Experimental Determination of the Electrical Axis in TESLA Cavities

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Abstract Determining the center and eccentricities of the individual cells in a nine-cell TESLA cavity for quality control in cavity production has been a manual and rather imprecise measurement. Accuracies of 0.1 mm are difficult to obtain because of angular inaccuracies as well as by the physics of the measurement technique. A new method is introduced which shows good promise to measure the electrical center of the TESLA cavity by utilizing intrinsic properties of the family of TE_{111} cavity modes through the use of a perturbing dielectric rod and MAFIA code.

I. Introduction and Background.

The current measurement technique and laboratory setup for finding the eccentricities of cells in the TESLA accelerating cavities is shown in Fig. 1. The cavity is laid upon two sets of two bearings at the ends of each beampipe. A bearing measurement mechanism is placed upon the equator of each cell and a distance/angle measurement is performed. In short, the geometry of the exterior of the cavity is measured and no measurement of the geometry of the interior of the cavity is performed. The goal of this measurement procedure is to determine whether the cavity was made according to the specification of roundness and concentricity. The interpretations of the results for this measurement are such that if an individual cell is far
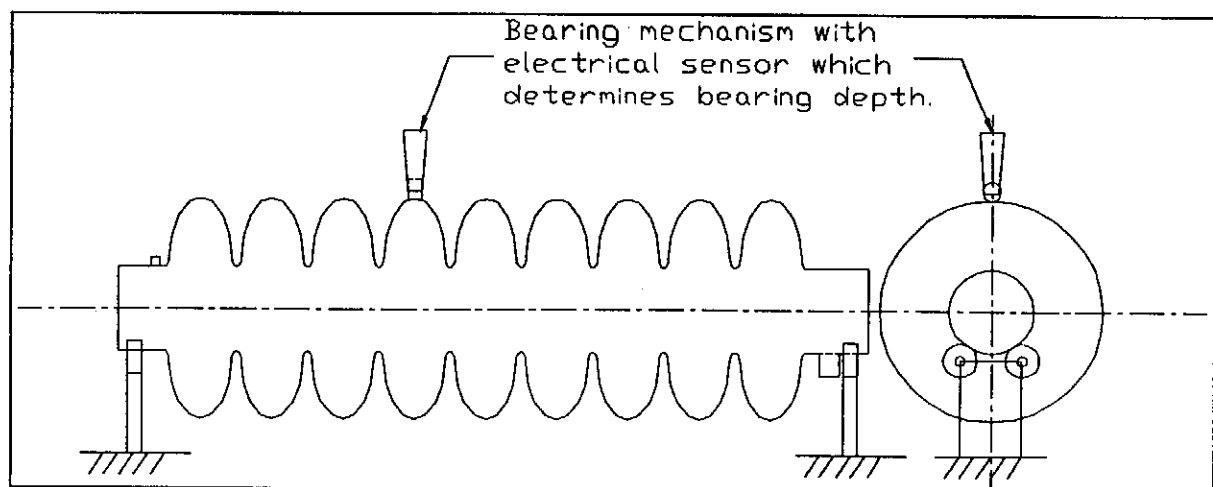

Figure 1. Schematic of the current measurement technique for finding the eccentricities of the individual cells of the TESLA accelerating structure.

enough off-axis then undesired coupling will occur between the beam and HOM's (Higher Order Modes) of the cavity. This accuracy of this measurement relies upon the following assumptions:

♦ the thickness of the niobium in each cell is constant,

♦ the quality of the weld from each ½ cell is ideal and smooth,

♦ the beampipes at each end of the cavity are concentric,

♦ the surface quality of the beampipes at the position where the four bearings hold the cavity in place is both smooth and round.

If any of these assumptions are violated then an incorrect interpretation of the eccentricity measurement can result. These assumptions are seemingly strong and can be readily violated in many of the cavities that have been studied.

An alternative to this measurement procedure is to make field measurements inside of the cavity to find the electrical axis for the cavity. The electrical axis is a position inside the cavity where the beam has minimum coupling to dipole modes. The electrical axis may or may not coincide with the geometrical axis because of imperfections in each cell of a cavity. Consider the laboratory setup shown in Fig. 2. Depicted in Fig. 2 is a vertically held TESLA cavity, a NWA (network analyzer), an amplifier, a long thin dielectric rod with a low permittivity, two antennaes, and a weight to
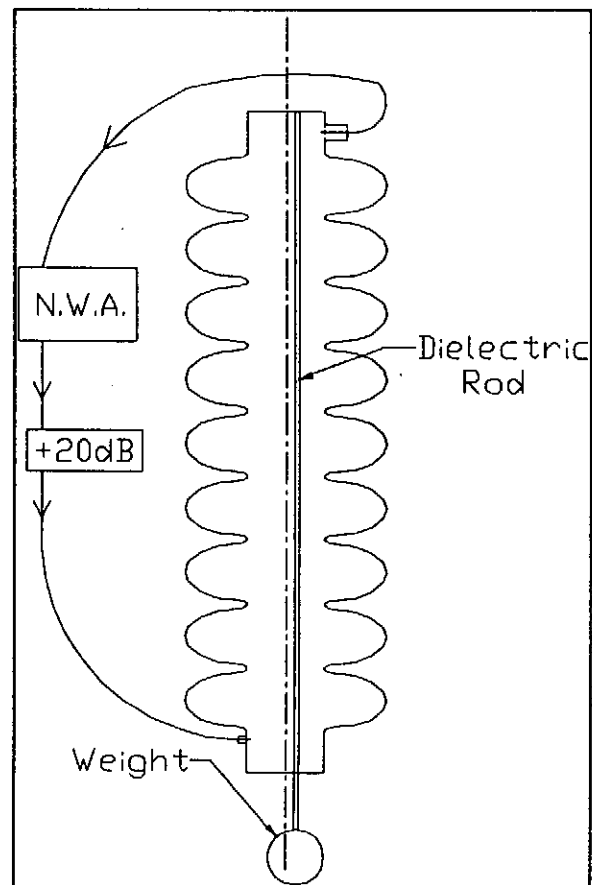

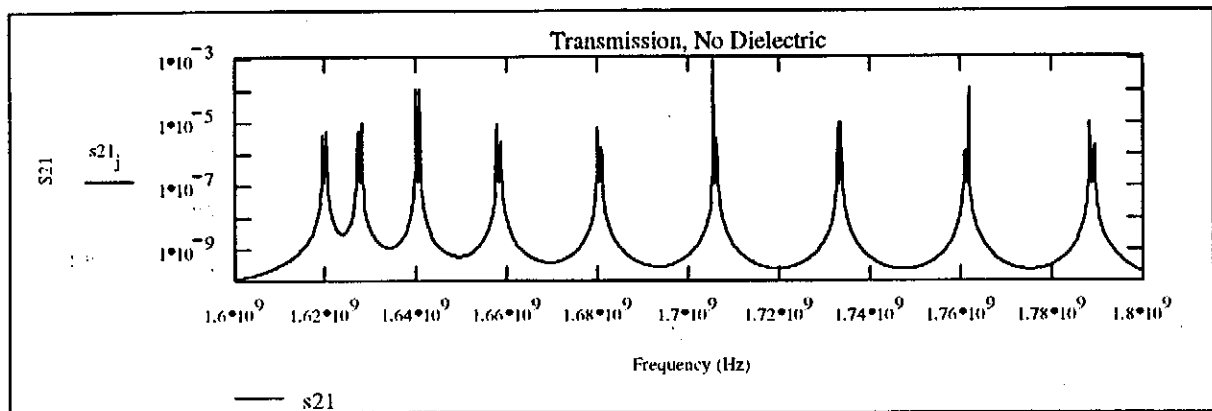**Figure 2.** Alternative to current eccentricity measurement.

Figure 3. Sample measurement on a NWA. Note the double peaks at each resonance caused by imperfections in the building of the cavity. The double peaks result from the imperfections in the cavity and the polarization of the family of TE_{111} modes.

hold the dielectric rod vertical. The cavity is held vertically so that the dielectric rod does not sag from the effect of gravity. The frequency span of the network analyzer is set so that the family of TE_{111} modes are excited in the cavity. A polarization in the angular, or φ direction, exists for the family of TE_{111} cavity modes. Since a cavity is never perfectly round, the resonant frequency of each polarization will be slightly different. The network analyzer will therefore show eighteen individual peaks because of the different frequencies caused by the aforementioned polarization inherent to TE_{111} modes. The double resonances from the different polarizations are shown in a typical measurement depicted in Fig. 3.

From simple perturbation theory [1] it is clear that placing a dielectric into a cavity decreases the resonant frequency for each resonant mode of the cavity. The frequency shift is dependant on the dielectric and the location that the dielectric is placed. If the dielectric is placed in a high field region, the frequency shift is greater than when the dielectric is placed into a low field region. Examine, for the moment, the first set of peaks in Fig. 3, depicted in Fig. 4, and imagine that the fields of the first peak depends on $\cos(\varphi)$ while the second peak depends on $\sin(\varphi)$. When the dielectric is inserted into the $\varphi=0°$ position, it is effectively inserted into a high-field region for the first peak and is likewise inserted into a low-field
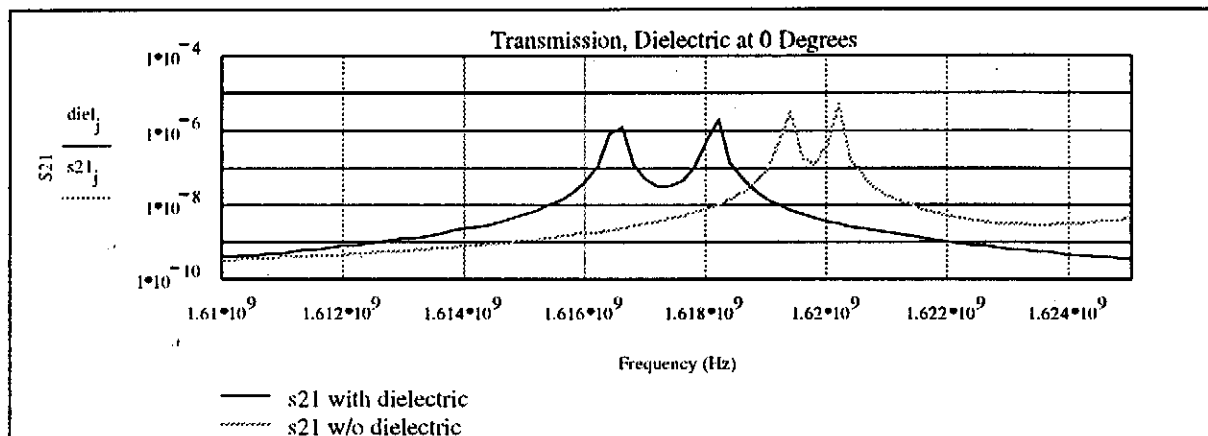
Figure 4. Detailed view of the first set of peaks from Fig. 3 for the case of a dielectric in the cavity and for no dielectric in the cavity. The dielectric is inserted into a high field region for the first peak and into a low field region for the second peak. Consequently the peaks of frequencies for the dielectric measurement separate further from the measurement without the dielectric.

region for the second peak. This causes the difference between the frequencies of the peaks to

separate more from their unperturbed state.

Now imagine that the dielectric is inserted into the φ=90° position depicted in Fig. 5.

For this scenario the dielectric is effectively inserted into a low-field region for the first peak

and is inserted into a high-field region for the second peak. This causes the difference between

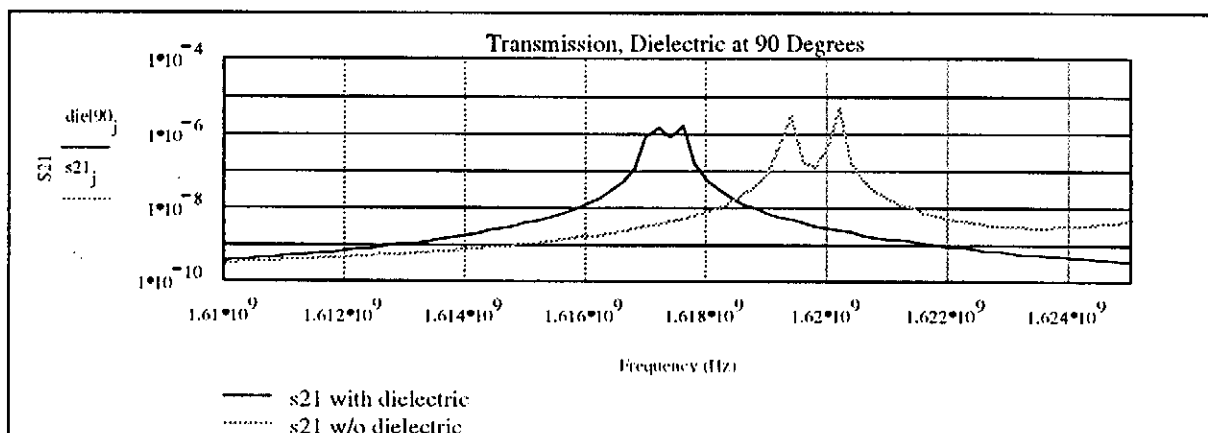the frequencies of the peaks to become smaller.

Figure 5. Detailed view of the set of peaks from Fig. 3 for the case of a dielectric inserted at a high field region for the second peak and in a low field region for the first peak. The resulting dielectric measurement shows that the frequencies of the two resonances come closer together than in the measurement without the dielectric.

From the data in Figs. 4 and 5, it is possible to use the resonant frequency data of the separated polarized peaks to find the electrical center of the TESLA cavities using simple perturbation theory derived from the following relationship:

$$\frac{\omega - \omega_o}{\omega_o} \approx -\frac{\Delta\varepsilon \int_{dielectric} \vec{E}_{dielectric} \cdot \vec{E}^*_o}{2\int_{cavity} \varepsilon_o |\vec{E}_o|^2}.$$

In this relationship, ω is the perturbed resonant frequency, ω_o is the unperturbed resonant frequency, $\vec{E}_{dielectric}$ is the electric field inside the dielectric, * represents the complex conjugation procedure, $\vec{E}_o$ is the unperturbed electric field, $\Delta\varepsilon$ is the difference between the relative permittivity of the dielectric rod and of free space, and ε_o is the permittivity of free space. The data and theory for this relationship may be found in [1].

The graphs shown in Figs. 3, 4, and 5 assume to have equal coupling with the antennae setup in Fig. 2 and results in a measurement of two modes overlapping. If one of the modes of a mode pair is more strongly coupled to than the other, then it is possible that the response of the more strongly coupled mode may completely overlap the weakly coupled mode because of the small frequency difference between the resonant peaks. The measurement procedure requires an accurate measurement of the resonant frequency of each mode to be performed as a function of position of the dielectric rod. The best results for the measurement of a single peak in a mode pair are obtained when the desired peak in the mode pair is tuned such that it completely overlaps the undesired peak of the mode pair. This is, of course, not important if the quality factor ("Q") is sufficiently high enough such that the frequency response of each mode pair do not overlap. The widths of the peaks are dependant on the conductivity of the

material of the resonant cavity and for higher conducting materials the overlapping of modes for each polarization becomes less of a practical problem.

II. Measurement Results and Interpretations.

The unperturbed fields for Eq. 1 were calculated using MAFIA with a model for the TESLA cavities. The perturbed fields were calculated from the theory included in [1] and assumed a ±1 mm offset at a diameter of 15 mm for a PVC dielectric rod with a 6 mm diameter. The niobium TESLA cavities A13, A14, and S9 were chosen for this measurement procedure. This procedure was iterated for each of the set of the 18 peaks of the TE_{111} modes and the experimental results are shown in Figs. 6, 7, 8, 9, 10, and 11.

The data in Figs. 8 and 9 only show the first seven modes for the A14 cavity. For this particular cavity, it was not possible to make a suitable measurement of the highest two mode pairs. This was the result of two particular problems. First, the mode separation for these modes was too small and it was not possible to excite a single mode of each pair. This is to say that the measurement showed that the cavity began to have an appearance of very "round." Second, the two highest mode pairs have high fields in the end cells and therefore it was difficult to only excite a single mode in each mode pair.
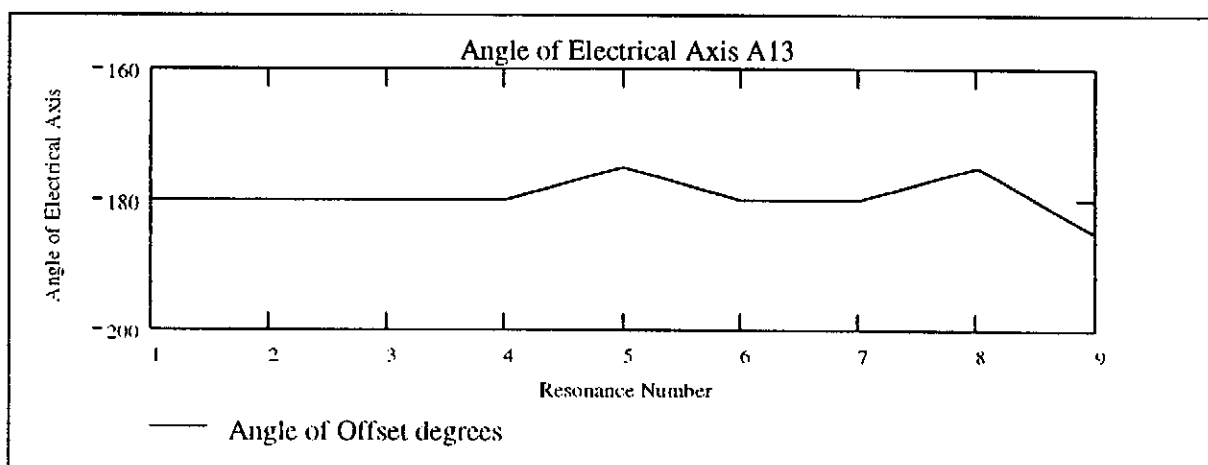

Figure 6. The angular position for the electrical axis for each resonances for the family of TE_{111} modes for the TESLA A13 cavity.
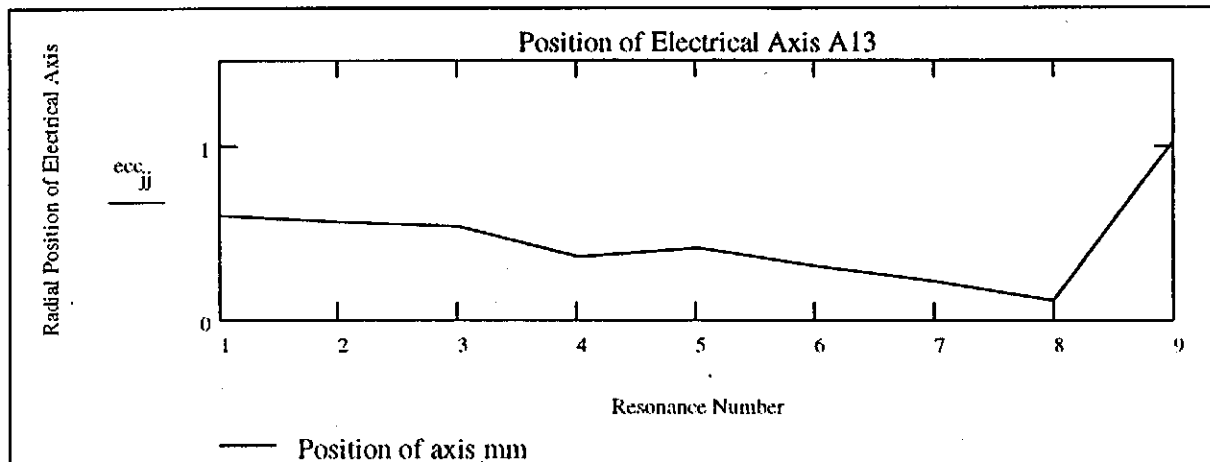
Figure 7. The radial position for the electrical axis for each resonance for the family of TE_{111} modes for the TESLA A13 cavity.

The family of TM_{110} modes were also measured and studied. Introducing a dielectric to a cavity tuned to this family of modes strongly perturbed the fields of these modes. This is because these modes have relatively strong fields in all of its radial, axial, and azimuthal directions. It was therefore difficult to find a simple linear relationship which can de-embed
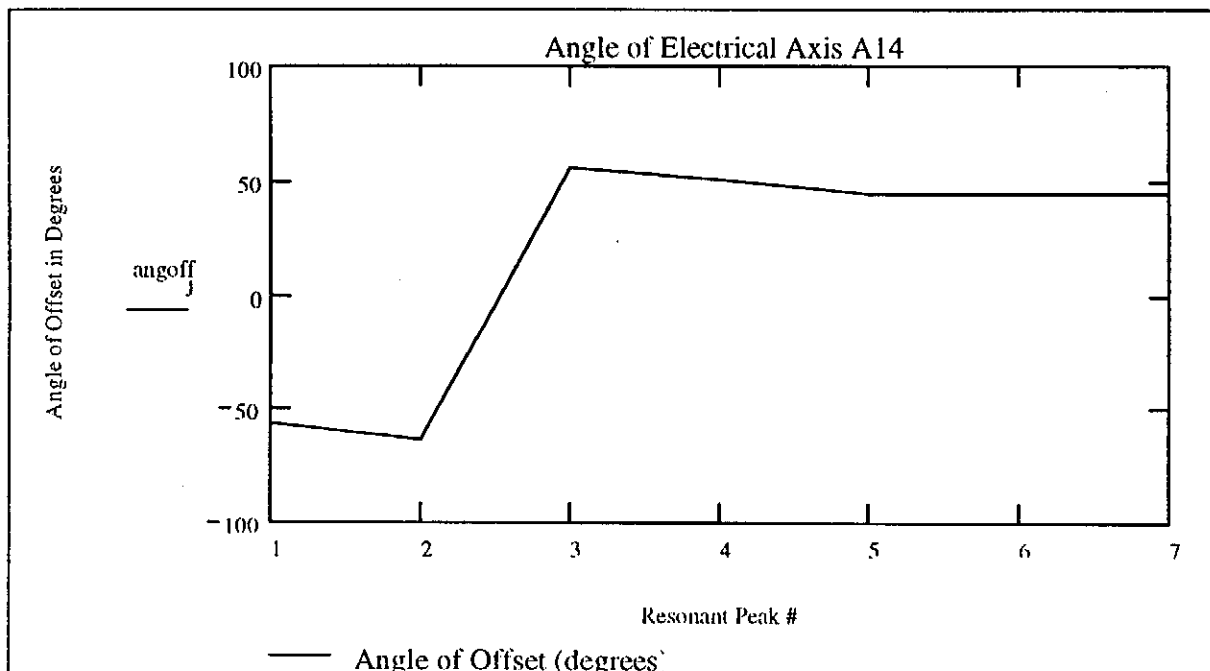
Figure 8. Angle measurement for the TESLA A14 cavity. The highest two peaks are not included because the cavity measurement showed that the cavity was very round and it was not possible to tune to a single peak of these mode pairs.
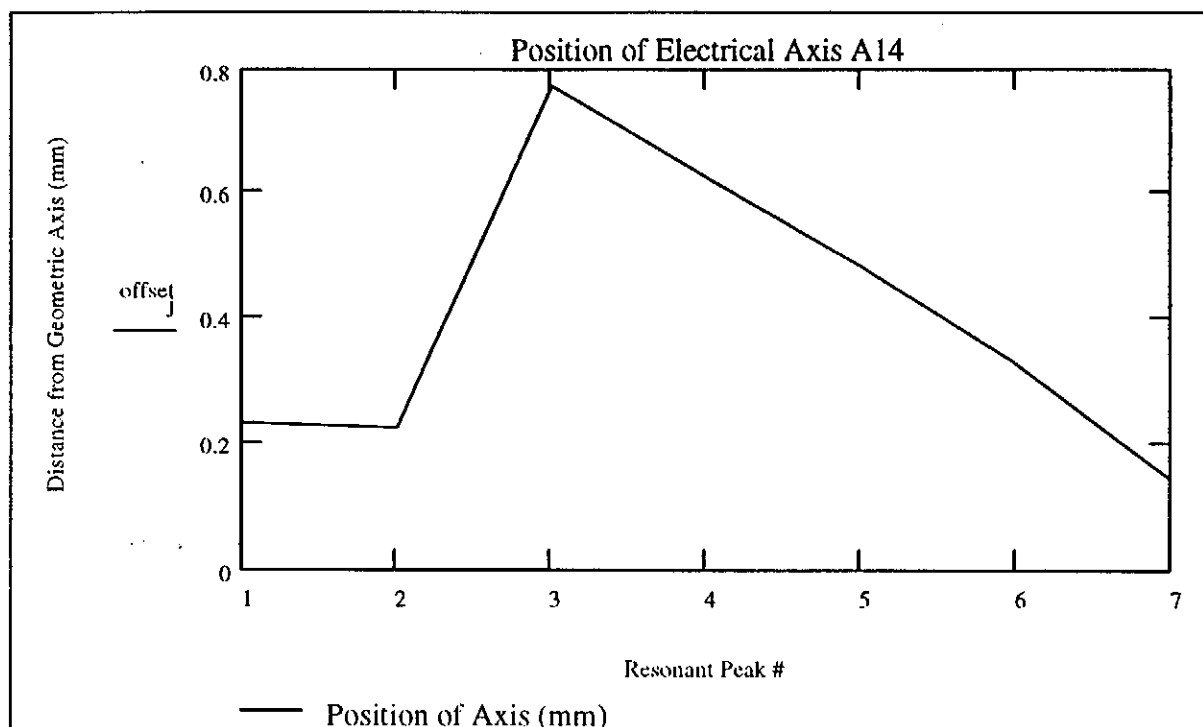
Figure 9. Radial position measurement for the TESLA A14 cavity. The highest two peaks are not included because the cavity measurement showed that the cavity was very round and it was not possible to tune to a single peak of these mode pairs.

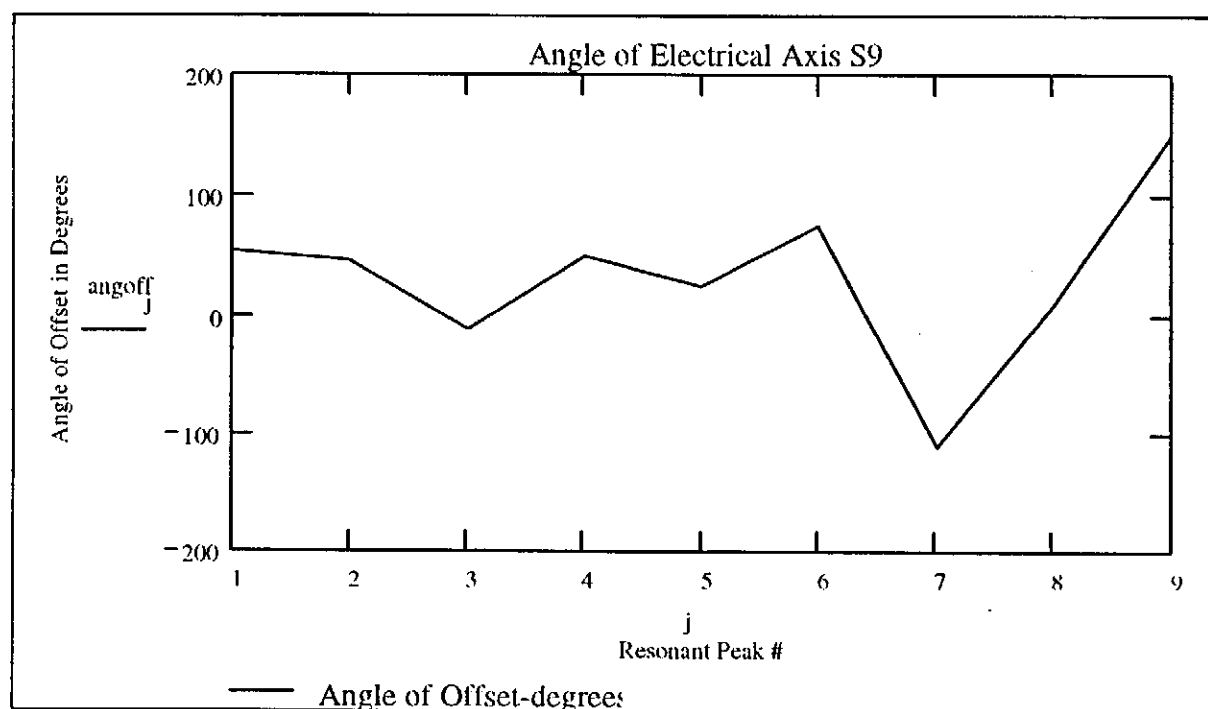the frequency measurements to find the electrical axis. Using a smaller dielectric and placing it

Figure 10. Angular measurements on the TESLA S9 cavity. This cavity had no fixed HOM couplers.
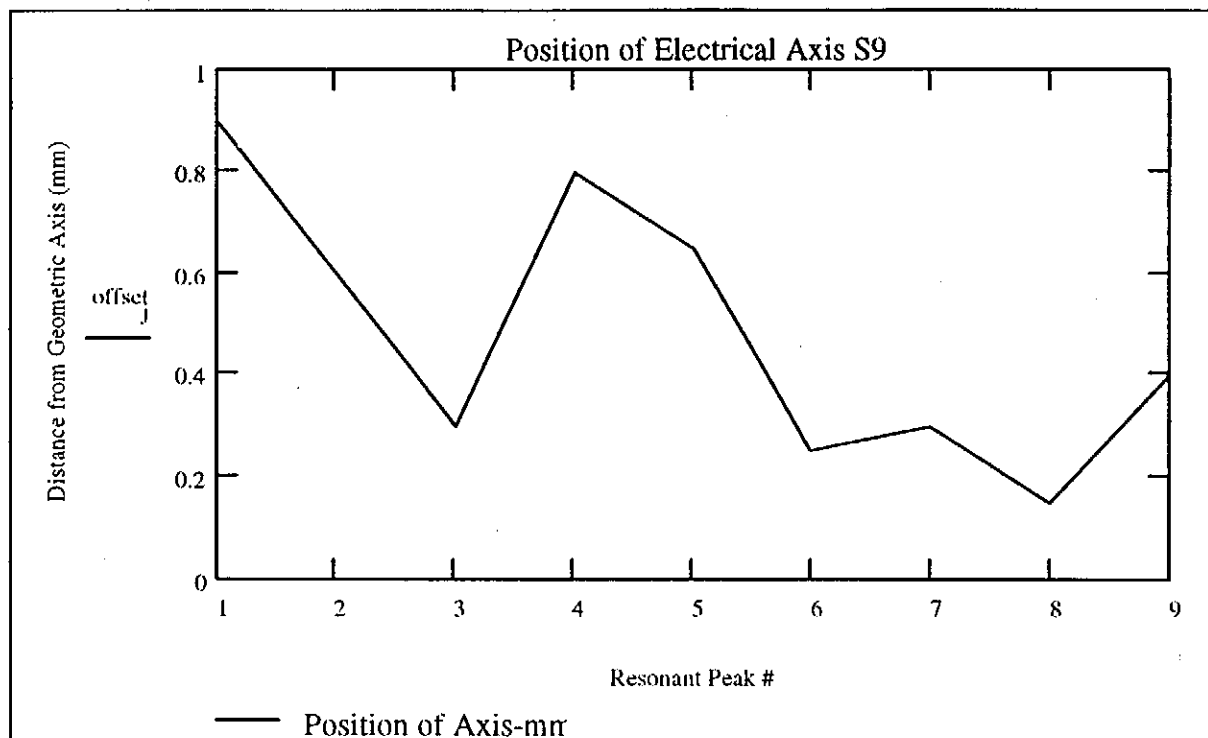
Figure 11. Radial measurement of the TESLA cavity S9. This cavity had no fixed HOM couplers.

closer to the geometrical axis may circumvent this problem.

III. Conclusions and Future Anticipations.

Using a dielectric rod to measure the electrical axis for a cavity is a direct and relatively simple way to find the electrical axis of a cavity. The position of the electrical axis is not, in general, constant for each family of modes for a particular cavity. The experimental results using the rod compare to the results of the manual measurement technique shown in Fig. 1. The errors in measurement can be minimized by temperature control of the cavity as well as by more a more automated process. The automated process can be quicker than a manual measurement and therefore temperature effects can be reduced. Controlling the swing or pendulum effect of the weight will also expedite the measurement process. Use of HOM couplers is important to hold the different polarized fields from rotating when the dielectric is moved through the cavity. The family of TM_{110} modes had particular problems with the

introduction of the dielectric to the cavity since the dielectric severely perturbed the fields in

the cavity. These modes may be also used to measure the electrical axis if a smaller dielectric

is used and placed closer to the geometrical axis.

[1] Roger F. Harrington: "Time-Harmonic Electromagnetic Fields," pp. 317-380, McGraw-Hill Book Co, New York, 1961