TRANSITION DYNAMICS OF THE WAKE FIELDS OF ULTRA SHORT BUNCHES

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Abstract

In the cavities and finite cell structures, ultra short bunches excite very high frequency electromagnetic fields. A fraction of these fields stay in the structure for a very long time. After several reflections another part leaves the structure. The rest part is chasing the bunch. In a time, this field will catch the bunch and take its kinetic energy. The time and the distance, when and where the bunch is caught, is inversely proportional to the bunch length. The time and the distance can be very long for a very short bunch. The analyses of the wake fields in this transient region is given for the Linear Colliders accelerating structure.

1 INTRODUCTION

While passing an acceleration section or a multi-cell cavity a short bunch creates high frequency electromagnetic fields in the cells. A fraction of the field, excited in one cell stays there for a long time. After several reflections, another part is leaving the cell. And the rest part is chasing the bunch. In a time this field will catch the bunch and take its kinetic energy. Naturally, this part is also responsible for the excitation of the fields in the next cells. The time or the distance, where the bunch is caught, is inversely proportional to the bunch length. It can be very long for a very short bunch. And the fields in the cells will be also different along this distance. Later, the superposition of the chasing fields will create the "steady state" wake field.

To study the dynamics of the wake fields in this transient region, we used the codes MAFIA[1] and NOVO. The latter was used for the short bunch calculations in the TESLA cavities [2]. A brief description of the algorithm of this code is presented in the last chapter.

2 TRANSFORMATION OF THE WAKE FIELDS IN THE SEMI-INFINITE PERIODICAL STRUCTURE

Short bunches interact with single cavity and periodical structure in a different way. In the cavity the loss factor is inversely proportional to the square root of the bunch size. And in the periodical structures loss factor fairly depends upon the bunch size. Usually accelerating sections consist of some number of cavities or cells. And this cavities and cells can be different. Thus the constant gradient NLC accelerating section contains 206 different cells, when the TESLA accelerating cryo - module is the chain of separated 9 cell- cavities. These accelerating structures are not really periodical.

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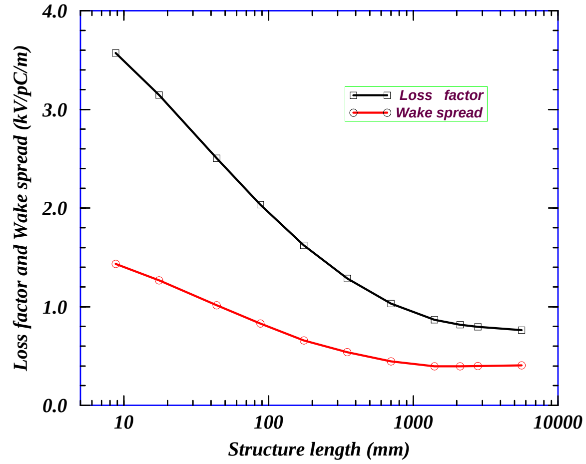

Figure 1: Loss factor and energy spread per unit length of the 10μ bunch in the periodical iris-loaded structure.

We can assume, that when the bunch comes inside the accelerating section, it has its transverse "self field" and interacts with the cells as with lonely cavities. But in time the fields from excited cells catch the bunch and partially compensate the field, coming into the next cell together with the bunch. For very short bunches this compensation is very strong. The interaction and hence, the loss factor decreases many times. On the Fig.1 the evaluation of the energy loss of the 10 μ bunch in the periodical structure is presented. The period of the structure is 8.75 mm, the gap is 6.89 mm and the aperture is 4.92 mm. The loss factor of the bunch in the periodical structure of TESLA cells is shown on the Fig.2. The bunch length is 200 μ.

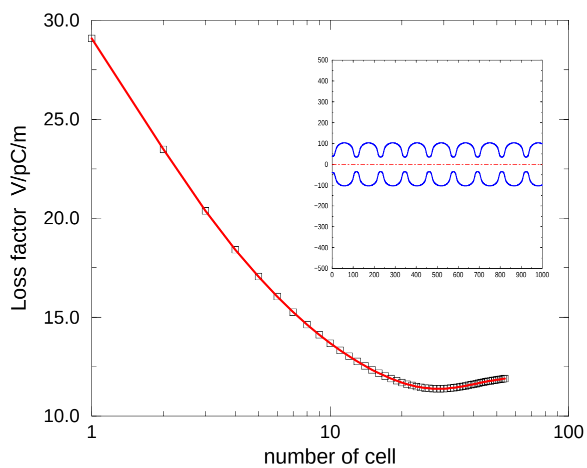

Figure 2: The loss factor of the 0.2 mm bunch in the periodical structure of the TESLA cells.

Transformation of the wake fields is shown on the Fig.3 and Fig.4 consequently.

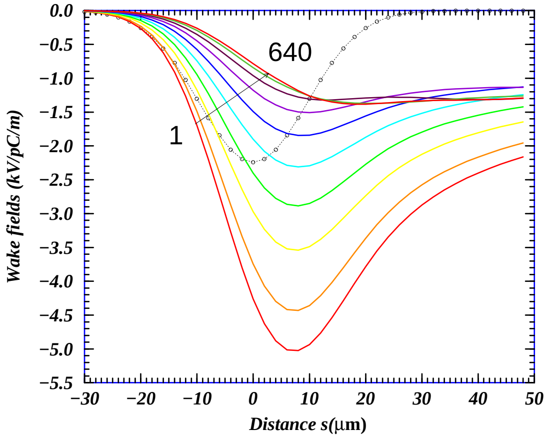

Figure 3: Wake fields of the 10μ bunch in the periodical iris-loaded structure.

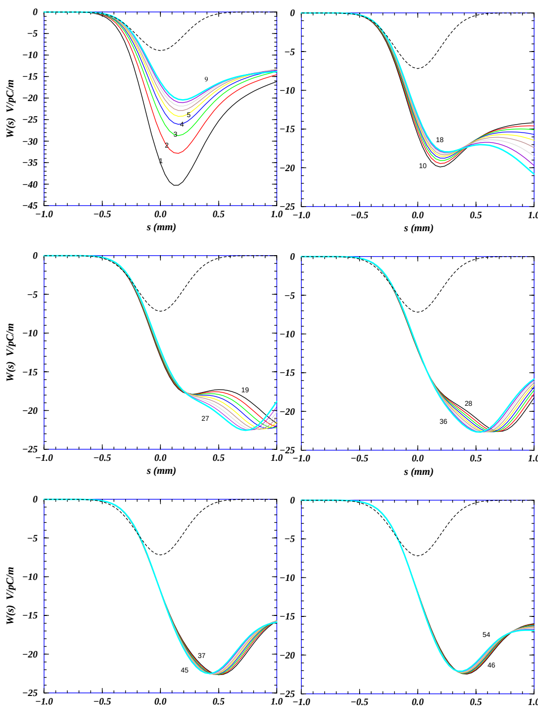

Figure 4: The wake fields of the 0.2 mm bunch in the periodical structure of TESLA cells.

For the iris-loaded structure the wake fields per unit length are given after 1,2,5,10,20,40,80,160, 320 and 640 cells. And for the TESLA cavities results are given for 9 consecutive cells on each picture, making easier comparison with the 9-cell TESLA cavities. These pictures show strong modification of the wake fields along the accelerating structure. Starting from the first cells wake fields are decreasing in amplitude, and the shape is becoming more and more linear around the bunch center, very close to the integral of the charge density.

The same strong modification of the wake fields takes place in bellows as well. In the bellows we have large aperture and small size irises. In the bellows of small number of cells the wake field has inductive character (derivative of the bunch distribution), while in the case of very large number of cells the wake field takes capacitor character (integral of the bunch distribution).

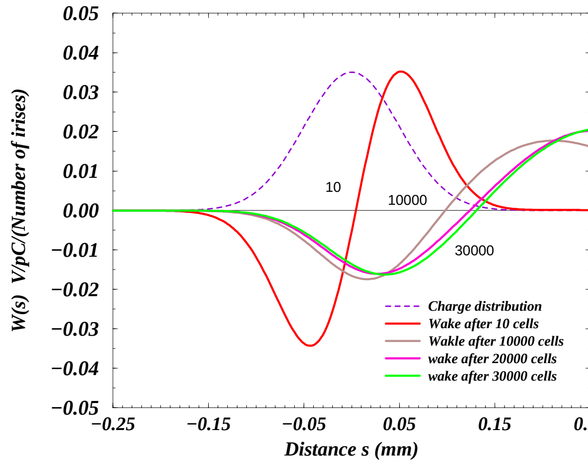

Figure 5: The wake fields of the 50 μ bunch in the bellow of the 5 mm aperture with 10 μ size irises. Wake fields are given after 10, 10000, 20000 and 30000 irises.

This is clearly seen on the Fig.5, where wake fields of the 50 μ bunch in the bellows with small size irises are presented. The aperture of the bellow is 5mm, the gap and the height is 10 μ and the period is 20 μ.

3 WAKE FIELD ENERGY

To find real fields, acting on particles, we split the full field E_{full} in the wake field E_{wake}, that really acts on the bunch particles and the "self" field E_{bunch} , that is moving together with this bunch, but does not interact with particles (in the relativistic case).

$$E_{wake} = E_{full} - E_{bunch}$$

The energy distribution of the wake field, following the bunch, can be described by the longitudinal energy density $\Lambda(s)$, that is the transverse integral of the energy density at a distance s from the center of the bunch

$$\Lambda(s) = \frac{\epsilon_0}{2}\int [E^2_{wake}(s) + H^2_{wake}(s)]d\varphi r dr$$

When the bunch comes out of the cavity, the integral of this density $\Lambda(s)$ along the bunch way, shows the energy $T(s_0)$, that is following the bunch at a distance s_0 behind

$$T(s_0) = \int_{-\infty}^{s_0} \Lambda(s)ds$$

This integral is approaching the loss factor K_{loss}, when $s_0 \longrightarrow \infty$

$$K_{loss} = \int_{-\infty}^{\infty} \Lambda(s)ds = T(\infty)$$

While decreasing the bunch length, the loss factor is increasing, and more and more high order modes are excited in a cell. On Fig. 6 the loss factor in one cell of the TESLA cavity is shown over the bunch length, together with the energy integral $T(s_0)$.

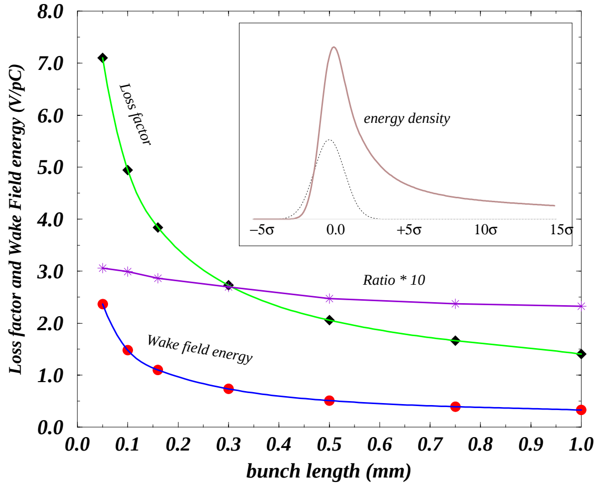

Figure 6: Loss factor and the wake field energy, following the bunch in the tube, after a single cell of the TESLA cavity over the bunch length. Ratio of field energy to the loss factor is multiplied by 10 (to use the same scale).

In this figure, inside the box, the energy density $\Lambda(s)$ in the tube and the bunch charge distribution are also shown. The energy density has a slowly vanishing tail. The energy of the following field is defined as $T(s_0 = 15\sigma)$. The ratio of the energy integral to the loss factor is slightly growing up while the bunch length is decreasing, coming to the value of 30%. So, one third of the "excited energy" immediately leaves the cell with the bunch.

How far will the wake field follow the bunch in the tube? One can predict, that at least, up to the distance L, where the field "catches" the bunch

$$L = \frac{a^2}{2\sigma}$$

where a is the radius of the tube, σ is the bunch length. For the regular cell of the TESLA cavity and for the bunch length $\sigma = 0.5$ mm, this distance $L = 1.225$ m, is equal to the length of the 9-cell cavity. For the bunch of $\sigma = 50\mu$, the distance is more than one accelerating cryomodule length.

As the wake field follows the bunch for a long time, in a multi-cell cavity this field is increasing with the number of cells, that the bunch meets on a distance L. Results of the computation of the energy integral $T(15\sigma)$ along the TESLA cavity are shown on Fig. 7.

It can be seen, that the energy of the following field is linearly growing up in the first cells. The number of cells, where the field approaches the asymptotic solution, is determined by the "catch up" length L and period of structure D

$$N = L/D = \frac{a^2}{2\sigma D}$$

Values for N for the TESLA cavity (9-cell) are shown in Fig. 7 for different bunch length.

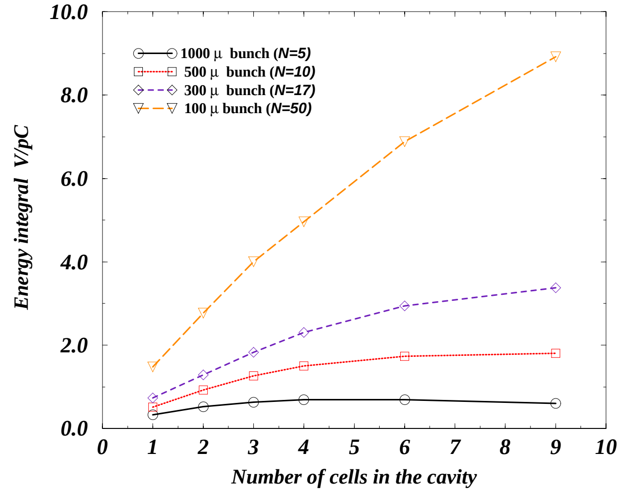

Figure 7: Wake field energy, following the bunch in the TESLA cavity, for bunchlength of 1 mm, 0.5 mm and 0.1 mm.

4 DIRECTED AND REFLECTED FIELDS IN THE TESLA CAVITY

To study the reflected and the directed fields in the TESLA cavity, the model of excitation of one cell was used. The full electromagnetic field is separated and we do calculations only for the wake field. On the surface of the "excited" cavity the wake field has to take the value of the bunch field with negative sign in accordance with boundary conditions. The geometry of the "excited" cell (N2) and the cells around (N1, N3) are shown in Fig. 8. Cell N2 is excited by the bunch of 0.5 mm length. The energy in the cells is calculated and presented in time. When the bunch leaves cell N2 the wake field energy is going with it and excites cell N3; then coming to the end of cell N3, one part of the energy goes to the next cell and the energy in cell N3 is going down. At the same time, the field, reflected from the iris between cell N2 and N3, is crossing cell N2 and coming to cell N1.

We can estimate the reflected and the directed coefficients comparing the field energy in adjoining cells. This coefficients are changing from cell to cell, as the frequency spectrum of the field is changing too. Low frequency modes are slowly moving along the structure, when high frequency modes are traveling with the bunch. The coefficient of transmitted energy from the first cell to the second is around 30 %. In the next cell it is approximately 50 %. From cell to cell the reflected energy is decreasing and finally high frequency modes are traveling along the structure almost without reflections. If we know the transmission coefficient of the energy Π, then we can estimate the time dependence of the energy in the "excited" cell by

$$\mathcal{E}(t) = \mathcal{E}_0(1-\Pi)^{ct/D} = \mathcal{E}_0 e^{-t/\tau}$$

and find the energy attenuation time τ of high frequency modes in one cell

$$\tau = -\frac{D}{c\ln(1-\Pi)}$$

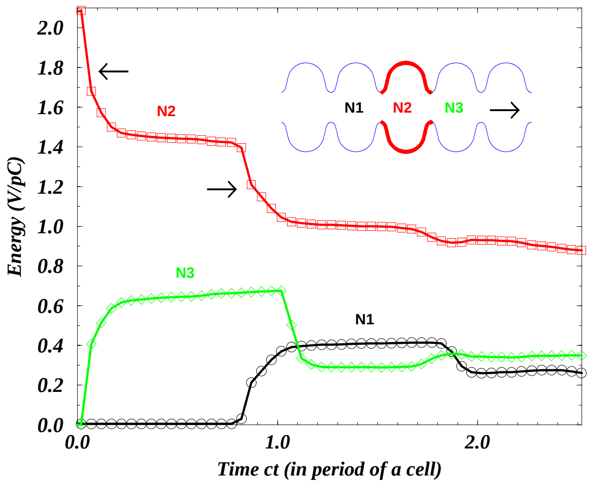

Figure 8: Field energy in cells of TESLA cavity.

Taking Π=0.3 we get τ = 1 ns.

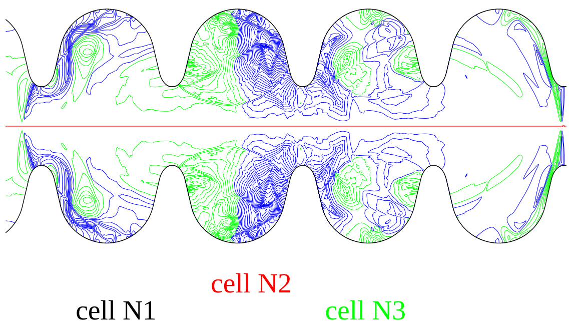

Figure 9: Electric force lines in the TESLA cavity with one excited cell.

The photo (Fig.9) of the electric force lines is made at the moment, when the bunch "must" be inside the last iris. The red point on the right shows bunch position. The corresponded time (previous picture) is equal to two periods of cell. Green and blue lines have opposite directions of electric field. One can see that the energy of the field is divided by excited cell and adjoining cells. High frequency modes are chasing the bunch, leaving "free gap" after. The backward fields have approximately the same structure.

5 SHORT BUNCH WAKE FIELD CALCULATION

Naturally the convenient equation for the wake field computer calculations can be derived from the Maxwell equations. We will consider the case for the cylinder symmetry structures.

For the wake potential calculations we usually assume, that the beam is moving with the constant velocity. Therefore the charge density ϱ and current density $\overrightarrow{j}$ can be described in the way

$$\varrho = \varrho(s,r)\delta(z - V_z t + s)$$

$$\overrightarrow{j} = j_z = \varrho(s,r)V_z\delta(z - V_z t + s)$$

where $\varrho(s,r)$ is the charge density inside the bunch.

In the relativistic case, when the velocity of the particles is equal to the speed of light ($V_z = c$), we can use the flux Φ for the description of the electromagnetic fields

$$\Phi(t,r,z) = \int_0^r E_z(t,r',z)r'dr'$$

Field components can be found from derivatives

$$E_z = \frac{1}{r}\frac{\partial}{\partial r}\Phi$$

$$E_r = \frac{1}{r}\frac{\partial}{\partial z}\Phi + \frac{2}{rc}I_z$$

$$H_\varphi = \frac{1}{r}\frac{\partial}{c\partial t}\Phi + \frac{2}{rc}I_z$$

Where I_z is the current of the traveling bunch

$$\overrightarrow{I} = I_z = cq(s)\delta(z - ct + s)$$

with the charge distribution $q(s)$ along the bunch.

This flux Φ satisfies the second order equation

$$\frac{\partial^2\Phi}{c^2\partial t^2} - \frac{\partial^2\Phi}{\partial z^2} = r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\Phi}{\partial r})$$

with boundary conditions

$$\overrightarrow{n} \bullet (\overrightarrow{grad}\Phi - \frac{2}{rc}\overrightarrow{I}) = 0$$

For the finite -difference approximation implicit scheme was used. This scheme gives not only stable solution, but also better "numerical dispersion curve" in comparison with the explicit scheme

$$\cos\omega\Delta t = \frac{cos\beta\Delta z - (1 - (\frac{c\Delta t}{\Delta z})^2)}{1 + 2(\frac{c\Delta t}{\Delta r})^2\sin^2\frac{\alpha\Delta r}{2}}$$

Good resolution, especially in the region of minimum critical wavelength ($\lambda_{cr} = 2\Delta_z$) is achieved, when the time step equal to coordinate step $c\Delta t = \Delta z$. In this case, the implicit scheme takes the form

$$\Phi_k^{n+1} - \frac{1}{2}(\frac{c\Delta t}{\Delta r})^2 r\Delta\frac{1}{r}\Delta\Phi_k^{n+1} =$$

$$= \Phi_{k+1}^n + \Phi_{k-1}^n - \Phi_k^{n-1} + \frac{1}{2}(\frac{c\Delta t}{\Delta r})^2 r\Delta\frac{1}{r}\Delta\Phi_k^{n-1}$$

For the given number of mesh points, the solution of the implicit equation needs approximately the same "computer" time, as for direct calculation in the explicit algorithm scheme. However, the implicit method has a great advantage in the short bunch calculation, as it does not need large number of the mesh points on the bunch length.

To show how it works, we present the examples of the wake potential calculations for two cavities structure with tapers and the NLC accelerating section. In the first example, the bunch of 1mm length is passing 5 m long structure. The comparison of the wake potentials, calculated with ten,

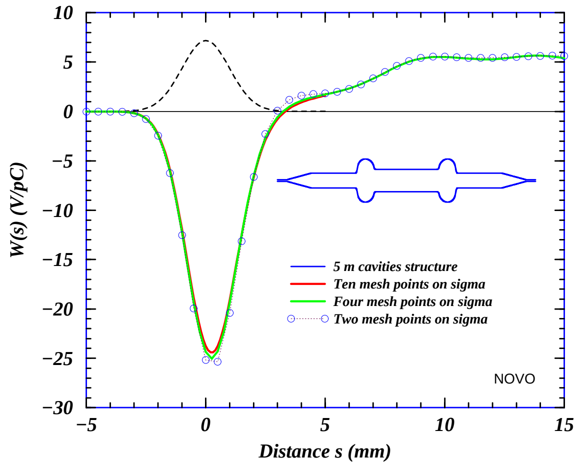

Figure 10: The wake fields of the 1mm bunch in two cavity accelerating structure with tapers. Comparison of the results of calculations with different number of mesh points on the bunch length.

four and two mesh points on the bunch length is given on the Fig.10. It is easy to see, that there is no ”mesh dispersion”, no modulation with the critical wavelength and no diffusion. To achieve acceptable accuracy even two mesh points on the bunch length will be enough.

The wake potential of a 0.1mm bunch in the constant gradient NLC structure of 206 ellipse cells is shown on Fig.11. Calculations were done for 10 and 5 mesh points on the bunch length. No difference can be noticed between two results. The long distance wake potential is shown on Fig.12. The comparison of the results of 5 and 2 mesh points on the bunch length is shown.

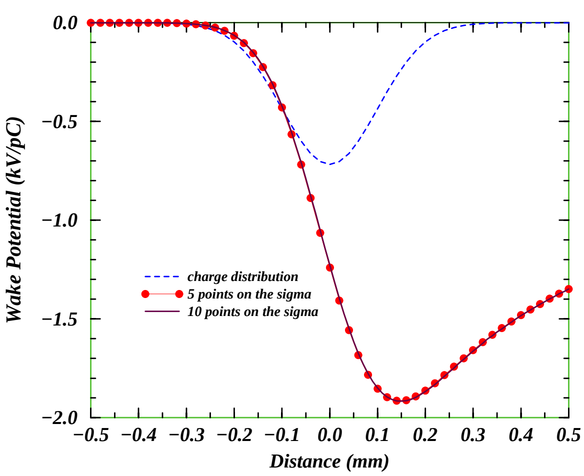

Figure 11: The wake potential of the 0.1 mm bunch in the NCL accelerating section. Comparison of the results of calculations with 10 and 5 mesh points on the bunch length.

The presented scheme can be easily transformed for computation of the fields on the moving mesh points, which is one more advantage for the short bunch wake field calculation. Also it is very easy to make pictures of electric force lines, just to find the lines, where flux of electric field

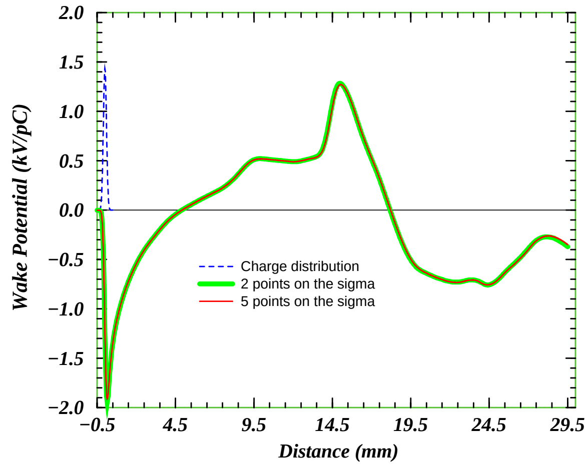

Figure 12: The long distance wake potential of the NLC section, calculated by 5 and 2 mesh points on the bunch length.

is constant

$$\Phi(r, z) = const$$

As an additional example, on Fig.13 the picture of electric force lines of the 100 μ bunch at the end of the NLC section is presented.

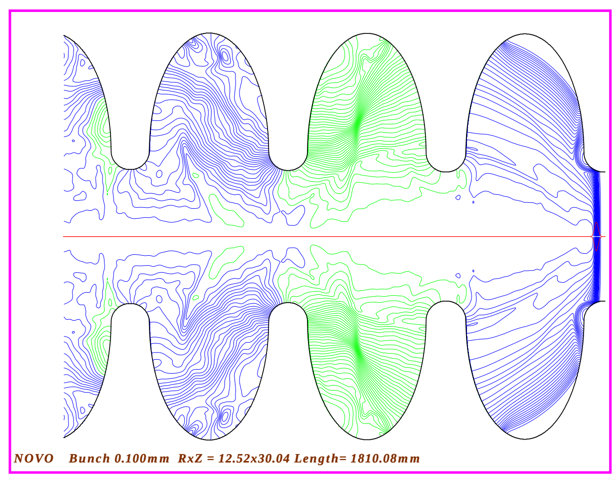

Figure 13: End of the NLC section. Electric Force lines of the field, excited by the 100 μ bunch.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

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[2] A.N.Novokhatski and A.Mosnier, ”Short Bunch Wake Potentials for a Chain of TESLA Cavities”, DAPNIA/SEA-96-08, 1996.