Terahertz Wakefields in the Superconducting Cavities of the TESLA-FEL Linac

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1 Introduction

The operation of a Free Electron Laser (FEL) in the vacuum ultraviolet or X-ray regime requires the acceleration of electron bunches with an rms length of 25 to 50 μm. The wake fields generated by these subpicosecond bunches extend into a frequency range well beyond the threshold for Cooper pair breakup (about 750 GHz) in superconducting niobium. For this reason there has been considerable concern whether the acceleration of such short bunches would at all be possible using the 9-cell TESLA superconducting cavities. A first approach to the problem ([1], [2], [3]) in which the wake field generation was based on a diffraction model, led to the result that the superconducting cavities could indeed be operated with 25 μm bunches without suffering a breakdown of superconductivity (quench), however at the price of a reduced quality factor and an increased heat transfer to the superfluid helium bath. There has been a debate on the applicability of the diffraction model and in particular on the question whether the ultra high frequency wake fields inside the superconducting structure can be considered as a gas of independent photons undergoing absorption or reflection at the vacuum-superconductor interface. In the meantime the more conventional method of computing wake fields in the time domain by numerical methods has been extended into the very short bunch regime and has been applied for single cavities as well as a sequence of multicell cavities. As will be shown below, both methods lead to comparable results in case of a single multicell cavity but the second approach predicts moreover a significant reduction of wake field losses in a long string of cavities.

2 Diffraction model calculation

Following Bane and Sands ([4]) we consider a highly relativistic particle bunch travelling along the axis of a perfectly conducting beam pipe of radius a and passing through a pill-box cavity of gap length g. The electric field accompanying the bunch can be taken as radial. Its value at the pipe wall at $r = a$ is given in terms of the bunch charge q by

$$E(a) = \frac{q}{2\pi\varepsilon_0 a} \tag{1}$$

while the magnetic field is azimuthal and of magnitude $B(a) = E(a)/c$. For radiation in the THz regime the wavelength is much shorter than the pipe radius, hence close to the wall the electromagnetic field of the bunch can be replaced locally by a plane wave. This wave is diffracted at the entrance aperture of the cavity which we approximate as a straight edge. The outward diffracted wave will then hit the cavity wall close to the exit aperture. For a monochromatic wave of intensity $I_0 = I_0(\omega)$, the intensity distribution in a distance g from the aperture is given by [5]

$$I(y) = \frac{I_0}{2}\left[(C(u) - 1/2)^2 + (S(u) - 1/2)^2\right] \quad \text{with } u = \sqrt{\frac{\omega}{\pi c g}}\, y\,. \tag{2}$$

where y is the transverse distance y from the edge of the geometric boundary and $y > 0$ means the shadow region, $C(u)$ and $S(u)$ are the Fresnel integrals. At $y = 0$ one gets $I = I_0/4$ and $I(y)$ drops smoothly to zero with increasing y. To compute the radiation power hitting the cavity wall near the exit aperture one has to integrate formula (1) from zero to infinity. Using the relation

$$\int_0^\infty \left[(C(u) - 1/2)^2 + (S(u) - 1/2)^2\right] du = 1/(2\pi) \tag{3}$$

one obtains for the power scattered into the shadow region

$$P = I_0 \pi a \sqrt{\frac{cg}{\pi\omega}}\,. \tag{4}$$

The field of a short bunch has a wide frequency spectrum. The longitudinal charge distribution in the bunch is taken to be a Gaussian of variance $\sigma_z = c\sigma_t$

$$\rho(z - ct) = \frac{q}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{(z - ct)^2}{2\sigma_z^2}\right)\,. \tag{5}$$

The Fourier transform is

$$\tilde{\rho}(\omega) = \frac{q}{c} \exp\left(-\frac{\omega^2}{2\,\omega_c^2}\right) \exp(-i\omega z/c) \tag{6}$$

with the "cut-off" frequency $\omega_c = 1/\sigma_t$. Then

$$I_0(\omega) = \frac{1}{\mu_0 c}\left|\frac{\tilde{\rho}(\omega)}{2\pi\varepsilon_0 a}\right|^2$$

and the energy per frequency interval diffracted into the shadow region becomes

$$U(\omega) = \frac{q^2\sqrt{cg}}{4\pi^{5/2}a\varepsilon_0 c}\frac{\exp(-\frac{\omega^2}{\omega_c^2})}{\sqrt{\omega}} \; . \tag{7}$$

The total energy per bunch diffracted onto the cavity wall is obtained by integration over all frequencies, making use of the relation

$$\int_0^\infty \exp(-u^2)/\sqrt{u}du = \Gamma(1/4)/2 = 1.815 \; .$$

The energy deposited in the pill-box cavity by a single bunch is therefore

$$W_1 = \int_0^\infty U(\omega)d\omega = \frac{1.82q^2}{4\pi^{5/2}a\varepsilon_0 c}\sqrt{gc/\sigma_t} \; . \tag{8}$$

In the context of the diffraction model the nine-cell TESLA cavity is simply treated as a structure with nine apertures at which independently the diffraction takes place. In FEL operation there will be $N_b = 11315$ bunches per 800 μs long macropulse with a bunch charge of $q = 1\,\mathrm{nC}$ and $\sigma_z = 25\,\mu\mathrm{m}$. The macropulse repetition rate is $f_{rep} = 5\,\mathrm{Hz}$. Hence the time-averaged power scattered into the cavity is

$$P_{tot} = N_b \cdot W_1 \cdot f_{rep} = 2.91 \text{ W} \; . \tag{9}$$

According to Babinet's Theorem, the same amount of power is diffracted into the illuminated region behind the diffracting aperture. This implies that 2.91 W of wake field power travel along the beam pipe out of the 9-cell cavity. This radiation will eventually catch up with the relativistic electrons and reduce their energy.

For an rms bunch length $\sigma_z = 25\,\mu\mathrm{m}$ the "cut off" frequency is $f_c = \omega_c/2\pi = 1.9$ THz. By integration of Eq. (7) one finds that 32% of the intensity is above 750 GHz, the threshold frequency for Cooper pair breakup in niobium. Most of the diffracted radiation hits the next iris of the multicell cavity in a narrow, ring-shaped region close to the smallest diameter. The radiation pulse has a time duration of less than a picosecond and hence the power density of the radiation impinging on the ring region at the next iris is so high that one could easily image an immediate breakdown of superconductivity in that region. Fortunately, this is not the case since most of the radiation is not absorbed but reflected by the superconductor.

To compute the energy absorption in high frequency fields it is convenient to describe the superconductor by its frequency dependent surface resistance. The surface resistance of the high-purity niobium used in the TESLA cavities has been computed within the Eliashberg model. An outline of this theoretical approach is given in the appendix. For an operating temperature of 2 K the surface resistance is plotted in Fig. 1 as a function of frequency. Starting from a value of about 20 nΩ at the fundamental mode frequency of 1.3 GHz the resistance rises to a few $\mu\Omega$ at 600 GHz and

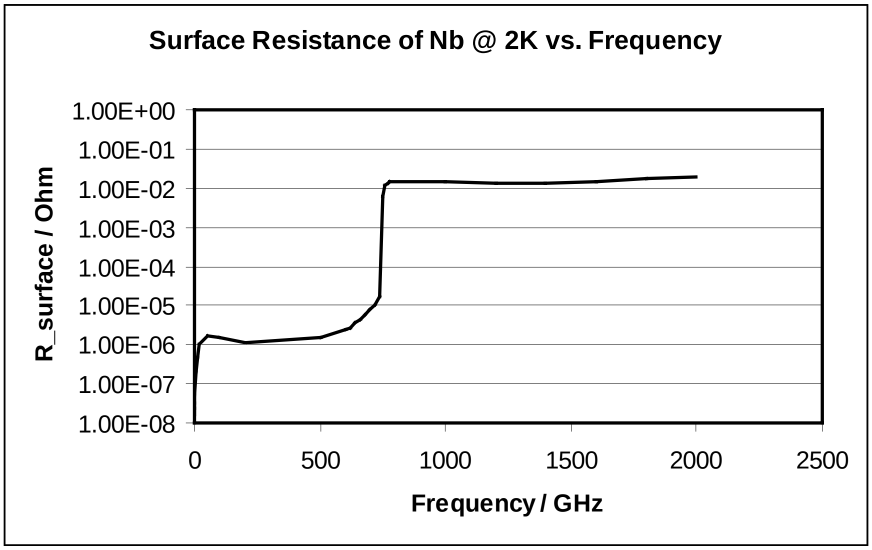

Figure 1: Surface resistance of Nb at 2 K as a function of frequency.

then exhibits a large step at 750 GHz to a value of 15 mΩ. The surface resistance is related to the reflectivity r of the metal by

$$R_s = 0.25 Z_0 (1 - r) \; . \tag{10}$$

where $Z_0 = 1/\varepsilon_0 c = 377\,\Omega$ is the characteristic impedance of the vacuum.

For frequencies below the Cooper pair threshold of 750 GHz the reflectivity is almost indistinguishable from unity but even above 750 GHz the reflectivity is more than 99.98%. This implies that the potentially dangerous THz radiation undergoes thousands of reflections before being absorbed by the niobium. The time of flight between two reflections is in the order of a nanosecond, hence the original picosecond radiation pulse is stretched in time by many orders of magnitude before absorption takes place. Also the spatial distribution is greatly altered by the reflections.

To study the process of multiple reflections in the 9-cell cavity we resort to the photon picture which appears justified since the wavelength of Terahertz radiation is much smaller than the cavity dimension. Figure 2 shows 12 successive reflections of two photons starting at the same point of the first aperture but with slightly different angles with respect to the beam axis. From the totally different behaviour of the resulting trajectories it is evident that chaotic motion is present which is of course no surprise for elastic scattering at a boundary of elliptical shape. Hülsmann and Klein [3] have carried out a Monte Carlo simulation of many thousand reflections which underline the results already visible in Fig. 2: almost every point on the cavity surface is hit by the photons, and there is a high chance that the photons remain trapped in the 9-cell structure until they are absorbed. The large number of reflections and their chaotic nature have the beneficial effect that the radiation power which eventually has to be absorbed by the niobium is distributed over the whole cavity surface and smeared out in time by many orders of magnitude.

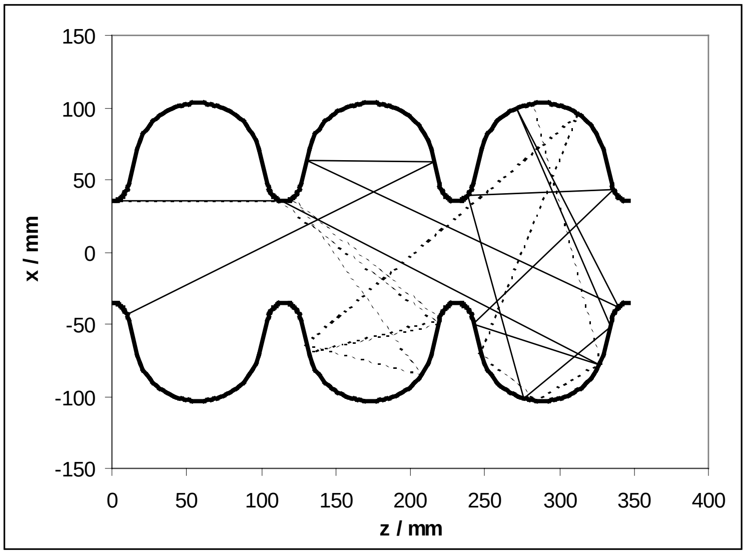

Figure 2: Simulation of two photons scattering in part of a TESLA cavity (full and dashed lines, respectively). They start from the same point on the left-most iris, but with slightly different initial angle.

Wake field radiation below 750 GHz can undergo millions of reflections before being absorbed in the superconductor. Intuitively, trapping in the multicell structure looks much less likely because the radiation will have a high chance of escaping into the beam pipe. Suitable absorbing materials in the beam pipe sections between the cavities should therefore be rather effective in removing this radiation [6].

To get an estimate on the wake field load in a TESLA cavity we therefore make the simplifying assumption that all wake field intensity above 750 GHz is absorbed in the cavity while all radiation below 750 GHz eventually leaves the cavity and is absorbed by a suitable material in the beam pipe sections. The power absorbed in the 9-cell cavity is taken to be uniformly distributed over the whole inner surface of 0.8 m^2. According to eq. (9), $0.3 \cdot 2.9 \approx 0.9\,\mathrm{W}$ of average and 200 W of instantaneous power are dissipated in the inner niobium surface of the cavity. An important question is how much the temperature of the inner cavity surface rises and how fast the temperature rise occurs. First we consider the stationary case with an incident power density of $\Phi = 200/0.8 = 250\,\mathrm{W/m^2}$. The wall thickness is 2.5 mm and the heat conductivity of our high-purity niobium at 2 K amounts to $\approx 7\,\mathrm{W/m^2K}$ [7], hence the temperature rise is about 0.19 K. Solving the time-dependent heat equation one finds that the stationary state is achieved in about $100\,\mu$s, see Fig. 3, so stationary heat conduction applies for most of the $800\,\mu$s long macropulse and the inner surface assumes a temperature of 2.19 K for cooling with superfluid helium of 2 K. If the initial quality factor at 2 K is $Q_0 = 10^{10}$ then Q_0 drops to $\approx 9 \cdot 10^9$ at 2.19 K. The consequence is an increased surface heating by the fundamental 1.3 GHz mode. Taking that into account we finally arrive at an even lower effective Q_0 of about $7 \cdot 10^9$.

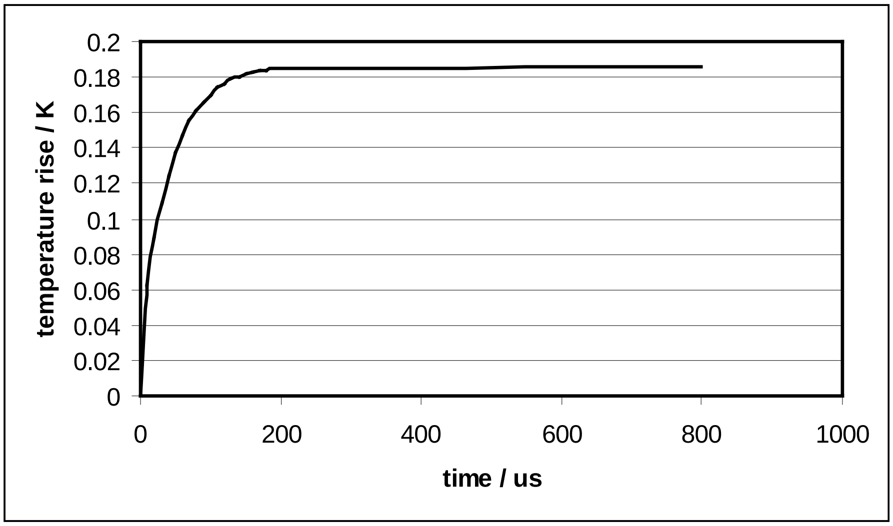

Figure 3: Calculated temperature rise at the inner surface of the cavities during the $800\,\mu$s long FEL beam pulse.

In summary, the TESLA cavities are not quenched by the wake fields of ultrashort bunches, but there is a significant increase in the heat load on the cryogenic system. We will show in the next chapter that a refined treatment of the wakefields in a string of multicell cavities leads to much lower loss factors.

3 Numerical calculation of the loss factor

In case of a periodic array of resonators, the induced wake fields can not be simply calculated as the sum of the single cavity contributions, because the field traveling with the bunch is modified due to the presence of the upstream discontinuities. It can be shown that in an infinite periodic system the steady state longitudinal loss factor becomes independent of bunch length and the high-frequency part of the impedance ($Z_{periodic}(\omega) \propto \omega^{-3/2}$) drops much faster than for a single cavity ($Z_{single} \propto \omega^{-1/2}$) [8, 9]. One can apply simple geometric arguments (see [8]) in the framework of the diffraction model to estimate the steady-state loss factor in a periodic array of cavities. This has been done for the TESLA accelerator [10], yielding a reduction of the loss factor for short (25μm) bunches by a factor of six with respect to the single-cavity estimate. To which extent the transition to the periodic regime actually applies in the TESLA linac can more accurately be answered by numerical calculations. For short bunches the interaction length of the inward deflected wake traveling with the bunch can become very large (up to tens of meters). Wakefield calculations in the time domain have therefore been performed for up to 2 entire TESLA accelerator modules, each about 12 m long and containing eight 9-cell resonators, bellows and beam pipes [11]. The minimum possible bunch length for reasonable numerical effort is $\sigma_z = 50\mu$m. For this case, the results show still a significant difference between the wakes calculated for the first and the second module, indicating that the transition length to the steady state exceeds one module length.

In Fig. 4 the calculated loss factor per module (results for second module in a string of two) as a function of bunch length is shown. One can clearly recognize that the dependence on σ_z is much weaker than $\propto 1/\sqrt{\sigma_z}$, in contrast to the numerical result for a single-cell calculation, Fig. 5, so that the condition of an infinite periodic structure seems to be relatively well fulfilled for the TESLA linac. From the numerical results an analytical approximation to the (point charge) wake potential per module has been derived [11]:

$$w(s) = 315 \cdot \frac{V}{pC} \cdot \left(1.165 \cdot \exp(-\sqrt{s/3.65mm}) - 0.165\right) \tag{11}$$

Fourier transformation of the wake potential yields the impedance $Z(\omega)$. From $Z(\omega)$ we obtain the relevant high-frequency contribution to the loss factor, $k_{loss}(\omega_l)$, by integration together with the bunch spectrum and the lower integration boundary ω_l as a variable. The resulting $k_{loss}(\omega_l)$ is shown in Fig. 6 for the case of $\sigma_z = 25\mu$m. We find that the contribution to the loss factor above the Niobium gap frequency ($\omega_l = 2\pi \cdot 750\,$GHz) amounts to only about 6%, almost one order of magnitude less than the diffraction model estimate given in the previous section. The total loss factor, extrapolated to $\sigma_z = 25\mu$m from the numerical results (Fig. 4), amounts to 165 kV/nC per module and we get for the average wakefield power $P_{tot} = 1.17\,$W per 9-cell cavity, considerably less than the estimate given above (Eq. (9)). The power dissipated in the cavities at the 2 K level is then only about 0.07 W, on average or less than 2 W during the beam pulse, which does not lead to a significant increase of the Niobium surface temperature.

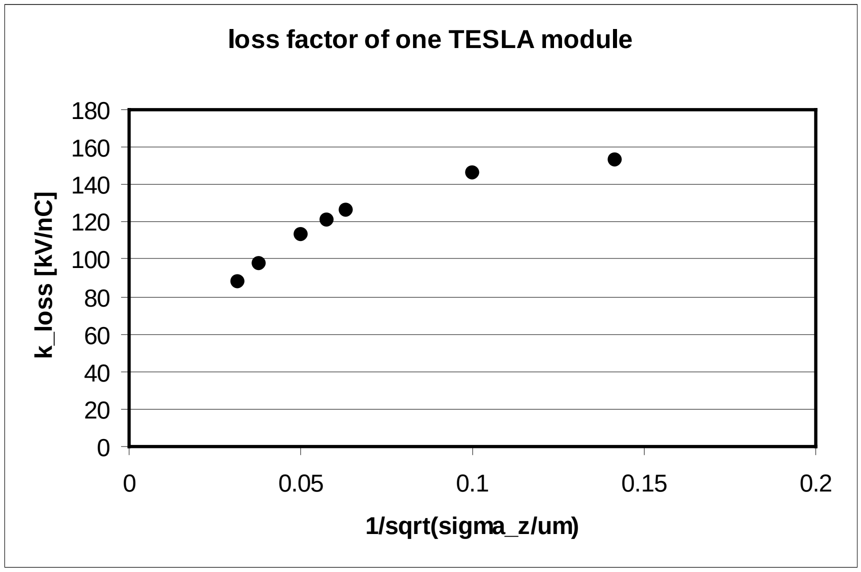

Figure 4: Calculated loss factor per TESLA module versus $1/\sqrt{\sigma_z}$ ($\sigma_z = 50...1000\mu$m).

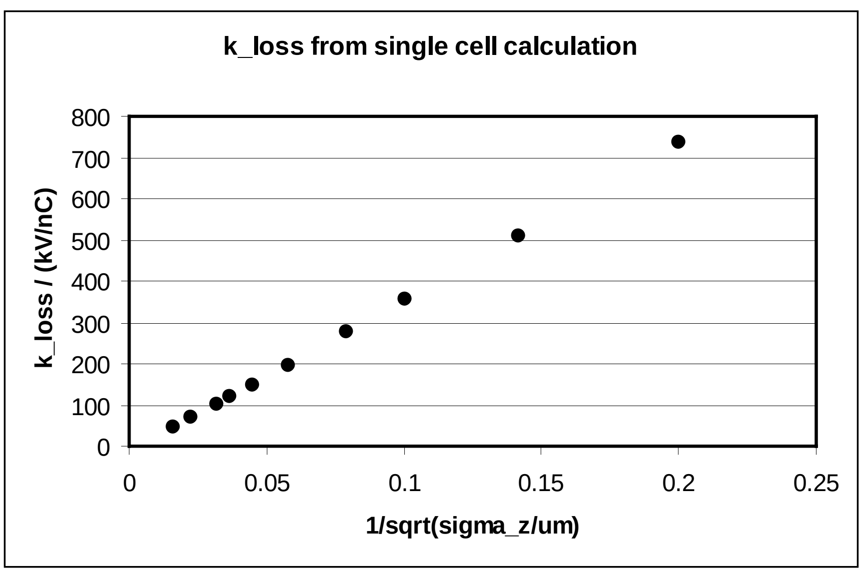

Figure 5: Calculated loss factor of 72 single cells versus $1/\sqrt{\sigma_z}$ ($\sigma_z = 25...4000\mu$m).

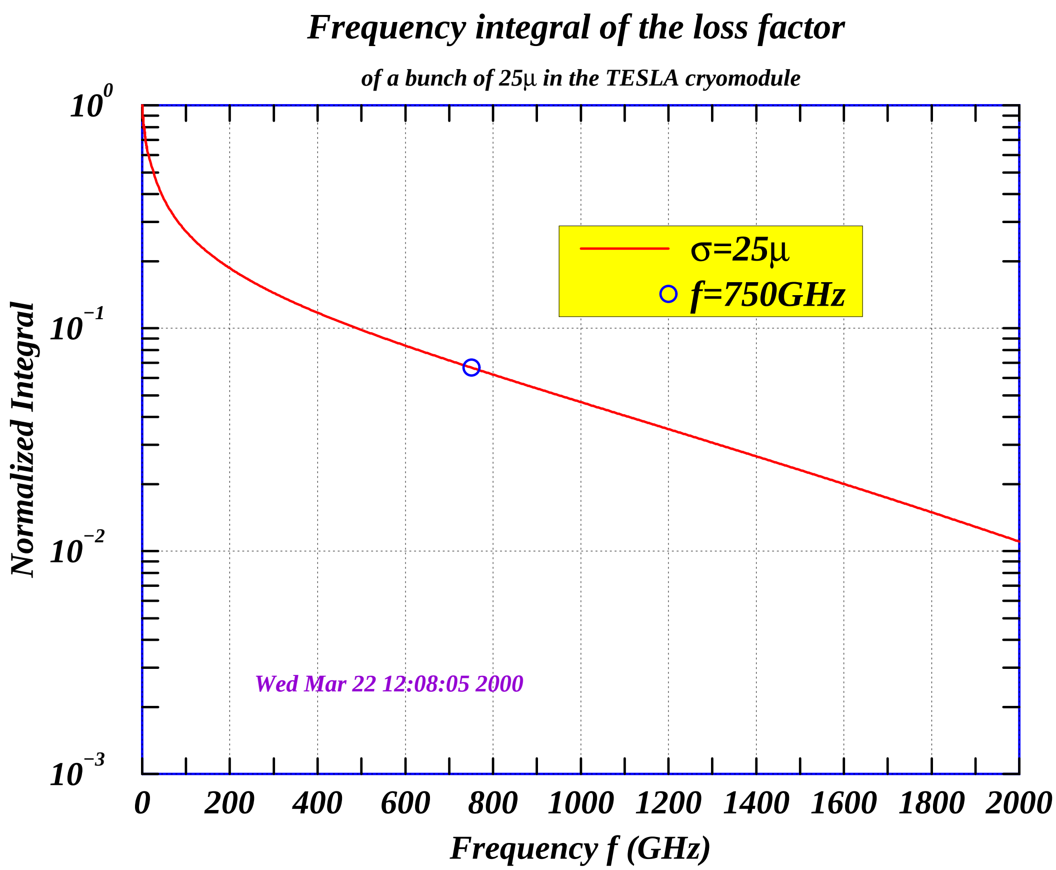

Figure 6: Fractional part of the loss factor above a lower frequency boundary.

4 Appendix

In this appendix we shall outline the calculation of the surface impedance in the strong coupling approximation. Strong coupling effects change the amplitude and the temperature dependence of the superconducting order parameter, they lead to a renormalization of the quasiparticle mass, which in turn affects the London penetration depth, and they result in temperature and energy dependent quasiparticle lifetimes. At sufficiently low temperatures and low frequencies of the incident radiation, scattering from inelastic processes is negligible and it is important to keep disorder induced elastic scattering, parametrized by a normal state scattering rate $\Gamma_N = 1/2\tau$. This quantity is not measurable directly. We deduce it from a fit of our calculations to the surface resistance measured at $\nu = 1.3\,\mathrm{GHz}$ on Niobium sheets similar to those used in the construction of our cavities. The result is $\Gamma_N \approx 1\,\mathrm{meV}$ which corresponds to a lifetime $\tau \approx 3 \cdot 10^{-13}\,\mathrm{sec}$. A value $\Gamma_N \approx 0.3\,\mathrm{meV}$ is deduced from the residual resistance ratios, which fall in the range 300 to 500. With $v_F \approx 3 \cdot 10^7\,\mathrm{cm/sec}$ for Niobium (see Blaschke *et al.* [14] for references to Nb material parameters), one obtains quasiparticle mean free paths in excess of the penetration depth. For R_s at least this would necessitate taking nonlocal effects into account [15]. These, however, are important only in the few GHz range and could affect our estimate of Γ_N, resolving the discrepancy between the two values just discussed.

Using linear response theory, the complex nonlocal transverse conductivity of a homogeneous system can be expressed in terms of a current-current correlation function:

$$\sigma(\vec{q},\omega) = \sigma_1(\vec{q},\omega) + i\sigma_2(\vec{q},\omega) = -\frac{e^2}{i\omega}\left\{\langle[j_x, j_x]\rangle(\vec{q},\omega) + \frac{n}{m}\right\} \tag{12}$$

In the local limit $\vec{q} = 0$, the surface impedance is simply given by

$$R_s(\omega) - i\omega\mu_0\lambda(\omega) = \sqrt{\frac{\omega\mu_0}{i\sigma_1(\omega) - \sigma_2(\omega)}}\,, \tag{13}$$

irrespective of the nature of the boundary scattering ($\lambda(\omega)$ denotes the frequency dependent penetration depth). Deep in the superconducting state and for frequencies below the gap frequency the approximation $\sigma_1(\omega) \ll \sigma_2(\omega)$ holds. In this case, R_s can be approximated as

$$R_s(\omega) = \frac{1}{2}\omega^2\mu_0^2\lambda^3(\omega)\sigma_1(\omega)\,. \tag{14}$$

This results shows very clearly, that a very high degree of purity does not necessarily lead to low losses: An increase in quasiparticle mean free path increases σ_1 but reduces the penetration depth λ.

We assume that Niobium can, for the present purpose, be described by the nearly free electron model which implies a single band with spherical Fermi surface. Any $\vec{k}$-dependence of the electron-phonon interaction, is assumed to be averaged out, and hence the superconducting energy gap will be isotropic. Impurities, providing a temperature-independent elastic scattering mechanism, are described by δ-function potentials, so that the resulting selfenergies are also momentum independent. With these assumptions, vertex corrections to the current-current correlation function vanish exactly, so that the conductivity can be expressed in terms of single particle Green's functions. With regard to superconductivity we make the additional assumption that pairing only involves electron states very close to the Fermi surface and that the density of states in the vicinity of the Fermi surface can be approximated by a constant. The integrations with respect to quasiparticle momenta can then be performed analytically and one is left with

$$
\begin{aligned}
\sigma(\omega) \;=\; \frac{e^2 N(0) v_F^2}{3\omega} \int\limits_{-\omega/2}^{+\infty} d\Omega \Bigg\{ & \tanh\frac{\Omega}{2T}\, M(\Omega_+ + \omega, \Omega_+) - \tanh\frac{\Omega+\omega}{2T}\, M(\Omega_- + \omega, \Omega_-) \\
& - \left[\tanh\frac{\Omega}{2T} - \tanh\frac{\Omega+\omega}{2T}\right] M(\Omega_+ + \omega, \Omega_-) \Bigg\} \qquad (15)
\end{aligned}
$$

where

$$
M(\Omega_\pm + \omega, \Omega_\pm) = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} d\varepsilon \mathrm{Tr}[\, \hat{G}(\varepsilon, \Omega_\pm + \omega))\, \hat{G}(\varepsilon, \Omega_\pm)] \qquad (16)
$$

$\pm$ denotes a positive (negative) infinitesimal imaginary part. With the help of some symmetry relations we have reduced the interval for the Ω-integral to $[-\omega/2, \infty)$. In a weak coupling theory [12], this integration is further restricted by the presence of an energy gap, which necessitates separate consideration of absorption by thermally excited quasiparticles and by pair breaking. In strong coupling, no energy gap exists except at $T = 0$ so that we numerically evaluate the formally much simpler integral (15). The rapid variation with temperature of the superconducting density of states does not cause numerical problems.

The single particle Green function $\hat{G}(\varepsilon, \Omega_\pm)$ is written as a 2×2 matrix in particle-hole space [13]. It's form is familiar from BCS-theory

$$
\hat{G}(\varepsilon, \Omega_\pm) = -\frac{\Omega Z(\Omega_\pm)\tau_o + \varepsilon\tau_3 + \phi(\Omega_\pm)\tau_1}{-\left(\Omega Z(\Omega_\pm)\right)^2 + \varepsilon^2 + \phi^2(\Omega_\pm)}\,, \qquad (17)
$$

which is a consequence of the mean field approximation. The τ_i are Pauli matrices. The strong coupling theory affects only the calculation of the complex normal and anomalous selfenergies Z and ϕ, for which the following selfconsistency equations

$$\begin{aligned}
\frac{Z(\omega)}{\eta(\omega)} &= 1+\frac{\pi T}{\omega}\sum_{\omega_{n'}}\left[\lambda(\omega-i\omega_{n'})\right]\,\frac{i\omega_{n'}Z(i\omega_{n'})}{\sqrt{\phi^2(i\omega_{n'})+[\omega_{n'}Z(i\omega_{n'})]^2}} \\
&\quad+\frac{\pi}{\omega}\int_0^\infty d\Omega\,\alpha^2F(\Omega)\left\{\frac{[N(\Omega)+f(\Omega-\omega)]\;(\omega-\Omega)Z(\omega-\Omega)}{\sqrt{\phi^2(\omega-\Omega)-[(\omega-\Omega)Z(\omega-\Omega)]^2}}\right. \\
&\quad\left.+\frac{[N(\Omega)+f(\Omega+\omega)]\;(\omega+\Omega)Z(\omega+\Omega)}{\sqrt{\phi^2(\omega+\Omega)-[(\omega+\Omega)Z(\omega+\Omega)]^2}}\right\}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\phi(\omega)}{\eta(\omega)} &= \pi T\sum_{\omega_{n'}}\left[\lambda(\omega-i\omega_{n'})-\mu^*(\omega_c)\Theta(\omega_c-|\omega_{n'}|)\right]\,\frac{\phi(i\omega_{n'})}{\sqrt{\phi^2(i\omega_{n'})+[\omega_{n'}Z(i\omega_{n'})]^2}} \\
&\quad+\pi\int_0^\infty d\Omega\,\alpha^2F(\Omega)\left\{\frac{[N(\Omega)+f(\Omega-\omega)]\;\phi(\omega-\Omega)}{\sqrt{\phi^2(\omega-\Omega)-[(\omega-\Omega)Z(\omega-\Omega)]^2}}\right. \\
&\quad\left.+\frac{[N(\Omega)+f(\Omega+\omega)]\;\phi(\omega+\Omega)}{\sqrt{\phi^2(\omega+\Omega)-[(\omega+\Omega)Z(\omega+\Omega)]^2}}\right\}
\end{aligned} \tag{19}$$

can be derived from the usual eqations of strong coupling theory [13] by performing one of the frequency integrals analytically. This mixed representation involving real and Matsubara frequencies has first been derived by Marsiglio *et al.* [16] and is found to give very satisfactory convergence of the iterative solution of these equations. We found this method more reliable than the calculation of the selfenergies on the imaginary frequency axis, followed by analytic continuation using Padé approximants [14].
The quantity λ characterizing the electron-phonon interaction is defined as usual [13]

$$\lambda(i\omega_n-i\omega_{n'})=\int_0^\infty d\Omega\,\alpha^2F(\Omega)\,\frac{2\Omega}{\Omega^2-(i\omega_n-i\omega_{n'})^2}\ , \tag{20}$$

and N and f are the Bose and Fermi functions

$$N(\Omega)+f(\Omega\pm\omega)=\frac{1}{2}\left[\coth\frac{\Omega}{2T}-\tanh\frac{\Omega\pm\omega}{2T}\right]\ . \tag{21}$$

The effect of impurity scattering is contained in

$$\frac{1}{\eta(\omega)}=1-\frac{\Gamma_N}{\sqrt{\phi^2(\omega)-[\omega Z(\omega)]^2}}\ . \tag{22}$$

Clearly, the quantities

$$\tilde{Z}=Z/\eta \qquad \text{and} \qquad \tilde{\phi}=\phi/\eta \tag{23}$$

are solutions of the above equations in the absence of scattering. This means that there is no need to take scattering into account when solving the coupled integral equations (18) and (19) for $\tilde{Z}$ and $\tilde{\phi}$. The complete selfenergies are obtained from (22) with η given in terms of $\tilde{Z}$ and $\tilde{\phi}$ as

$$\eta(\omega) = 1 + \frac{\Gamma_N}{\sqrt{\tilde{\phi}^2(\omega) - [\omega\tilde{Z}(\omega)]^2}} \, . \tag{24}$$

Input parameter for the calculation are, in addition to Γ_N discussed at the beginning of the appendix, the Eliashberg function $\alpha^2F(\omega)$, which we take from tunneling experiments [17]. For the Coulomb pseudopotential μ^* a value 0.17 has been used and the Coulomb cut-off ω_c has been set equal to 240 meV [13, 14]. The prefactor in Eq. (16) can be expressed in terms of the plasma wavelength $e^2N(0)v_F^2/3 = 1/2\mu_0\lambda_p^2$, which in weak coupling and in the clean limit is identical to the London penetration depth λ_L. Both impurity scattering and strong coupling effects cause λ_L to exceed λ_p. So we choose $\lambda_p = 230$Å to arrive for typical parameter sets at the experimentally measured $\lambda_L = 330$Å [14].

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