RECTANGULAR WAVEGUIDE COUPLER FOR TWO TESLA SUPERCAVITIES

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Abstract

A rectangular waveguide coupler design (RWG coupler) for two M×N-cells TESLA supercavities is described. The geometrical dimensions for different RWG couplers are determined. The electromagnetic field distribution under the beam loading condition is investigated and the transverse kick caused by the RF fields of the RWG coupler is estimated.

I. Introduction

The design of coaxial coupler for 9-cells TESLA cavity operating at π-mode 1.3 GHz frequency is described in [1]. Due to nonsymmetrical design of a coaxial input coupler, a transverse kick caused by electromagnetic fields is inevitable. The fill factor defined as a ratio of active accelerating system length to total accelerating system length is less than 0.75.
To increase the fill factor and to decrease the number of input and HOM couplers an M×N-cells supercavity was suggested in [2]. It consists of M subcavities coupled by $\lambda/2$-length beam pipes. Each subcavity has N cells and field oscillation in the neighbouring cells has $\pi$-phase shift. Cell-to-cell coupling coefficient in each subcavity is equal to $K_c=0.019$. Field oscillation in the neighbouring cells connected by $\lambda/2$-length beam pipe has 0-phase shift. Thus, the supercavity has an operational 0-$\pi$-mode and, as consequence, possess some field amplitude stabilization along the cavity [3]. The beam pipes between subcavities have diameter $2R_{bpc}=114$ mm and provide the coupling coefficient $K_{c,s}=0.002$ between neighbouring cells connected by those beam pipes. A coaxial coupler for two supercavities is considered in [4].

To avoid a transverse kick caused by nonsymmetrical electromagnetic fields a symmetrical coaxial coupler was proposed to drive two supercavities [5]. Therefore this setup has three planes of symmetry. Such couplers require additional $\lambda/2$-length beam pipe due to $\pi$ phase shift in the oscillation of the fields in the neighbouring cells of the supercavities connected to the coupler. Due to the longitudinal symmetry of this coupler, only monopole modes with the odd symmetry can be stimulated by the coupler. This improves the field nonuniformity. On the other side, all modes with the even symmetry stimulated by the beam cannot couple to the input waveguide. The transient beam loading of one side stimulated 4×7-cells supercavity is investigated in [2,6,8]. An investigation of the double 4×7-cells supercavity stimulation with even or odd symmetry has not yet been done.

We propose to use a rectangular waveguide coupler to drive two supercavities, which further exploits the effective accelerating length of TESLA and has certain other advantages. The transverse kick due to the coupler asymmetry is estimated.

II. Parameters of TESLA cavities

Parameters of the 9-cells TESLA cavity are presented in Table1. These cavity parameters were calculated with formulae (1) and correspond to bunch parameters listed in the same table.
\[ Q_{\text{ext}} = \frac{\pi E_{\text{acc eff}} N_{\text{cell}} c}{4 K_{\text{loss}} N_{\text{be}} e f_{\text{b}}} \]
\[ \tau_{\text{op}} = \frac{E_{\text{acc eff}} N_{\text{cell}} c}{4 K_{\text{loss}} N_{\text{be}} e f_{\text{op}}} \]
\[ t_1 = \tau_{\text{op}} \ln 2 \]
\[ P_{\text{gen}} = \frac{E_{\text{acc eff}} N_{\text{cell}} c N_{\text{be}} e f_{\text{b}}}{2 f_{\text{op}}} \]
\[ K_{\text{loss}} \left( \frac{V}{pC} \right) = \begin{cases} 0.2131 & 4 \times 7 \text{ cells} \\ 0.2337 & 9 \text{ cells} \end{cases} \]
\[ c = 2.9979245 \times 10^8 \text{ m/sec} \]
\[ e = 1.6021892 \times 10^{-19} \text{ C} \]

**Table 1**

Parameters of the 9-cells TESLA cavity and bunch parameters.

Here: \( E_{\text{acc eff}} \) is an average effective accelerating field gradient in the cavity,
\( T_{\text{RF}} \) is RF pulse duration,
\( f_{\text{op}} \) and \( f_{\text{b}} \) are operational radio frequency and bunch repetition frequency,
\( N_{\text{be}} \) is the number of particles in the bunch,
\( q_b \) and \( I_b \) are bunch charge and pulse beam current,
\( K_{\text{loss}} \) and \( K_{\text{loss}}/N_{\text{cells}} \) are loss parameter of the cavity and loss parameter per cell,
$Q_{ext}$ and $Q_{ext(1+1)}$ are external Q-factor of the cavity and external Q-factor of the first (input) cell,

$\tau_{op}$ is time decay of the operational mode,

t is time moment of the first bunch passage the cavity,

$P_{gen}$ is an input power.

Parameters of 2x4x7-cells and 2x4x9-cells supercavities are summarized in Table 2.

**Table 2a**

<table>
<thead>
<tr>
<th>$E_{ccent. \ Vm}$</th>
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<th>$N_{cell}$</th>
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<td>$b, \ Hz$</td>
<td>$q_{b, \ C}$</td>
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<tr>
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**Table 2b**

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<th>$N_{cell}$</th>
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<td>$f_{op/Hz}$</td>
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<td>5.705341719E-4</td>
<td>4.28973E-1</td>
<td>8.218186E-3</td>
<td>1.705673E+6</td>
</tr>
</tbody>
</table>

**Table 2**

Parameters of 2x4x7-cells and 2x4x9-cells TESLA supercavities.
Table 2 represents parameters of doubled supercavities. Here $Q_{ext(1+1)}$ is external Q-factor of two neighbouring cells connected to the coupler. Of course the input power must be increased proportionally to the total number of cells in the cavity $N_{cells}$. It may add multipacting and breakdown problems in increasing the number of cells in the supercavities.

III. Rectangular waveguide coupler for two 4×7-cells TESLA supercavities

The schematic design of a RWG coupler for a double M×N-cells TESLA cavities is shown in Fig.1.

The RWG coupler consists of a 165.1×30 mm² rectangular waveguide short-circuited at the one end and coupled with two M×N-cells TESLA supercavities by beam pipes ($2R_{bp}$=78 mm). The position of the short circuiting plane surface $L_{short}$ is chosen to provide $Q_{ext}=3.361626\times10^6$ ($Q_{ext(1+1)}=120058$) for 4×7-cells TESLA cavities (see Table 2). To calculate the $Q_{ext(1+1)}$ dependence on short circuiting plane surface position $L_{short}$ we use MAFIA CODE and slightly modified Kroll-Yu-method [7]. This method permits us to calculate $Q_{ext}$ and a reference plane position in the input part of RWG where the steady state reflection coefficient is equal to +1. To simplify MAFIA calculation we use a half of RWG (15-mm height) and only one cell in our calculation. Table 3 and Fig.2 show $Q_{ext(1+1)}$ and reference plane position dependence on the short circuiting plane surface position $L_{short}$ measured from the cavity axes.

One can see that $Q_{ext(1+1)}=120000$ corresponds to $L_{short}=139$ mm. It means that length of the short-circuited part of the rectangular waveguide is close to $\lambda/2=161.1$ mm and we can expect high electric field strength in this part of the RWG. Moreover, $Q_{ext(1+1)}$ dependence on short-circuiting plane position has high sensitivity to $L_{short}$ ($dQ_{ext(1+1)}/dL_{short}$ is very large).

To avoid an overvoltage in the short-circuited part of the rectangular waveguide we considered the other form of the short-circuited surface shown in Fig.3.

In this design short-circuiting surface consists of cylindrical surface with fixed radius $R_{sh}=49$ mm and two plane surfaces tangential to the cylindrical one. Changing angle $\alpha$ we can obtain the necessary value of $Q_{ext}$.

Table 4 and Fig.4 show $Q_{ext(1+1)}$ and reference plane position dependence on angle $\alpha$ calculated with Kroll-Yu-method and MAFIA CODE.
Fig. 1. RWG coupler for two M×N-cells TESLA supercavity with the short-circuiting plane surface.

Table 3

| Q_{ext}^{(1+1)} and reference plane position dependence on the short-circuiting plane surface position |
Fig. 2. $Q_{\text{ext}(1+1)}$ and reference plane position dependence on the short-circuiting plane surface position
Fig.3. RWG coupler for two M×N-cells TESLA supercavity

Table 4
$Q_{ext}(1+1)$ and reference plane position dependence on $\alpha$
Fig. 4. $Q_{ext}^{(1-1)}$ and reference plane position dependence on $\alpha$. 
From these data one can obtain angle $\alpha_{op}=58.5^\circ$ and $Q_{ext(1+1)}=121733$ (angle between plane surfaces is equal to $\beta_{op}=63^\circ$).

Such short-circuiting surface provides the necessary value of $Q_{ext(1+1)}$ and low level of the electric field strength in the rectangular waveguide. Due to 0-phase shift in the field oscillation of the neighbouring cells coupled with the RWG there is no additional $\lambda/2$-length beam pipe as in the case of symmetrical coaxial coupler.

### IV. Electromagnetic field distribution in the RWG coupler and the first cell

The first bunch passes the cavity at the time moment $t_1=570.534$ $\mu$s corresponding to zero input reflection coefficient (see Table 2a). After this time moment input reflection coefficient is very close to zero under the beam loading condition for a given input power $P=1.327$ MW, bunch parameters $q_b=5.815$ nC, $f_b=1.413$ MHz and cavity parameters $Q_{ext}=3.362\times10^6$, $K_{loss}/N_{cell}=2.13071\times10^{11}$ V/C ($E_{acc\,eff}=25\times10^6$ V/m).

To study a field distribution along the beam line we calculate the field distribution under the travelling wave regime in the beginning part of the input RWG. The geometry used for the field distribution simulation is shown in Fig.5. We use half RWG and one cell. The plane $y=0$ is a symmetry plane of the investigated system and there are no $E_y$ and $B_y$ field components in this plane.

![Fig.5. One cell geometry used for the simulation of the field distribution in the RWG coupler and cavity](image-url)
We use two planes with e-e (electric-electric) or m-m (magnetic-magnetic) boundary conditions in our calculation to simulate travelling wave regime in the input RWG. We choose a position of the plane in the input RWG in such way to obtain the same frequency for both e-e and m-m boundary conditions $f_{ee}=f_{m-m}=f_{op}=1.3$ GHz. The proper linear combination of the two electromagnetic fields permits us to simulate travelling wave condition in the input RWG and correspondingly field amplitude and phase distribution along the beam line (z-axis) and x-axis. Three field components $E_z$, $E_x$ and $cB_y$ (here $c$ is velocity of light) are of great interest ($E_y=cB_x=0$ in the y=0 plane).

Fig.6 and Fig.7 show the amplitude and phase distribution along x-axis for $E_z$ and $cB_y$ components at $z=1.5$ mm and $y=0$. Here and later field components correspond to the input power $P=1.327$ MW. One can see that there is travelling wave in the beginning part of the input RWG (linear dependence of the phase on x-coordinate and constant amplitude) and standing wave regime in the vicinity of the beam pipe ($R_{b,p}=39$ mm).

The amplitude of $E_z$-component in the beginning part of the input RWG is equal to $7.509 \times 10^5$ V/m and increases up to $2.077 \times 10^6$ V/m on the beam line in the middle plane of the RWG. The coordinates $x=\pm39$ mm correspond to the beam pipe radius. The amplitude of the $cB_y$-component is equal to $1.446 \times 10^5$ V/m on the beam line at $z=1.5$ mm and $cB_y=5.374 \times 10^5$ V/m in the beginning part of the input RWG. Both $E_z$ and $cB_y$ components in the beginning part of the input RWG correspond to the power $P=1.327$ MW of the travelling wave in the RWG with cross section $165.1 \times 30$ mm$^2$. The $E_z$-component increases only in the beam pipe region and is not too large out off this region. Thus there is no overvoltage in the RWG and electromagnetic field has a good symmetry.

Fig.8, Fig.9, Fig.10 show the amplitude and phase distribution along z-axis for the $E_z$, $cB_y$ and $E_x$ components. One can see that $E_{z,\text{max}}=49$ MV/m on the cavity axis at $z=115.3$ mm (in the middle plane of the cavity cell), $cB_y$ and $E_x$ components have not too large values on the z-axis ($cB_{y,\text{max}}/E_{z,\text{max}}=0.00295$, $E_{x,\text{max}}/E_{z,\text{max}}=0.000251$ and $cB_{y,\text{max}}/E_{x,\text{max}}=11.78$).

The $cB_y$ and $E_x$ components strongly decrease in the RWG region and are negligible in the beam pipe and in the first cell of the cavity (see Fig.9 and Fig.10).

Now we have all the necessary data to estimate the transverse kick caused by electromagnetic fields in the RWG coupler.
Fig. 6. $E_z$ amplitude and phase distribution along x-axis
(y=0 and z=1.5 mm)
Fig. 7. $cB_y$ amplitude and phase distribution along x-axis
(y=0 and z=1.5 mm)
Fig. 8. $E_z$ amplitude and phase distribution along z-axis 
(x=y=0)
Fig. 9. $cB_y$ amplitude and phase distribution along z-axis
(x=y=0)
**Fig. 10.** $E_x$ amplitude and phase distribution along $z$-axis 
($x=y=0$)
V. Estimation of RF kicks through the RWG coupler and two 4×7-cells supercavities

First of all let us estimate RF transverse kick through the RWG coupler caused by the \( E_x \) and \( cB_y \) components on the beam line. There are only these two components creating x-component of the kick in the plane \( y=0 \).

\[
\Delta P_x = \frac{q}{c} \int_{-L}^{+L} \left\{ E_x(0,0,z) - j \frac{\lambda}{2\pi} \left( \frac{\partial E_x(0,0,z)}{\partial z} - \frac{\partial E_z(0,0,z)}{\partial x} \right) \right\} e^{\frac{j2\pi x}{\lambda}} \, dz \tag{2}
\]

where \( E_x \) and \( E_z \) are complex quantities characterized by amplitudes (real positive value) and phases,

\( q \) is a charge of the test particle,
\( c \) is light velocity,
\( L = 3\lambda/4 \),
\( j \) is imaginary unit.

In our estimation we can suppose \( E_x(0,0,\pm L)=0 \) (see Fig.10) and expression (2) takes the following form

\[
\Delta P_x = \frac{q}{c} \frac{j \lambda}{2\pi} \int_{-L}^{+L} \frac{\partial E_z(0,0,z)}{\partial x} e^{\frac{j2\pi x}{\lambda}} \, dz
\]

Here \( E_z(x,y,z) = E_{z0}(x,y,z)e^{j\phi(x,y,z)} \) \tag{3}

\( E_{z0}(x,y,z) \) is an amplitude and
\( \phi(x,y,z) \) is a phase of \( z \)-component of the electric field

Fig.8 shows \( E_{z0}(0,0,z) \) and \( \phi(0,0,z) \) distribution along the \( z \)-axis. Using last expression one can obtain

\[
\Delta P_x = \frac{q \lambda}{c 2\pi} \int_{-L}^{+L} \left\{ \frac{\partial E_{z0}(0,0,z)}{\partial x} + jE_{z0}(0,0,z) \frac{\partial \phi(0,0,z)}{\partial x} \right\} e^{\frac{j(2\pi z + \phi(0,0,z) + \pi)}{\lambda}} \, dz \tag{4}
\]
Fig. 11 shows absolute value and phase of the quantity \( j(\lambda/2\pi) \times [\partial E_x(0,0,z)/\partial x] \) as function of \( z \).

Estimated transverse kick is equal to \( 9.804 \times 10^3 \) -j3.647 eV/c. At the same time the longitudinal kick in our case is equal to \( 1.6142 \times 10^8 \) eV/c. Thus \( \text{Re}(\Delta P_x)/\Delta P_x = 6.074 \times 10^5 \) (56 cells cavity).

The next figures show \( E_z \) amplitude and phase distribution along x-axis for different values of z-coordinate:

- \( z=10.5 \) mm corresponds to RWG region,
- \( z=17 \) mm corresponds to the beginning part of the beam pipe,
- \( z=54.7 \) mm corresponds to the end of the beam pipe,
- \( z=114.3 \) mm corresponds to the middle region of the cell.

In all cases \( y=0 \) (the symmetry plane of the investigated system). The points with \( E_z=0 \) and phase=0 lie out off the studied system.

One can see that \( E_z \)-component has good amplitude and phase symmetry at \( z>54.7 \) mm (end of the beam pipe).
Fig. 12b  \( z = 17 \text{ mm} \)
Fig. 12c

\[ z = 54.7 \text{ mm} \]
Fig. 12d  \( z = 114.3 \text{ mm} \)

Fig. 12.  \( E_z \) amplitude and phase distribution along x-axis for different values of \( z \)

In the vicinity of the cavity axis (|x|<2 mm) \( E_z \)-component has a maximum 0.517% asymmetry in the x-direction (in the RWG middle plane) and there is no asymmetry in the cell (|z|>54.7 mm).
VI. Test cavity

A one cell test-set up was designed for a high power test of the RWG coupler. The schematic design of the RWG coupler and the single resonator is shown in Fig.13.

![Diagram of RWG coupler for one cell TEST-cavity](image)

**Fig.13. RWG coupler for one cell TEST-cavity**

To allow a test of the high power travelling wave regime without beam and superconducting environment, the test cavity may be fabricated of copper. $Q_0$-factor of the copper cavity is equal to 24220 and external $Q$ factor $Q_{ext} = Q_0$ is needed to provide the travelling wave condition in the input waveguide. The RWG coupler has the same shape and dimensions as in the RWG coupler for the two TESLA supercavities. To provide the necessary value of $Q_{ext} = 24220$ we change the length of the beam pipe connecting RWG and one cell cavity.

The $Q_{ext}$ dependence on the beam pipe length calculated with Kroll-Yu-method and MAFIA CODE is shown in Fig.14.

One can see that $Q_{ext} = 25336$ (close to necessary value) corresponds to the beam pipe length $L_{b.p.} = 24.3$ mm. Other dimensions of the RWG coupler
and one cell cavity are shown in Fig.14. The shape and dimensions of the one cell cavity correspond to the 1-st cell of the TESLA cavity.

![Graph showing the dependence of external Q-factor on beam pipe length](image)

**Fig.14. The dependence of external Q-factor of the one cell TEST-cavity on the beam pipe length \( L_{b.p.} \).**

### VII. Conclusion

A rectangular waveguide coupler for a double 4×7-cell TESLA supercavity (56 cell) is developed. To provide the necessary external Q-factor of the 4×7-cell TESLA supercavities the dimensions of the short-circuiting surface are determined. Such coupler provides a transmission of the high power level at low field strength and good field symmetry.

Field asymmetry, longitudinal and transverse kicks are estimated. It is shown that the ratio \( \text{Re}(\Delta P_z)/\Delta P_z=4.9\times10^{-5} \) for the 56-cell cavity. This estimation was carried out under the travelling wave regime in the beginning part of the rectangular waveguide corresponding to the beam loading effect.
A one-cell copper test cavity with RWG coupler was developed for the high power test of the RWG coupler at room temperature.

VII. References

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