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Transverse Autophasing Solution for Low Chromaticity Linear Accelerators

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Abstract: The transverse autophasing conditions for low chromaticity high energy linear accelerators to prevent single bunch emittance enlargement in wakefields have been studied. The independent solution for a machine with constant beta lattice has been treated. The matching of the amplitudes of dispersive and wakefield terms is accomplished by accelerating the bunch behind the crest of the radio-frequency wave at the low energy part of the linac. The analytical predictions are confirmed by particle tracking simulations in TeV Energy Superconducting Linear Accelerator (TESLA).

1 Introduction

An emittance enlargement of a single bunch in short range transverse wakefields is one of the unwanted features in high intensity linear accelerators that prevents the acceleration of beams with small emittances [1-4]. The technique to control this emittance enlargement is known as the Balakin-Novokhatsky-Smirnov (BNS) damping[5] that has been successfully demonstrated at the Stanford Linear Collider (SLC) [6]. The BNS damping technique is based on the introduction of an incoherence in transverse oscillations of the head and tail particles of the bunch by producing the correlated energy spread in the bunch to prevent the resonant beam blow-up in transverse wakefields. The special solution of the BNS damping, known as the particle transverse autophasing [7], provides the average cancellation of the transverse kick of the trailing particles in wakefields by corresponding additional focusing in quadrupoles that contain the lattice. This is accomplished by the lowering the energy of the trailing particles relative to the head so that they are more strongly focused by the quadrupole lattice.

In this paper we give the results of our study on particle transverse autophasing in low chromaticity linear accelerators to suppress the single bunch emittance enlargement in wakefields when the beam performs coherent betatron oscillations.

It is well known, that if the accelerating radio-frequency (RF) phase is fixed, the interaction of the particles with accelerating structures of the machine leads to constant correlated energy spread in the bunch, that is independent of actual energy. The net cancellation of the transverse displacement of trailing particle caused by transverse wakefields and betatron phase shift (autophasing), is then possible, if the focusing lattice is scaled along the linac via the betatron function [8] or phase advance per cell [9] with an additional requirement to actual correlated energy spread. However, for a machine with low chromaticity focusing lattice, the dispersive emittance dilution is negligible, and it opens a possibility to provide the autophasing conditions by producing the initial extra correlated energy spread only. Note, that for a constant beta lattice, both the transverse wakefield term and the extra induced dispersive term in the equation of motion then decrease inversely proportional to the beam energy. The initial correlated energy spread is induced at the origin of linac by acceleration of the bunch behind the crest of the RF wave similar to that developed at the SLC [6]. After the matching of the amplitudes of the dispersive and wakefields terms, the RF phase is switched to its optimal value ahead of the RF crest to eliminate the energy spread at the end of linac and to reduce an additional dispersive emittance dilution of the beam. Thus, as the autophasing condition is satisfied at the low energy part of the linac, the bunch stays in the autophasing regime during the beam acceleration to high energies thus preventing the further emittance degradation in transverse wakefields. We apply this approach for the main linear accelerator of high luminosity TESLA option [10], where the single bunch emittance enlargement in wakefields is a critical factor for acceleration of the beams with very small vertical emittance when the beam performs coherent betatron oscillations.

2 Chromaticity of the machine

Consider a relativistic electron bunch traversing a linear accelerator with cylindrically symmetric accelerating structure and a FODO (F-focusing quadrupole, O-drift space, D- defocusing quadrupole) focusing lattice. The longitudinal distribution of the bunch is assumed to be rigid and Gaussian shaped. We suppose that an injected beam has a zero initial correlated energy spread and the linac is fully filled with uniform accelerating sections. In addition, the bunch performs a coherent free betatron oscillations with initial amplitudes x_0, x'_0

$$\gamma_{x0} x_0^2 + 2\alpha_{x0} x_0 x'_0 + \beta_{x0} x'^2_0 = a_0^2, \tag{1}$$

with $\alpha_{x0}, \beta_{x0}, \gamma_{x0}$ being the initial Twiss parameters of the machine, a_0^2 the area (divided by π) of machine initial central phase ellipse that is determined by an initial transverse jitter of the beam. As a short bunch travels through the accelerator, it gains energy from externally driven accelerating mode and loses energy due to the longitudinal wakefields. The net energy deviation of the particles at the position s within the bunch and position z along the linac is then given by

$$\Delta E(z,s) = eQzW_z(s) + \hat{G}z\cos(\phi_{RF} + \frac{2\pi}{\lambda_{RF}}s) - Gz = U(s)z, \tag{2}$$

where W_z is the longitudinal wake potential per unit length, $\hat{G}$, G the maximum and the average accelerating gradient, Q the total charge of the bunch, ϕ_{RF}, λ_{RF} the RF phase and wavelength, and $U(s)$ is then the particle net energy deviation per unit length. The relative correlated energy spread induced in accelerating sections is then vary with energy as

$$\delta_{cor}(s,z) = \frac{\Delta E(s,z)}{E(z)} = \frac{U(s)}{G}\left[1 - \frac{\gamma_0}{\gamma(z)}\right], \tag{3}$$

where $\gamma_0, \gamma(z)$ are the initial and actual Lorenz factors of the design particle. The rms value of the parameter $\delta_c(s) = U(s)/G$ is determined by the longitudinal impedance of accelerating structure, amplitude, frequency and RF phase of accelerating mode, the length and total charge of the bunch. As the beam propagates in the linac with nonzero initial offset, the different longitudinal slices of the bunch will filament in transverse phase plane due to the difference in betatron oscillation frequencies of off- and on-energy particles in the linear focusing lattice of the linac. The average betatron phase shift of the off-energy particle at the actual position z along the linac is given by

$$\Delta\mu(z) = -\frac{1}{2}\int_0^z \delta_{cor}(z')\beta(z')K(z')dz', \tag{4}$$

where K is the normalized quadrupole strength. In thin lens approximation, the formula can be rewritten as the sum over the linac cells, which in turn may be replaced by an integral in energy range. Using the well known relation for a symmetric FODO lattice

$$\frac{1}{4}KL_q(\beta_{max} - \beta_{min}) = \tan\frac{\mu}{2}, \tag{5}$$

the betatron phase shift of the off-energy particle in the constant beta lattice machine is then given by

$$\Delta\mu(z) = -2\delta_c\tan\frac{\mu}{2}\sum_{n=1}^{N}\left(1 - \frac{\gamma_0}{\gamma_n}\right) \approx -2\delta_c\tan\frac{\mu}{2}\frac{\gamma_0}{\Delta\gamma}\left(\frac{\gamma}{\gamma_0} - \ln\frac{\gamma}{\gamma_0} - 1\right), \tag{6}$$

which is valid for $\gamma_n = \gamma_0 + n\Delta\gamma$ that slowly varies with the cell number n; $\Delta\gamma$ is the energy gain per a single FODO cell, μ is the betatron phase advance per cell.

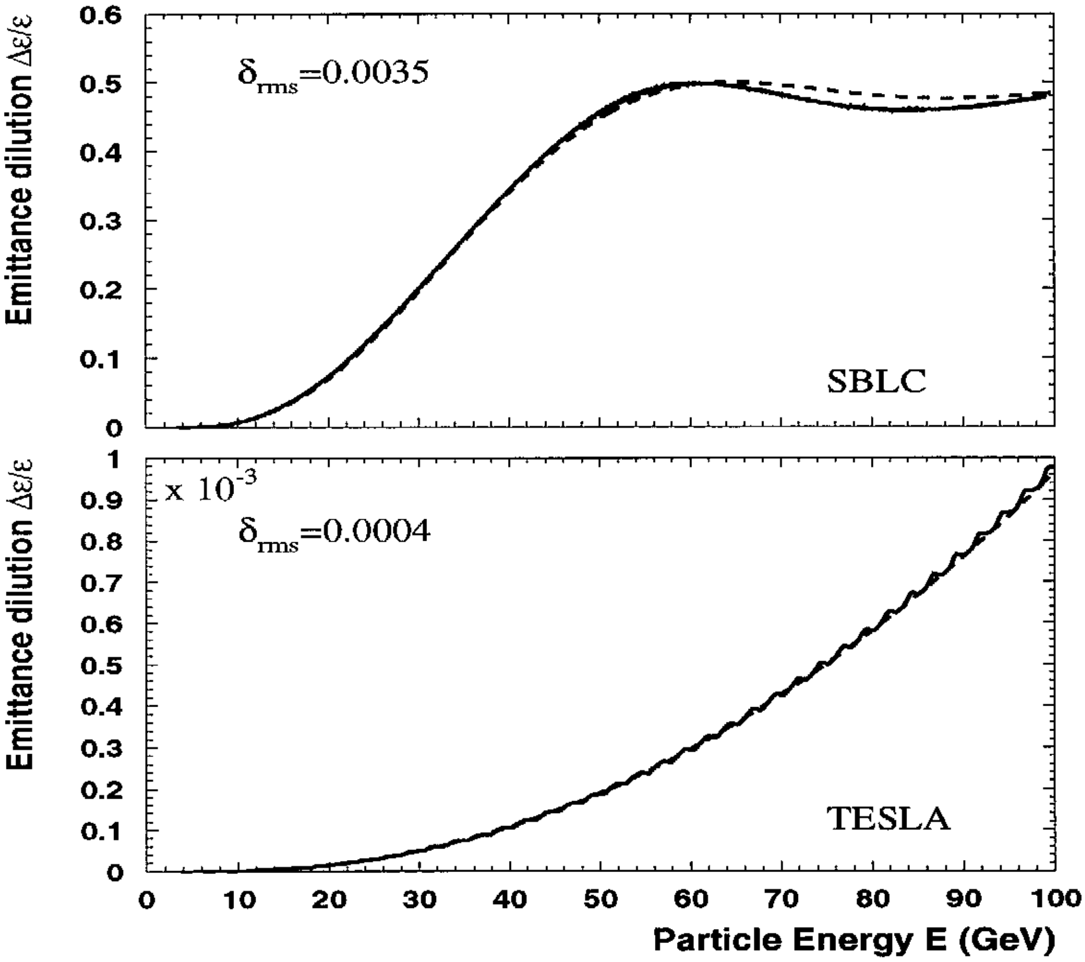

Fig.1 *The dispersive emittance dilution of the bunch due to particle correlated energy spread in SBLC (top) and TESLA (bottom) main linacs. The bunch performs coherent betatron oscillations with one standard initial offset. The solid lines - tracking calculations, dashed lines - analytical predictions.*

A good approximation for emittance dilution, caused by the correlated energy spread, can be found in rectangular longitudinal shape model of the bunch with linear variation of the correlated energy spread [11]

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{1}{2}\frac{a_0^2}{\varepsilon_0}\left(1 - \frac{\sin^2\Delta\hat{\mu}}{\Delta\hat{\mu}^2}\right), \tag{7}$$

where ε is the actual natural emittance of the bunch, $\Delta\hat{\mu}$ is the phase shift of the particle with maximum energy deviation $\hat{\delta}$. In our beam model this value is related to the rms correlated energy spread within the bunch δ_{rms} by $\hat{\delta} = \sqrt{3}\delta_{rms}$. For small chromaticity of the machine $\Delta\hat{\mu} << 1$, we obtain the following emittance dilution

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{1}{6}\frac{a_0^2}{\varepsilon_0}\Delta\hat{\mu}^2 = 2\frac{a_0^2}{\varepsilon_0}\left[\delta_{crms}\frac{\gamma_0}{\Delta\gamma}\tan\frac{\mu}{2}\left(\frac{\gamma}{\gamma_0} - \ln\frac{\gamma}{\gamma_0} - 1\right)\right]^2 . \tag{8}$$

Thus , the emittance dilution of the bunch due to correlated energy spread strongly depends on the energy gain per cell $\Delta\gamma$, phase advance per cell μ and the rms value of the parameter δ_c. Note, that by means of proper choice of accelerating RF phase for a short bunch to be accelerated in low impedance machine one can provide the small correlated energy spread. In addition, the small transverse wake fields in such a machine allows to increase the number of accelerating modules per single FODO cell and to reduce the betatron phase advance per cell. All this provide the low chromaticity of the machine and these arguments underlie the TESLA project cell arrangement [12]. For comparison we show in Fig.1 the emittance dilution caused by correlated energy spread in high chromaticity S-Band Linear Collider and low chromaticity TESLA designs [12] when the beam performs coherent betatron oscillation in the main linear accelerator with one standard initial offset.

3 Autophasing

As is well known [1-4], during coherent betatron oscillations, the off-axis beam induces the transverse wake fields in accelerating sections that reciprocally affect the trailing particles within the bunch and cause the emittance enlargement. The simplest approach to simulate the essentials of the process is the two-particle model for the bunch [1]: the beam is modeled by two macroparticles with the charge $eN_e/2$ that are separated by the longitudinal distance $\Delta s = 2\sigma_s$, where σ_s is the rms length of the bunch, e is the electron charge and N_e is the number of particles in the bunch. The first, heading particle feels no transverse wakefield and thus undergoes free betatron oscillations with initial amplitudes x_0, x_0' . The second, trailing particle experiences the dipole wakefields W_d due to the off-axis motion of the leading particle. With neglect of the correlated energy spread, the emittance enlargement of the bunch by transverse wake fields in the constant beta lattice machine is then given by [3]

$$\frac{\Delta\varepsilon}{\varepsilon}(z) = \frac{1}{2}\frac{a_0^2}{\varepsilon_0}\frac{C^2}{\sin^2\mu}\left(\frac{E_r W_d L_c}{4G}\right)^2 \ln^2\frac{\gamma(z)}{\gamma_0}, \tag{9}$$

where $C = 4\pi\epsilon_0 r_e N_e$, ϵ_0 is the dielectric constant, r_e the classical electron radius, L_c the focusing cell length, E_r the particles rest energy. Thus, for a very small vertical emittance, the single bunch emittance enlargement can be a critical factor even for low transverse wake fields if $\varepsilon_0 < a_0^2$.

In the presence of the energy spread, the trailing particle in two-particle model of the bunch has the energy deviation δ_{cor}. The relative transverse displacement

$\Delta x = x_2 - x_1$ of the tail particle with respect to the head will then obey the equation of motion

$$\Delta x'' + \frac{\gamma'}{\gamma}\Delta x' + K(1-\delta_{cor})\Delta x = \delta_{cor} K x_1 + C_w \frac{\gamma_0}{\gamma} x_1, \tag{10}$$

with $C_w = CW_d/2\gamma_0$ for δ_{cor} small. Note , that if the bunch has initial relative correlated energy spread δ_0, its contribution to actual relative energy spread decreases inversely proportional to the design energy

$$\delta_{cor}(z) = \delta_c \left[1 - \frac{\gamma_0}{\gamma(z)}\right] + \delta_0 \frac{\gamma_0}{\gamma(z)}. \tag{11}$$

To calculate the autophasing conditions for low chromaticity machine, we neglect the contribution to emittance dilution due to correlated energy spread δ_c induced in the main accelerating modules, and keep only the terms caused by the initial correlated energy spread and the transverse wake fields. We suppose that the head particle performs free betatron oscillations and use the M_{12} matrix element to solve the equation

$$\Delta x = \int_0^z M_{12}(z', z)\left[\delta_0 \cdot K(z') + C_w\right] \frac{\gamma_0}{\gamma(z')} x_1(z') dz'. \tag{12}$$

Since the autophasing cancellation is quasi-local, one can neglect the chromatic phase advance that becomes significant only after a large number of cells. In the thin lens approximation, the driving term in the solution is presented as the sum over the cells and we find the following expression for the relative transverse displacement after N cells

$$\Delta x = \frac{1}{2} a_0 \left[\frac{\gamma_0}{\gamma(z)} \beta(z)\right]^{1/2} \sum_{n=1}^{N} \frac{\gamma_0}{\gamma_n} \left[\delta_0 K_n L_q (\beta_{nmax} - \beta_{nmin}) + C_w L_A \overline{\beta}_n\right] \sin\left[\psi(z) - \theta_0\right], \tag{13}$$

where a_0, θ_0 define the initial amplitudes of coherent betatron oscillations, $\psi(z)$ is the actual betatron phase, L_A is the total length of accelerating sections per a FODO cell. The beta function in the accelerating sections is approximated by the average betatron function of a symmetric FODO cell

$$\overline{\beta} = \frac{1}{2}(\beta_{max} + \beta_{min}) = \frac{L_c}{\sin\mu}. \tag{14}$$

Thus, the autophasing condition in n-th cell is satisfied for a constant beta lattice ($L_c = const, \mu = const$) and the initial correlated energy spread is given by

$$\delta_0 = -\frac{C_w L_A L_c}{8 \sin^2(\mu/2)}. \tag{15}$$

Note, that the required correlated energy spread is negative, i.e. the tail particle has the lower energy with respect to the heading one.

In many particle model of the bunch the transverse autophasing solution directly follows from the first order solution, when the leading particles feels no transversely deflecting fields[2]. Indeed, the change of trailing particle transverse position in n -th FODO cell in the presence of energy spread $\delta_n(s)$ and transverse wake potential $W_d(s)$ induced by preceding charges, is given by

$$\Delta x(z,s) = \frac{1}{2}a_0 \left[\frac{\gamma_0}{\gamma(z)}\beta(z)\right]^{1/2} \frac{\gamma_0}{\gamma_n} \left[4\delta_0(s)\tan\frac{\mu_n}{2} + CL_A\overline{\beta}_n\frac{1}{\gamma_0}W_d(s)\right] \sin\left[\psi(z) - \theta_0\right], \tag{16}$$

where

$$W_d(s) = \int_{-\infty}^{s} w_d(s-s')\rho(s')ds' \tag{17}$$

with $w_d(s)$ the point dipole transverse wake potential of the accelerating structure, $\rho(s)$ the bunch normalized longitudinal distribution. Thus the transverse autophasing solution for physical bunch in low chromaticity machine read as

$$4\delta_0(s)\tan\frac{\mu}{2} = -\frac{C}{\gamma_0}L_A\overline{\beta}\int_{-\infty}^{s} w_d(s-s')\rho(s')ds' \tag{18}$$

and the the first order solution in our approach becomes an exact solution of the equation of motion. Transforming to rms beam parameter we obtain

$$4\delta_{0rms}\tan\frac{\mu}{2} = \frac{C}{\gamma_0}L_A\overline{\beta}\sigma_w \tag{19}$$

where

$$\overline{W}_d = \int_{-\infty}^{\infty} W_d(s)\rho(s)ds, \tag{20}$$

$$\sigma_w^2 = \int_{-\infty}^{\infty} \left[W_d(s) - \overline{W}_d\right]^2 \rho(s)ds. \tag{21}$$

The energy spread and the transverse wake in form. (15) is then replaced by their rms values: $\delta_0 = 2\delta_{0rms}$ and $W_d = 4\sigma_w$. The transverse autophasing rms initial energy spread in the bunch is then given by

$$\delta_{0rms} = \frac{CL_AL_c\sigma_w}{8\gamma_0\sin^2(\mu/2)}. \tag{22}$$

The negative required correlated energy spread can be produced at the beginning of the linac by acceleration of particles at the negative RF phase behind the crest of accelerating mode. If we now suppose that the autophasing condition is satisfied starting from the energy γ_1 after the bunch has passed the first N_1 cells, the required rms correlated energy spread induced at the origin of the linac is then given by

$$\delta_c^{in}(1-\frac{\gamma_0}{\gamma_1}) = \delta_{0rms}\frac{\gamma_0}{\gamma_1}. \tag{23}$$

The value of δ_c^{in} is then basically determined by the accelerating RF phase ϕ_s at the low energy part of the linac. In terms of accelerating RF phase ϕ_s, the bunch rms length σ_z that is small with respect to the RF wave length λ_{RF}, we obtain

$$\frac{2\pi}{\lambda_{RF}}\sigma_z \sin\phi_s = \frac{CL_c\sigma_w}{8\gamma_0\sin^2(\mu/2)}\frac{E_0}{N_1\hat{G}}. \tag{24}$$

Actually, the bunch is at BNS damping regime up to energy of γ_1, after which it stays in autophasing regime (the RF phase is switched to its optimal value). The rest diluted emittance due to the dispersive and wakefield terms is then given by

$$\frac{\Delta\varepsilon}{\varepsilon} \approx 2\frac{a_0^2}{\varepsilon_0}\left[\delta_c^{in}\frac{\gamma_0}{\Delta\gamma_1}\tan\frac{\mu}{2}\left(\frac{\gamma}{\gamma_0}-\frac{\gamma_1}{\gamma_0}\ln\frac{\gamma}{\gamma_0}-1\right)\right]^2, \qquad \gamma \le \gamma_1. \tag{25}$$

Note, that the parameter δ_c^{in} and the energy gain per cell at the beginning of the linac $\Delta\gamma_1$ are also strongly dependent on γ_1. For a reasonable value of energy $\gamma_1 \sim 2.72\gamma_0$, the rest emittance dilution is then

$$\frac{\Delta\varepsilon}{\varepsilon} \approx 2\frac{a_0^2}{\varepsilon_0}\left(\delta_c^{in}\frac{\gamma_0}{\Delta\gamma_1}\tan\frac{\mu}{2}\right)^2 \tag{26}$$

4 High luminosity TESLA

One of the high luminosity TESLA options [10] assumes the reduction of a vertical beam emittance about 50 times with respect to the conceptual design[12] to obtain the luminosity of an order of 4.2×10^{34} $cm^{-2}s^{-1}$ for a centre-of-mass energy 500 GeV with the number of particles per bunch $N_e \sim 1.4 \cdot 10^{10}$ and the bunch rms length $\sigma_s = 0.7$ mm. The reduction of the charge per bunch with respect to the conceptual design ($N_e \sim 3.6 \cdot 10^{10}$) is essentially due to single bunch effects in vertical plane caused by the transverse wakefields. Fig.2 (dashed line) shows the vertical emittance enlargement along the main linac due to short range wakefields, when the bunch performs coherent betatron oscillations with initial amplitudes $a_0^2 = 10\varepsilon_0$ that is corresponded to initial vertical jitter about 25 μm. The beam parameters are corresponded to conceptual design with modified low vertical emittance.

The autophasing regime treated in the previous section assumes that for TESLA conceptual design parameters the required initial rms correlated energy spread is about 2%. If the autophasing regime is started at the energy $E \sim kE_0$, the required negative correlated energy is reduced by a factor of k and can be reached by acceleration of bunch in the negative RF phase. For the TESLA design this is accomplished by accelerating the bunch in the first six cells at the RF phase $\phi_1 = -33$ degree with acceleration gradient $G = 21\ MeV/m$. The reduction in final energy is then 0.8 GeV. Starting from the seventh FODO cell the RF phase is switched to its basic value of $\phi_0 = 3.4$ degree that provides a low correlated energy spread (on the order of $\delta_c = 4 \cdot 10^{-4}$) in the high energy part of the linac. The correlated energy variation along the linac is then as is shown in Fig.2. Starting from the energy $E = 7.57\ GeV$ an exact autophasing regime is satisfied and, hence, the damping of the emittance enlargement is observed as it clearly follows from the Fig. 2 . The patterns of the bunch transverse shape and particle vertical phase space distribution at the energy of 100 GeV without and with autophasing solutions is shown in Fig3.

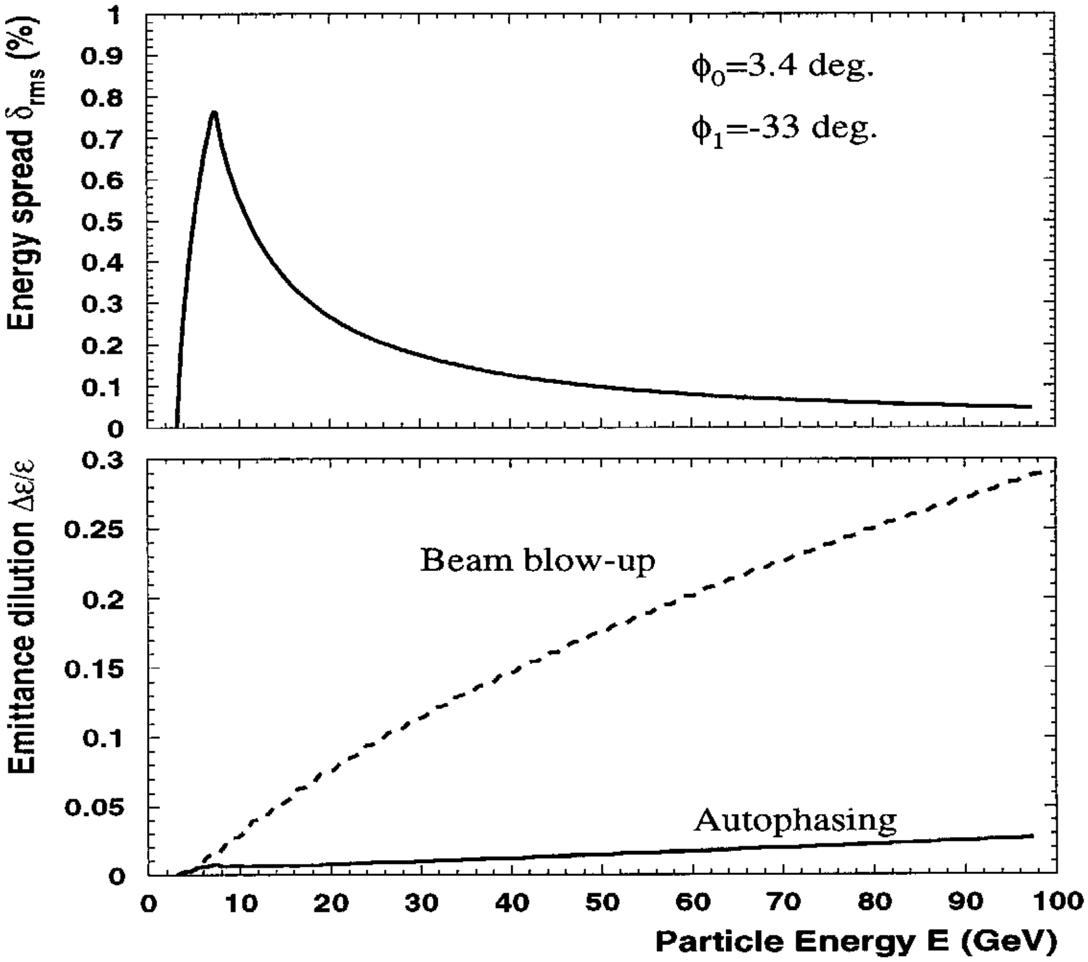

Fig.2 *The vertical beam emittance enlargement in TESLA main linac when the bunch performs coherent betatron oscillations (bottom, dashed line). The rms correlated energy spread variation along the TESLA main linear accelerator in the autophasing regime (top) and the damping of emittance enlargement (bottom, solid line).*

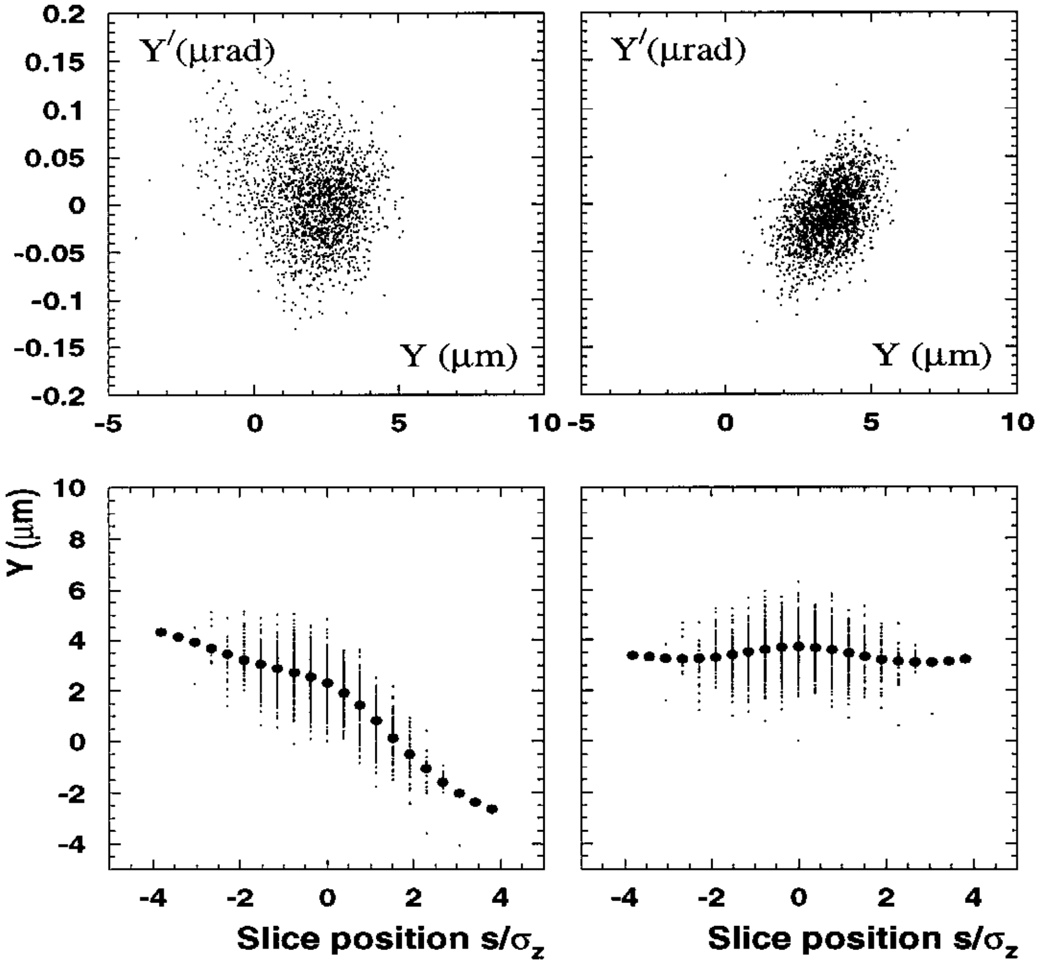

Fig.3 *The phase space distributions (top) and the transverse shapes (bottom) of the bunch at energy 100 GeV in the TESLA main linac (vertical plane). Shown are the patterns without (left) and with the autophasing (right). The marked points show the centers of longitudinal slices.*

5 Summary

The technique for particle transverse autophasing in low chromaticity linear accelerators similar to that developed for BNS damping at the SLC [6] have been treated . It was shown that in low chromaticity machine the exact autophasing can be provided without the scaling of the focusing lattice parameters. We call it an independent solution since, in addition to the aforesaid, the particle transverse autophasing is provided by initial extra correlated energy spread only. This approach was confirmed by particle tracking simulation in main linear accelerator of the TESLA project. Note, that the results open realistic outlooks for the low chromaticity TESLA project with modified low vertical beam emittance [10] to reach the luminosity close to the $10^{35}\ cm^{-2}s^{-1}$ level for a center-of-mass energy of 500 GeV.

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