is very close to E_{eff} of the present TTF structure. By doing some further optimizations of the end cells coupling k_{ss} the E_{eff} can be slightly increased. But a useful increase of the E_{eff} would need more precise manufacturing or more sophisticated tuning of the cavity chain.

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