Simulation of the Fast Beam-Ion Instability in the TESLA Electron Damping Ring

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Abstract

The Fast Beam-Ion Instability is considered potentially harmful in electron storage rings with short bunch spacing and high bunch charge, as it is the case in the proposed electron damping ring of the future linear collider TESLA. It arises from interaction between a stored bunch and an ion cloud previously created by all heading bunches during a single pass. To study this effect and to determine the required vacuum conditions, a simulation code has been developed. These simulation studies are presented in this paper.

1 Introduction

The Fast Beam-Ion Instability (FBII) [1] is caused by ionization of the residual gas during a single pass of the electron bunch train. At each longitudinal position in the machine, the n_bth bunch interacts with the ion cloud produced by the $n_b - 1$ bunches ahead. If this ion cloud is not perfectly transversally symmetric with respect to the center of the n_bth bunch, the interaction between bunch and ions will result in a transverse kick of both the bunch and the ion cloud.
Since in a real accelerator the bunches are always slightly displaced with respect to each other due to effects such as transverse wakefields, this initial distortion can result in large bunch oscillation amplitudes if the coupling between bunches due to the ion cloud is strong enough to counteract the damping due to synchrotron radiation or transverse feedback systems.
The number of ions created during a single bunch passage is proportional to the bunch charge as long as the number of molecules in the beam path is large compared to the number of ions created by the beam, which is usually the case. After an ion is created, it drifts freely in the gap between bunches, leading to a transverse blow-up of the ion cloud. Therefore, the larger the distance between bunches, the weaker the bunch-to-bunch coupling. For this reason, the Fast Beam-Ion Instability is expected to be very harmful in high-current, low-emittance machines with a large number of bunches, such as B-factories or linear collider damping rings, while it has never been observed for example in the HERA electron ring. Parameters of the TESLA electron damping ring as well as the HERA electron accelerator are given in Table 1.

The asymptotic growth rate of the Fast Beam-Ion Instability of the n_bth bunch

		HERA-e	TESLA DR
energy	E/GeV	27.5	5.0
damping time	τ/msec	13???	28
hor. emittance	ϵ_x/m	$22 \cdot 10^{-9}$	$1 \cdot 10^{-9}$
vert. emittance	ϵ_y/m	$4 \cdot 10^{-9}$	$2 \cdot 10^{-12}$
# of e^-/bunch	N_b	$4.2 \cdot 10^{10}$	$2 \cdot 10^{10}$
bunch spacing	L_{sep}/m	28.8	6.0
avg. hor. β-function	$\overline{\beta_x}/\mathrm{m}$	20.0	38.6
avg. vert. β-function	$\overline{\beta_y}/\mathrm{m}$	20.0	67.6

Table 1: Parameters of the HERA electron ring and the TESLA damping ring.

can be estimated at [1]

$$\tau_{e^-}^{-1}(n_b) = 6.7 \cdot \frac{N_b^{3/2} n_b^2 r_e r_p^{1/2} L_{\mathrm{sep}}^{1/2} c}{\gamma \sigma_y^{3/2} (\sigma_x + \sigma_y)^{3/2} A^{1/2} \omega_\beta} \cdot \frac{p}{\mathrm{mbar}}, \tag{1}$$

where r_e, r_p, and c are the classical electron radius, the classical proton radius, and the velocity of light, respectively, while $\omega_\beta = 1/\overline{\beta}$. A denotes the molecular mass number of the residual gas. In the case of nitrogen or CO_2, $A = 28$.
Using the parameters given in Table 1 and a vacuum pressure $p = 1 \cdot 10^{-8}$ mbar, we get for the tenth bunch in the bunch train an asymptotic growth rate of $\tau_{e^-}^{-1} = 5.3 \cdot 10^5\,\mathrm{sec}^{-1}$ in the case of the TESLA damping ring, while the corresponding number for HERA is $\tau_{e^-}^{-1} = 122\,\mathrm{sec}^{-1}$. Due to this small growth rate the FBII has never been observed in HERA.
To study the effect of the Fast Beam-Ion Instability in the TESLA electron damping ring in more detail, a simulation code has been developed. This paper describes the simulation method and results, leading to an estimate of the vacuum pressure required to avoid beam blow-up.

2 Simulation Method

The electron bunch is assumed to have a transverse gaussian charge distribution of fixed width. Therefore the interaction between bunch and ions is similar to the beam-beam interaction. Thus the change in transverse ion velocity, $\Delta v_{x,i}$, $\Delta v_{y,i}$, of the ith ion is given as

$$\Delta v_{y,i} + i \Delta v_{x,i} = -2 N_b r_e c \frac{m_e}{M_i} f(x_i, y_i), \tag{2}$$

where $f(x, y)$ is given by the Bassetti-Erskine formula [2],

$$\begin{aligned} f(x,y) &= -\sqrt{\frac{\pi}{2(\sigma_x^2 - \sigma_y^2)}} \left[w\left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right. \\ & \left. \quad - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) w\left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]. \end{aligned} \tag{3}$$

Here

$$\begin{aligned} w(z) &= \exp(-z^2)[1-\mathrm{erf}(-iz)], &(4)\\ \mathrm{erf}(x) &= \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\,\mathrm{d}t, &(5)\end{aligned}$$

denotes the complex error function, while N_b, r_e, c, m_e, and M_i are the number of electrons per bunch, the classical electron radius, the velocity of light, the electron rest mass and the ion rest mass, respectively. x_i, y_i are the transverse distances of the ion with respect to the bunch center.

Since the number of ions created per bunch is usually very high, the electron cloud is represented by macroparticles. While the expression for the transverse velocity change of an ion (Eq. (2)) remains correct also for macroparticles, this macroparticle representation has to be taken into account when calculating the kick on the electron bunch from the reaction force,

$$\Delta y' + i\Delta x' = \frac{2N_b r_e}{\gamma}\sum_i N_i \cdot f(x_i, y_i). \tag{6}$$

Here γ is the relativistic factor of the electrons, while N_i is the number of ions represented by the ith macroparticle.

The number of ionization points around the ring is chosen to be the same as the number of optical elements of the accelerator lattice. The number of ions created per single bunch at each of these points is computed as

$$N_{\mathrm{ions}} = N_b \cdot \sigma_{\mathrm{ion}} \cdot n_{\mathrm{mol}}. \tag{7}$$

Here σ_{ion} denotes the ionization cross section, while n_{mol} is the area density of residual gas molecules. The latter is calculated from the vacuum pressure p and the length L_j of the element under consideration as

$$n_{\mathrm{mol}} = L_j \cdot 6.023 \cdot 10^{26} \cdot \frac{p}{1.013\,\mathrm{bar}}. \tag{8}$$

Initially the first bunch is transversally displaced by $\Delta x = 0.1\sigma_x$, $\Delta y = 0.1\sigma_y$, while all consecutive bunches start on the closed orbit. The entire bunch train is tracked around the machine from one ionization point to the next using linear optics.

At each ionization point each bunch creates N_{ions} ions according to Eq. (7), which are represented by N_{macro} macroparticles. These ions (macroparticles) initially have the same transverse distribution as the creating bunch. To avoid additional effects from any asymmetries of the initial macroparticle distribution, the initial coordinates of the macroparticles are generated such that they result in a symmetric distribution with respect to the center of the creating bunch in both x and y. Therefore the number of macroparticles is an integer multiple of 4.

The Nth bunch interacts with $(N-1)\cdot N_{\mathrm{ions}}$ previously created ions, i. e. $(N-1)\cdot N_{\mathrm{macro}}$ macroparticles. This changes the transverse velocity of these ions (macroparticles), Eq. (2), and also the transverse momenta of the bunch, Eq. (6). After the passage of the Nth bunch, the new ion coordinates x_{N+1}, y_{N+1} and velocities $v_{x,N+1}$, $v_{y,N+1}$ for all ions at the time of the passage of the $(N+1)$st bunch are calculated as

$$
\begin{aligned}
v_{x,N+1} &= v_{x,N} + \Delta v_{x,N}, & (9)\\
v_{y,N+1} &= v_{y,N} + \Delta v_{y,N}, & (10)\\
x_{N+1} &= x_N + v_{x,N+1} \cdot \Delta t, & (11)\\
y_{N+1} &= y_N + v_{y,N+1} \cdot \Delta t, & (12)
\end{aligned}
$$

where $\Delta t = L_{\mathrm{sep}}/c$ is the bunch spacing.
The transverse kick on the electron bunch is calculated according to Eq. (6), and the bunch is tracked to the next ionization point.

3 Results

The TESLA electron damping ring basically consists of two arc sections with an average bending radius of 145 m and two long straight sections of 7.5 km length each, leading to the so-called "dog-bone" design [4]. Each arc is composed of 60 TME cells, while each straight section consists of 70 identical FODO cells. Figure 1 shows the optics of these two cells [5].

For the simulation, an ionization cross section of $\sigma_{\mathrm{ion}} = 2\,\mathrm{Mbarn}$ [3] is assumed for nitrogen gas with molecular mass number $A_{N_2} = 28$, thus $M_i = 28 \cdot 938\,\mathrm{MeV}/c^2$. At each ionization point in the ring the ion cloud produced by a single bunch is represented by 52 macroparticles. To limit the total number of macroparticles to be handled, only 20 bunches were considered. Figure 2 shows the evolution of the amplitude of the 5th, the 10th , the 15th and the 20th bunch in the train for a pressure of $1 \cdot 10^{-8}$ mbar. For a vacuum pressure of $1 \cdot 10^{-8}$ mbar the oscillation amplitude of the 5th bunch equals the vertical beam size after one damping time (500 revolutions), while it is even larger for consecutive bunches. Due to this fast growth of the Fast Beam-Ion Instability a vacuum pressure of $1 \cdot 10^{-8}$ mbar is not sufficient

At a reduced pressure of $1 \cdot 10^{-9}$ mbar the transverse oscillation amplitude of the 5th bunch is still smaller than one tenth of the vertical beam size, see Figure 3. After one transverse damping time, i. e. 500 turns, the oscillation amplitude of the 10th bunch is still smaller than the oscillation amplitude of the 5th bunch. Therefore even for a longer bunch train the oscillation amplitude of bunches in the tail of the train is expected to be tolerable.

4 Conclusion

As the simulation studies presented in this paper indicate, a vacuum pressure of $1 \cdot 10^{-9}$ mbar should be sufficient to keep the rise time of the Fast Beam-Ion Instability well above the damping time of the TESLA electron damping ring. Though only 20 bunches have been tracked to limit the CPU time to reasonable values, it is expected that even for longer bunch trains the rise time should be sufficiently large.

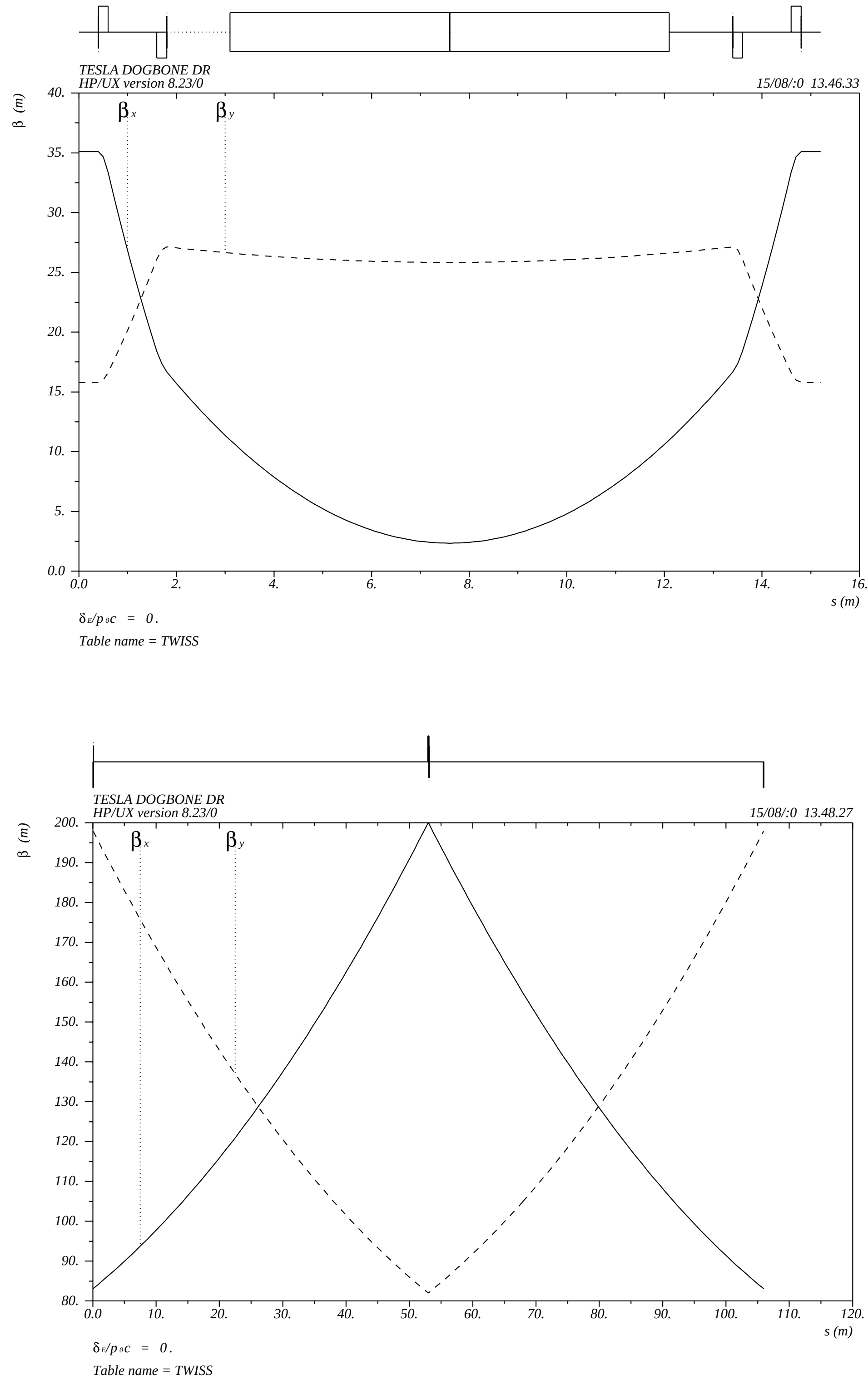

Figure 1: Optics of one TME cell in the arc (top) and one FODO cell in the straight section (bottom) of the TESLA electron damping ring.

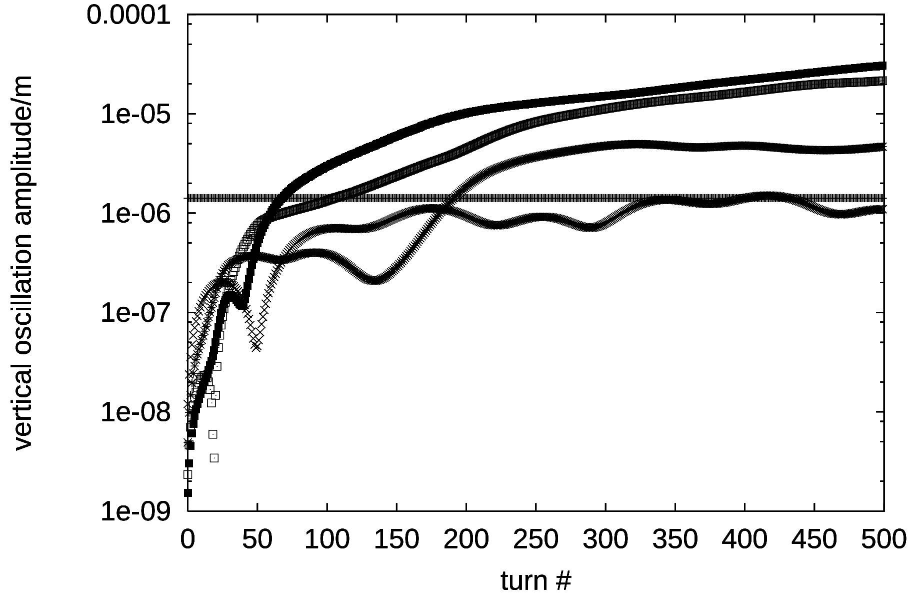

Figure 2: Oscillation amplitude of (from bottom to top) the 5th, the 10th , the 15th and the 20th bunch in the bunch train for a vacuum pressure of $1 \cdot 10^{-8}$ mbar. The horizontal straight line corresponds to the vertical beam size.

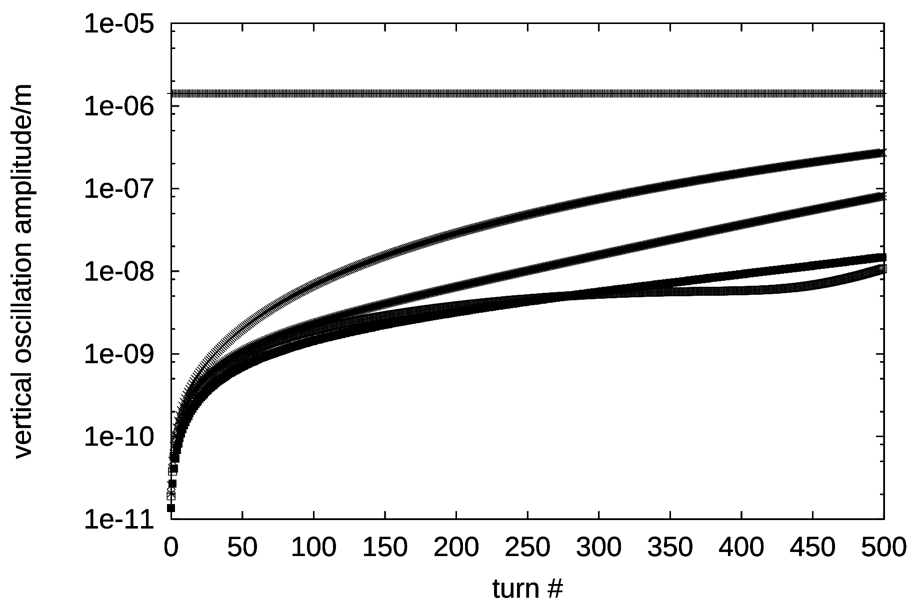

Figure 3: Oscillation amplitude of (from top to bottom) the 5th, the 10th, the 20th, and the 15th bunch in the bunch train for a vacuum pressure of $1 \cdot 10^{-9}$ mbar. The horizontal straight line corresponds to the vertical beam size.

5 Acknowledgements

I would like to thank W. Decking and R. Brinkmann for stimulating discussions and for reading the manuscript.

References

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