Simulation of the Fast Beam-Ion Instability in the TESLA FEL Transfer Line

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Abstract

The Fast Beam-Ion Instability arises from interaction between a bunch and an ion cloud previously created by all heading bunches during a single pass. To study this effect and to determine the required vacuum conditions for the TESLA FEL transfer line, a simulation code has been developed. The results of these simulation studies are presented in this paper.

1 Introduction

The Fast Beam-Ion Instability is a single-pass effect arising from the interaction between a bunch and an ion cloud which is produced by all leading bunches in the train. If this ion cloud is not perfectly symmetric with respect to the transverse bunch center, this interaction results in a transverse kick of the bunch and the ion cloud.

The bunches in a real accelerator are always slightly transversally displaced with respect to each other due to effects like wakefields. Such a small initial distortion can result in large oscillation amplitudes by means of the Fast Beam-Ion Instability especially in long bunch trains. The asymptotic FBII growth rate of the oscillation amplitude of the n_bth bunch in the bunch train can be estimated at [1]

$$\tau_{e^-}^{-1}(n_b) = 6.7 \cdot \frac{N_b^{3/2} n_b^2 r_e r_p^{1/2} L_{\text{sep}}^{1/2} c}{\gamma \sigma_y^{3/2} (\sigma_x + \sigma_y)^{3/2} A^{1/2} \omega_\beta} \cdot \frac{p}{\text{mbar}}, \tag{1}$$

energy	E/GeV	35
hor. emittance	ϵ_x/m	$1.5 \cdot 10^{-11}$
vert. emittance	ϵ_y/m	$1.5 \cdot 10^{-11}$
bunch length	$\sigma_s/\mu\text{m}$	25
# of e^-/bunch	N_b	$6.25 \cdot 10^9$
# of bunches/train	n_b	10000
bunch spacing	L_{sep}/m	30.0
avg. hor. β-function	$\overline{\beta_x}/\text{m}$	200.0
avg. vert. β-function	$\overline{\beta_y}/\text{m}$	200.0

Table 1: Parameters of the TESLA FEL transfer line.

where r_e, r_p, and c are the classical electron radius, the classical proton radius, and the velocity of light, respectively, while $\omega_\beta = 1/\beta$.
Using the parameters given in Table 1, we get for a vacuum pressure $p = 1 \cdot 10^{-8}$ mbar nitrogen ($A = 28$) an asymptotic growth rate of $\tau_{e^-}^{-1}(n_b) = 2.9 \cdot 10^6\,\mathrm{sec}^{-1}$ for the $n_b = 100$th bunch in the train. As a comparison with the time a bunch needs to travel along the 12 km long beam line shows, this growth rate leads to untolerable oscillation amplitudes. On the other hand, Eq. (1) is based on a simple linear model which does not exhibit any saturation effects due to nonlinearities at large amplitudes. Therefore the required vacuum conditions can only be determined by simulations.

2 Simulation Method

For the simulation, it has been assumed that all bunches have a gaussian shape of fixed width in all three dimensions, i. e. longitudinal, horizontal, and vertical. To investigate possible head-tail oscillations, each bunch is cut into an odd number of slices, each of them containing the same number of electrons. Each of these slices is longitudinally represented by the position of its center of gravity. In the following, the term "bunch" instead of "slice" will be used unless otherwise indicated.
The interaction between an ion and a bunch can be treated in analogy to the beam-beam interaction. The change in transverse ion velocity, $\Delta v_{x,i}$, $\Delta v_{y,i}$, of the ith ion is given as

$$\Delta v_{y,i} + i\Delta v_{x,i} = -2N_b r_e c \frac{m_e}{M_i} f(x_i, y_i), \tag{2}$$

where $f(x, y)$ is given by the Bassetti-Erskine formula [2],

$$\begin{aligned} f(x,y) \quad = \quad & -\sqrt{\frac{\pi}{2(\sigma_x^2 - \sigma_y^2)}} \left[w\left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right. \\ & \left. - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) w\left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]. \end{aligned} \tag{3}$$

Here

$$w(z) = \exp(-z^2)[1 - \mathrm{erf}(-iz)], \tag{4}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, \mathrm{d}t, \tag{5}$$

denotes the complex error function, while N_b, r_e, c, m_e, and M_i are the number of electrons per bunch, the classical electron radius, the velocity of light, the electron rest mass and the ion rest mass, respectively. x_i, y_i are the transverse distances of the ion with respect to the bunch center.
Due to the usually very high number of ions created during a single bunch passage, the ion cloud is represented by macroparticles. While the expression for the transverse velocity change of an ion, Eq. (2), remains correct also for macroparticles, this macroparticle representation has to be taken into account for the calculation of the kick on the electron bunch from the reaction force,

$$\Delta y' + i\Delta x' = \frac{2N_b r_e}{\gamma} \sum_i N_i \cdot f(x_i, y_i). \tag{6}$$

In this equation γ denotes the relativistic factor of the electron beam, and N_i is the number of ions represented by the ith macroparticle.
In these simulation studies the number of ionization points equals the number of optics elements in the beam line. The number of ions created by a single bunch at each of these points is computed as

$$N_{\text{ions}} = N_b \cdot \sigma_{\text{ion}} \cdot n_{\text{mol}}, \tag{7}$$

where σ_{ion} denotes the ionization cross section, while n_{mol} is the area density of residual gas molecules. The latter is calculated from the vacuum pressure p and the length L_j of the respective accelerator element as

$$n_{\text{mol}} = L_j \cdot 6.023 \cdot 10^{26} \cdot \frac{p}{1.013\,\text{bar}}. \tag{8}$$

Since each instability requires some initial distortion to start with, the first bunch in the bunch train (resp. the first slice in the bunch) is transversally displaced by $\Delta x = 0.1\sigma_x$, $\Delta_y = 0.1\,\sigma_y$. All consecutive bunches (slices) start on the closed orbit. The entire bunch train is tracked along the beam line by means of linear optics.
At each ionization point each bunch creates N_{ions} ions according to Equation (7), having the same transverse gaussian distribution as the creating bunch. The N_{macro} macroparticles representing these ions are initially created exactly symmetrically with respect to the bunch center in order to avoid any artificial effects due to asymmetries. For this reason the number of macroparticles is an integer multiple of 4.
The Nth bunch (slice) interacts with $(N-1)\cdot N_{\text{ions}}$ previously created ions, i. e. $(N-1)\cdot N_{\text{macro}}$ macroparticles. This interaction changes the transverse velocity of these ions (macroparticles), Eq. (2), and by means of the reaction force also the transverse momenta of the bunch, Eq. (6). After the passage of the Nth bunch, the new ion coordinates x_{N+1}, y_{N+1} and velocities $v_{x,N+1}$, $v_{y,N+1}$ for all ions at the time of the passage of the $(N+1)$st bunch are calculated as

$$v_{x,N+1} = v_{x,N} + \Delta v_{x,N}, \tag{9}$$
$$v_{y,N+1} = v_{y,N} + \Delta v_{y,N}, \tag{10}$$
$$x_{N+1} = x_N + v_{x,N+1} \cdot \Delta t, \tag{11}$$
$$y_{N+1} = y_N + v_{y,N+1} \cdot \Delta t, \tag{12}$$

where $\Delta t = L_{\text{sep}}/c$ is the bunch spacing or the spacing between slices, respectively. The transverse kick $\Delta x'$, $\Delta y'$, on the bunch (slice) is calculated according to Eq. (6).

3 Results

The TESLA FEL transfer line consists of a 12 km long 80-cell FODO structure, using permanent magnets of identical strength [3]. It is designed for an energy of 35 GeV but provides sufficient flexibility to suit beam energies between 15 and 50 GeV. The betatron phase advance per FODO cell is 42° for a 35 GeV beam. For the simulations a 35 GeV beam has been used.
The residual gas has been assumed as nitrogen or CO_2, with $A = 28$, therefore $M_i = 28 \cdot 938\,\text{MeV}$. The ionization cross section is $\sigma_{\text{ion}} = 2\,\text{Mbarn}$ [4]. The ion

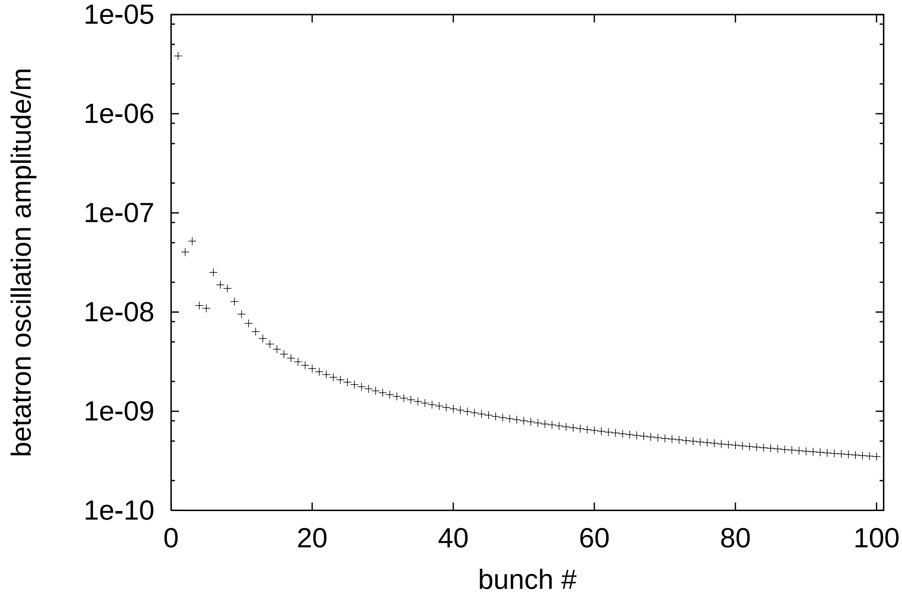

Figure 1: Vertical betatron oscillation amplitudes of 100 bunches at the end of the FEL transfer line for a vacuum pressure of $1 \cdot 10^{-6}$ mbar. The vertical beam size is indicated by the oscillation amplitude of the first bunch.

cloud produced by a single bunch (slice) per ionization point is represented by 52 macroparticles. Figure 1 shows the vertical betatron oscillation amplitude of all 100 bunches at the end of the transfer line. Since the emittances as well as the optics are equal in both transverse directions, the horizontal oscillation amplitudes are practically the same. All oscillation amplitudes are far below the beam size and therefore tolerable.

If the vacuum pressure is increased by one order of magnitude to $p = 1 \cdot 10^{-5}$ mbar the oscillation amplitudes of the first few bunches reach about $0.3\sigma_y$, while for the rest of the bunch train the oscillation amplitudes are much smaller, see Figure 2.

An even higher limit on the required vacuum pressure can be set from the investigation of possible head-tail oscillations. For this purpose a single bunch was longitudinally cut into 5 slices. The first slice was horizontally and vertically displaced by $\Delta x = 0.1\sigma_x$, $\Delta y = 0.1\sigma_y$. This leads to an almost linear increase of the betatron oscillation amplitudes of the following slices along the beamline. Due to the short bunch length of only $25\,\mu$m the ions practically do not change their transverse position between slices. Therefore only the ions created by the first, displaced slice lead to this amplitude growth, while ions created by other slices do not contribute. The resulting oscillation amplitudes of all slices but the first one are therefore identical. At the end of the FEL transfer line the oscillation amplitude of the trailing bunches correspond to 0.05σ for the $p = 1 \cdot 10^{-5}$ mbar case and 0.005σ for $p = 1 \cdot 10^{-6}$ mbar.

If we start with a "banana" shaped bunch with the initial displacement of the ith slice being

$$\Delta x_i \quad = \quad a_0 \cdot \sigma_x \cdot \left[1 - \cos\left(\frac{2\pi \cdot s_i}{8 \cdot l_{\mathrm{bunch}}}\right)\right], \tag{13}$$

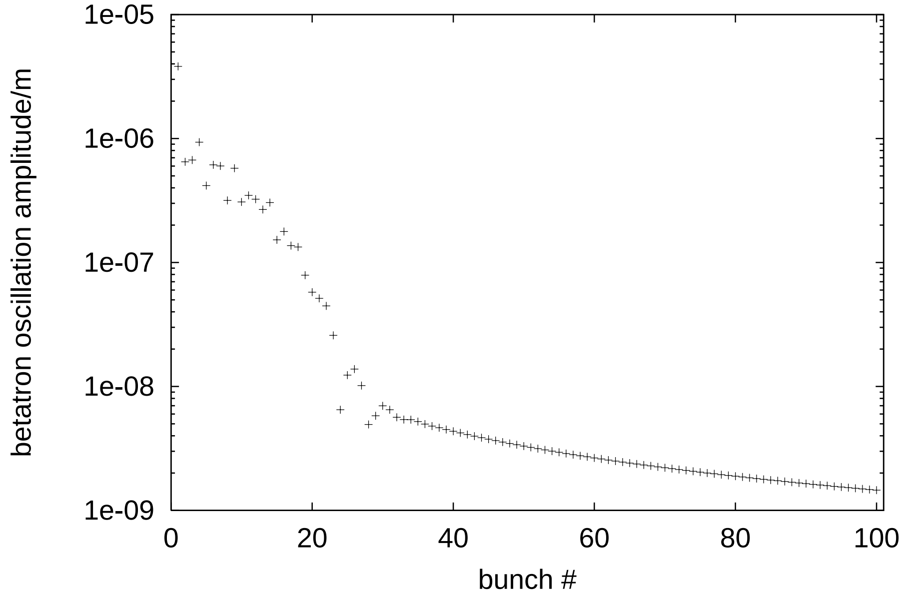

Figure 2: Vertical betatron oscillation amplitudes of 100 bunches at the end of the FEL transfer line for a vacuum pressure of $1 \cdot 10^{-5}$ mbar. The vertical beam size is indicated by the oscillation amplitude of the first bunch.

$$\Delta x_i \quad = \quad a_0 \cdot \sigma_x \cdot \left[1 - \cos\left(\frac{2\pi \cdot s_i}{8 \cdot l_{\text{bunch}}}\right)\right], \tag{14}$$

where $l_{\text{bunch}} = s_5 - s_1$ and s_i are the bunch length and the longitudinal position of the slice with respect to the head of the bunch, respectively, practically no change in the oscillation amplitude occurs along the transfer line for a_0 in the range $0.1 \ldots 0.5$.

4 Conclusion

According to the simulations presented in this paper, even a vacuum pressure as high as $1 \cdot 10^{-6}$ mbar would be sufficient to avoid the Fast Beam-Ion Instability in the TESLA FEL transfer line. The pressure in this beamline is expected to be lower by at least two orders of magnitude.

5 Acknowledgements

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