Relative Nonuniformity in The Amplitude of the Accelerating Field Along the M×N-cell TESLA Supercavities

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Abstract

The undesirable nonuniformity in the amplitude of the accelerating field along the M×N-cell TESLA supercavity caused by a random cells detuning is investigated. The dependence of the r.m.s. deviation of the relative accelerating field amplitude on the number of cells in the cavity is obtained for different M×N-cell TESLA supercavities. It is shown that M×N-cell TESLA supercavities possess a stabilization effect in the amplitude of the accelerating field distribution along the cavity. An analytical expression for an estimation of the nonuniformity in the amplitude of the accelerating field along the cavities with an operational π-mode is obtained.

I. Introduction

It is wellknown that the cavities with an operational π-mode have the biggest nonuniformity in the amplitude of the accelerating field along cavity due to random cells detuning. There are so called stabilized structures consisting of coupled cells, some of which are not excited and functions as a resonant coupling [1,2,3,4] between parts of the structure with an operational π-mode. The nonuniformity in the amplitude of the accelerating field along such structures has been investigated in [5] where analytical expressions for the nonuniformity in the amplitude of the accelerating field along the structure were obtained.

The M×N-cell TESLA supercavities [6] with an operational 0-π-mode consist of M subcavities coupled by λ/2-length beam pipes (λ=230.61 mm, operational frequency f_0=1.3 GHz). Coupling beam pipes are used for HOM damping with HOM couplers in each of these beam pipes. Field oscillations in neighbouring cells of the subcavity have π-phase shift while the field oscillations of neighbouring cells separated by beam pipes have 0-phase shift. Thus we can expect some field amplitude stabilization along the cavity using 0-π-accelerating mode.

To simulate field amplitude distribution along the cavity in the presence of random cells detuning and to estimate the nonuniformity in the amplitude of the accelerating field caused by random cell detuning we use an equivalent circuit of the investigated cavity which is described by the following system of equations

$$-\frac{K_{n-1}}{2}E_{n-1}+\left(1-\frac{f_0^2}{f_{cn}^2}\right)E_n-\frac{K_n}{2}E_{n+1}=0 \qquad (1)$$

Here E_n is field amplitude in the n-th cell,

$K_n/2$ is a coupling coefficient between n-th and (n+1)-st cell,

f_{cn} is a frequency of the n-th cell,

f_0 is a free oscillation frequency of the operational mode.

We consider a lossless cavity with the following cell-to-cell coupling coefficients

$K_n=K_{s\text{-}s}$=0.0020171 for n=N, 2N, 3N,…, (M-1)N,

$K_n=K_c$ =0.0188988 for n≠N, 2N, 3N,…, (M-1)N.

The cell frequencies of a perfectly tuned M×N-cells supercavity with 0-π-accelerating mode at f_0 have the following values:

$$f_{cn} = \frac{f_0}{\sqrt{1+\frac{K_c}{2}}}, \qquad \text{for } n = 1, MN$$

$$f_{cn} = \frac{f_0}{\sqrt{1+\frac{K_c}{2}-\frac{K_{s-s}}{2}}}, \text{ for } n = N, N+1, 2N, 2N+1, \ldots, (M-1)N, (M-1)N+1 \qquad (2)$$

$$f_{cn} = \frac{f_0}{\sqrt{1+K_c}}, \qquad \text{for any other } n$$

To simulate the amplitude nonuniformity of the accelerating field caused by random cells detuning we assume that each cell has a frequency $f_{cn}+\delta f_{cn}$, where δf_{cn} is a random frequency shift.

The amplitude nonuniformity of the accelerating field along the cavity is characterized by the r.m.s. quantity $\sigma_{\Delta E/E}$, which is defined as follows

$$\sigma_{\Delta E/E} = \lim_{J\to\infty}\left\{\frac{1}{J}\frac{1}{N_{cells}-1}\sum_{j=1}^{J}\sum_{n=1}^{N_{cells}}\left(\frac{E_{n,j}-E_{av\,j}}{E_{av\,j}}\right)^2\right\}^{\frac{1}{2}} \qquad (3)$$

$$E_{av\,j} = \frac{1}{N_{cells}}\sum_{n=1}^{N_{cells}} E_{n,j}$$

Where: $\sigma_{\Delta E/E}$ is r.m.s. of the relative nonuniformity in the amplitude of the accelerating field along the cavity,

- j is a random selection number,
- J is the total number of the random selections (J=5000 in our simulation),
- N_{cells} is the number of cells of the cavity (N_{cells}=MN),
- $E_{n,j}$ is a field amplitude in the n-th cell corresponding to the j-th random selection,
- $E_{av\,j}$ is average field amplitude in the cavity corresponding to the j-th random selection.

II. Relative nonuniformity in the amplitude of the accelerating field along the M×N-cell TESLA supercavities

We consider the relative nonuniformity in the amplitude of the accelerating field along the M×N-cell cavities with different numbers (M,N) of cells. We use a random cells detuning with uniform distribution in the frequency range $\pm\delta f_c$ ($-\delta f_c<\delta f_{cn}< +\delta f_c$, δf_c=10 and 50 kHz). The coupling coefficients $K_{s\text{-}s}$=0.0020171 and K_c =0.0188988 correspond to the radius of the coupling beam pipes R_{cbp}=57 mm and the radius of the irises opening R_i=35 mm. The cells have a Q_0-factor $Q_c=10^{10}$.

Table 1 and Fig.1 show simulation results for such M×N-cell TESLA cavities.

Table 1

Kc: 0.018898800 Ks-s: 0.002017100
Qc: 1.000000E+10 dfc, Hz: 1.000000E+4

Nonuniformity in the Amplitude of the Accelrating Field
Along the M*N-TESLA Cavity
fop=1.3 GHz

Nselections: 5000

M	N	Ncells	$\sigma_{\Delta E/Eav}$	$\sigma_{\Delta E/Eav\ max}$	$\sigma_{\Delta E/Eav\ min}$	ampl-min
1	7	7	0.002079958719	6.732084019409E-3	1.845487321899E-4	9.834E-1
1	9	9	0.002927379736	9.032394171811E-3	1.909511707138E-4	9.769E-1
1	13	13	0.005019134399	1.697926395938E-2	4.135270930947E-4	9.553E-1
1	18	18	0.007928519897	2.678702350263E-2	5.993422786154E-4	9.299E-1
1	27	27	0.014496609273	5.032908132627E-2	9.471909364422E-4	8.656E-1
1	36	36	0.022613546550	8.004923273269E-2	1.279135856718E-3	7.873E-1
1	45	45	0.030866339567	1.297624836269E-1	1.938108809007E-3	6.939E-1
1	54	54	0.040498917238	1.562835411848E-1	2.295197193482E-3	6.423E-1
3	7	21	0.008744109145	2.690246142728E-2	8.956785963236E-4	9.228E-1
3	9	27	0.008709411928	2.253447417332E-2	1.653765600241E-3	9.212E-1
3	11	33	0.009412271818	2.188478271399E-2	1.050306730092E-3	9.147E-1
3	13	39	0.011413241417	3.278717960373E-2	1.847872228263E-3	8.922E-1
3	15	45	0.014709281511	5.283645773748E-2	1.873701953020E-3	8.415E-1
3	17	51	0.018903452849	6.797687534758E-2	2.897774317313E-3	8.103E-1
3	19	57	0.023651924466	8.245671076525E-2	2.768026583754E-3	7.756E-1
4	7	28	0.011459406813	4.054458961069E-2	9.903397055655E-4	8.838E-1
4	9	36	0.010438250135	2.982129580305E-2	1.772427173496E-3	8.998E-1
4	11	44	0.010965931912	2.699838887122E-2	1.675245698883E-3	8.844E-1
4	13	52	0.014388941294	4.252914372289E-2	2.379917823672E-3	8.661E-1
4	15	60	0.019629463141	6.490912357646E-2	2.792243620891E-3	8.019E-1
5	7	35	0.014034073732	4.380634054730E-2	2.640938422756E-3	8.765E-1
5	9	45	0.011889287489	3.376631853490E-2	2.773329565806E-3	8.868E-1
5	11	55	0.012528010465	3.089355638998E-2	2.827331206678E-3	8.779E-1
5	13	65	0.017135871786	5.578365234378E-2	3.319570530779E-3	8.318E-1
6	7	42	0.017170459546	5.348327803368E-2	2.860232096424E-3	8.477E-1
6	9	54	0.013780823772	3.912104978347E-2	3.294420214081E-3	8.725E-1
6	11	66	0.013778116111	3.844182950173E-2	4.167455808293E-3	8.529E-1
6	13	78	0.020610130322	6.121711884174E-2	4.218663629417E-3	8.018E-1
7	7	49	0.020103752452	6.496274210222E-2	3.042573369752E-3	8.270E-1
7	9	63	0.015298899516	4.837527829168E-2	3.402007736221E-3	8.454E-1
7	10	70	0.014110416443	3.908480767672E-2	3.849686906015E-3	8.401E-1
7	11	77	0.015046146902	3.715629575581E-2	4.688105586537E-3	8.580E-1
7	13	91	0.023924059863	8.496721177460E-2	4.659563248134E-3	7.676E-1

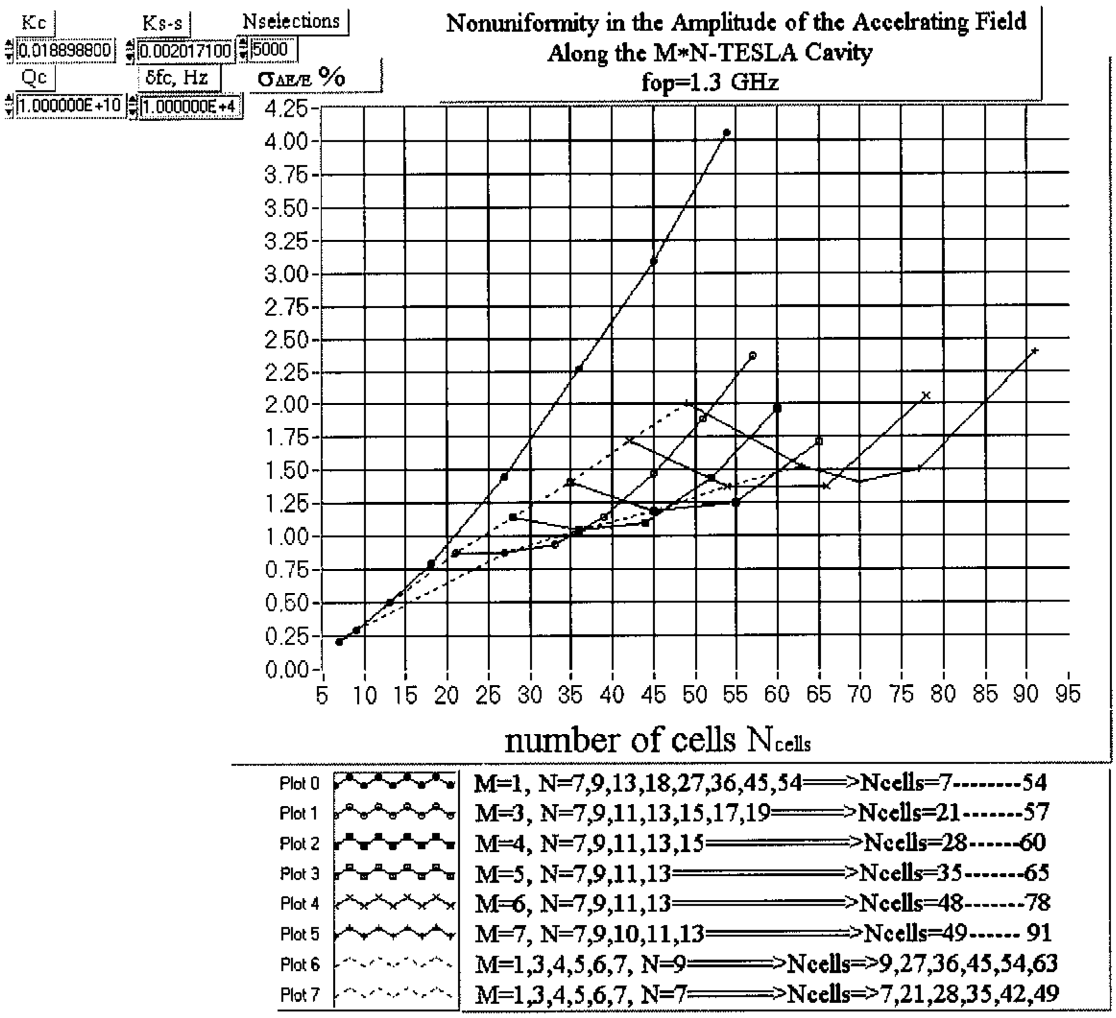

Fig.1. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=10000 Hz, $K_{s\text{-}s}$=0.0020171)

M=1 corresponds to a usual cavity without coupling beam pipes and with an operational π-mode (Plot 0 in Fig.1). One can see that the relative nonuniformity in the amplitude of the accelerating field along the cavities is the biggest for this case. The plots 1-5 in Fig.1 correspond to M=3, 4, 5, 6 and 7 subcavities. The phase distribution along the cavity was checked during the nonuniformity calculation and corresponded to an operational 0-π-mode field oscillation in the cavity.

One can see that nonuniformity in the amplitude has a minimum corresponding to N=9 for M=3, 4, 5, 6 and N=10 for M=7. Plot 6 and 7 (dashed lines) show a dependence of $\sigma_{\Delta E/E}$ on number of subcavities for N=7 and N=9.

As it can be shown that $\sigma_{\Delta E/E}$ is proportional to the relative cells detuning $\sigma_{\delta fc/fo}$ ($\sigma_{\delta fc/fo}=\delta f_c/(f_0 3^{1/2})$ in our case). We have an ability to recalculate $\sigma_{\Delta E/E}$ for other random cells detuning. But in the case of the large random cells detuning and unsuccessful combination of the cells detuning the operational 0-π-mode phase distribution can be broken.

Tables 2 - 6 and Fig.2 - 6 show simulation results for the other values of the coupling coefficient $K_{s\text{-}s}$.

Table 2

Kc: 0.018898800 Ks-s: 0.001000000
Qc: 1.000000E+10 dfc, Hz: 1.000000E+4

Nonuniformity in the Amplitude of the Accelrating Field Along the M*N-TESLA Cavity fop=1.3 GHz

Nselections: 5000

	M		N		Ncells		$\sigma_{\Delta E/Eav}$		$\sigma_{\Delta E/Eav\ max}$		$\sigma_{\Delta E/Eav\ min}$		ampl-min
0	1	0	7	0	7	0	0.002079958719	0	6.732084019409E-3	0	1.845487321899E-4	0	9.834E-1
	1		9		9		0.002927379736		9.032394171811E-3		1.909511707138E-4		9.769E-1
	1		13		13		0.005019134399		1.697926395938E-2		4.135270930947E-4		9.553E-1
	1		18		18		0.007928519897		2.678702350263E-2		5.993422786154E-4		9.299E-1
	1		27		27		0.014496609273		5.032908132627E-2		9.471909364422E-4		8.656E-1
	1		36		36		0.022613546550		8.004923273269E-2		1.279135856718E-3		7.873E-1
	1		45		45		0.030866339567		1.297624836269E-1		1.938108809007E-3		6.939E-1
	1		54		54		0.040498917238		1.562835411848E-1		2.295197193482E-3		6.423E-1
	3		7		21		0.023417482478		8.125554056965E-2		1.344737407273E-3		8.109E-1
	3		9		27		0.023838968015		8.067800082511E-2		1.682079441444E-3		7.947E-1
	3		11		33		0.023520782729		7.845244639395E-2		2.466040754339E-3		7.890E-1
	3		13		39		0.023119765309		6.682183958311E-2		2.398931789826E-3		8.038E-1
	3		15		45		0.022874704236		7.529496621222E-2		2.233574357156E-3		7.385E-1
	3		17		51		0.023075085760		6.449694211100E-2		2.823113340130E-3		7.864E-1
	3		19		57		0.023570414595		6.076414317248E-2		3.595207837787E-3		7.732E-1
	4		7		28		0.032575489297		1.111853624998E-1		2.116207601224E-3		7.406E-1
	4		9		36		0.033255384310		1.104611957117E-1		2.138938674270E-3		7.199E-1
	4		11		44		0.031853132489		1.102580209812E-1		3.170452528118E-3		7.025E-1
	4		13		52		0.030809458344		1.016198022894E-1		3.563031043529E-3		7.293E-1
	4		15		60		0.028792277290		8.765055996802E-2		3.794187879926E-3		7.538E-1
	5		7		35		0.043604188136		1.735133663202E-1		2.768456774919E-3		6.278E-1
	5		9		45		0.043101885572		1.679948256814E-1		3.780421330974E-3		6.032E-1
	5		11		55		0.040633915697		1.543695342248E-1		4.922430549966E-3		6.381E-1
	5		13		65		0.037825550715		1.270638249745E-1		4.400274983119E-3		6.667E-1
	6	?	7	?	42	?	0.110511730333	?	5.011634833697E-1	?	3.367606542530E-3	?	1.129E-2
	6	?	9	?	54	?	0.179129959404	?	5.092481373754E-1	?	6.132021496802E-3	?	3.191E-4
	6	?	11	?	66	?	0.184904608041	?	5.171800190058E-1	?	4.739092581068E-3	?	1.462E-5
	6	?	13	?	78	?	0.125837031914	?	5.223591976992E-1	?	7.087630852040E-3	?	5.693E-6
	7	?	7	?	49	?	0.412635061408	?	5.284494675654E-1	?	9.530266779123E-3	?	8.758E-6
	7	?	9	?	63	?	0.278499141293	?	5.227672635292E-1	?	7.383295506488E-3	?	8.304E-6
	7	?	10	?	70	?	0.240938871416	?	5.134480172581E-1	?	4.380814247126E-3	?	7.958E-6
	7	?	11	?	77	?	0.228328534844	?	5.114181340796E-1	?	8.199548476937E-3	?	2.590E-6
	7	?	13	?	91	?	0.267065883072	?	5.128669202952E-1	?	7.446904197030E-3	?	2.775E-5

Rows marked by the sign "?" correspond to the cases when one or more random selections of total number $N_{selections} = 5000$ don't have a 0-π-mode phase distribution along the cavity. Such r.m.s. $\sigma_{\Delta E/E}$ are not correct enough.

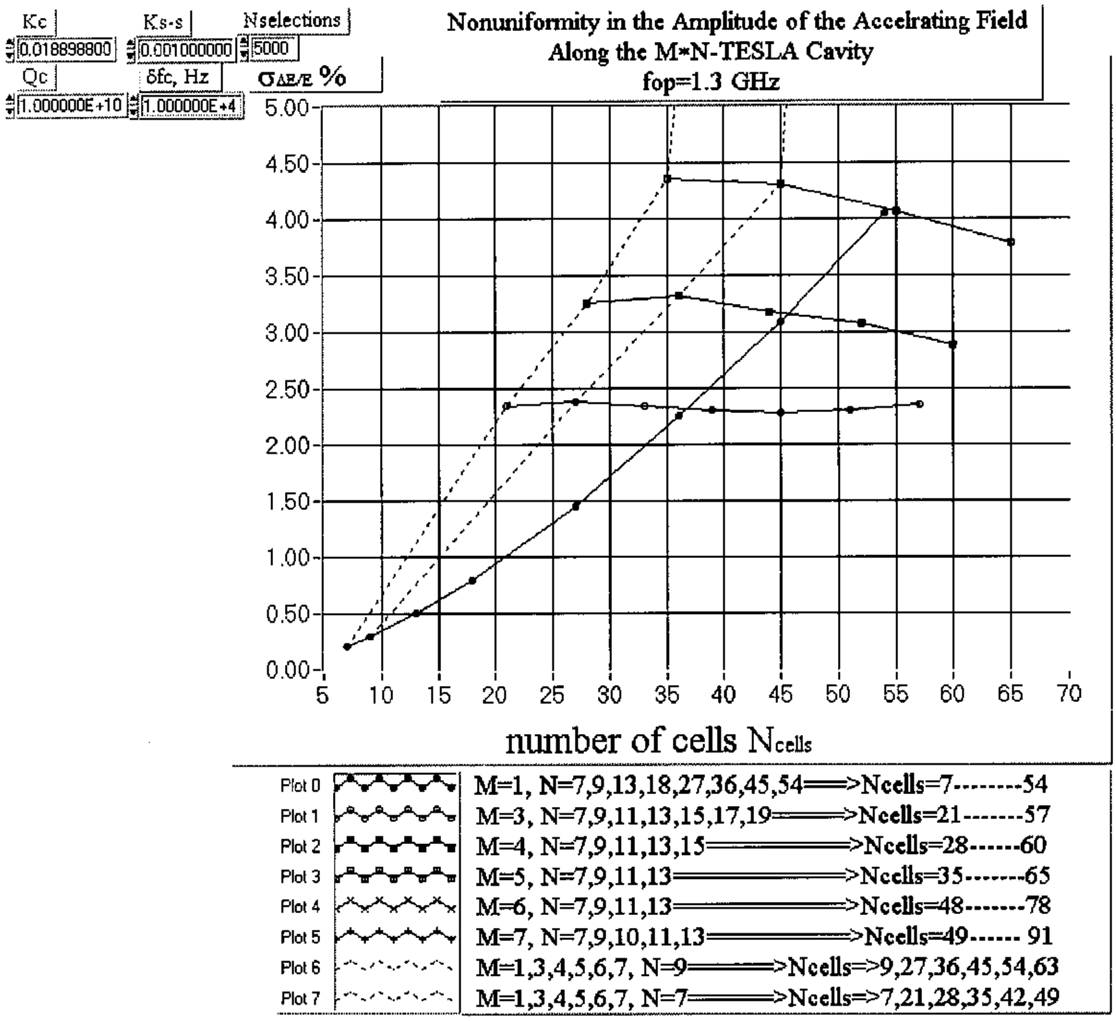

Fig.2. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=10000 Hz, K_{s-s}=0.001, plot 4 and 5 are not shown)

Table 3

Nonuniformity in the Amplitude of the Accelrating Field Along the M*N-TESLA Cavity fop=1.3 GHz

Kc: 0.018898800 | Ks-s: 0.001500000 | Qc: 1.000000E+10 | dfc, Hz: 1.000000E+4 | Nselections: 5000

M	N	Ncells	$\sigma_{\Delta E/Eav}$	$\sigma_{\Delta E/Eav\ max}$	$\sigma_{\Delta E/Eav\ min}$	ampl-min
1	7	7	0.002079958719	6.732084019409E-3	1.845487321899E-4	9.834E-1
1	9	9	0.002927379736	9.032394171811E-3	1.909511707138E-4	9.769E-1
1	13	13	0.005019134399	1.697926395938E-2	4.135270930947E-4	9.553E-1
1	18	18	0.007928519897	2.678702350263E-2	5.993422786154E-4	9.299E-1
1	27	27	0.014496609273	5.032908132627E-2	9.471909364422E-4	8.656E-1
1	36	36	0.022613546550	8.004923273269E-2	1.279135856718E-3	7.873E-1
1	45	45	0.030866339567	1.297624836269E-1	1.938108809007E-3	6.939E-1
1	54	54	0.040498917238	1.562835411848E-1	2.295197193482E-3	6.423E-1
3	7	21	0.013317368245	4.730004501980E-2	1.022576191006E-3	8.794E-1
3	9	27	0.013363300453	4.264661888410E-2	1.414156003692E-3	8.826E-1
3	11	33	0.012988236317	3.934992903608E-2	1.816670813377E-3	8.753E-1
3	13	39	0.013293889499	3.519278874470E-2	1.865191624112E-3	8.738E-1
3	15	45	0.014714200513	3.796030627373E-2	1.842980784433E-3	8.560E-1
3	17	51	0.017372206964	4.846095611633E-2	2.639456244088E-3	8.413E-1
3	19	57	0.020546946373	5.579651119028E-2	3.346204996170E-3	8.147E-1
4	7	28	0.018374249610	6.681778343352E-2	2.084956019288E-3	8.276E-1
4	9	36	0.016924363591	5.420650355721E-2	2.276064193434E-3	8.420E-1
4	11	44	0.015972682456	5.329682907109E-2	2.548798044822E-3	8.279E-1
4	13	52	0.015599374033	4.116754220841E-2	3.606004953131E-3	8.350E-1
4	15	60	0.017296718342	5.241146093439E-2	3.747142986376E-3	8.408E-1
5	7	35	0.023463833576	7.754460531451E-2	2.179787186370E-3	7.975E-1
5	9	45	0.021474112180	7.116291695794E-2	2.041630000925E-3	8.028E-1
5	11	55	0.018921811277	5.373100745896E-2	3.771821978697E-3	8.309E-1
5	13	65	0.017731405650	4.482521910830E-2	4.724850874728E-3	8.159E-1
6	7	42	0.030142139823	1.153005225513E-1	2.845909335870E-3	7.282E-1
6	9	54	0.025994881798	8.900175575152E-2	4.498278685671E-3	7.563E-1
6	11	66	0.021872837768	7.670869599533E-2	4.260815944479E-3	7.751E-1
6	13	78	0.019343007532	5.136482751861E-2	5.637287321431E-3	7.954E-1
7	7	49	0.036560060509	1.207112288609E-1	3.908016455210E-3	7.167E-1
7	9	63	0.031292474947	1.014400723012E-1	5.231264647344E-3	7.118E-1
7	10	70	0.028256535415	9.523154122492E-2	5.027589143562E-3	7.413E-1
7	11	77	0.025010687137	8.355458952931E-2	5.430839733721E-3	7.692E-1
7	13	91	0.021208599633	5.526489718176E-2	5.634503589261E-3	7.701E-1

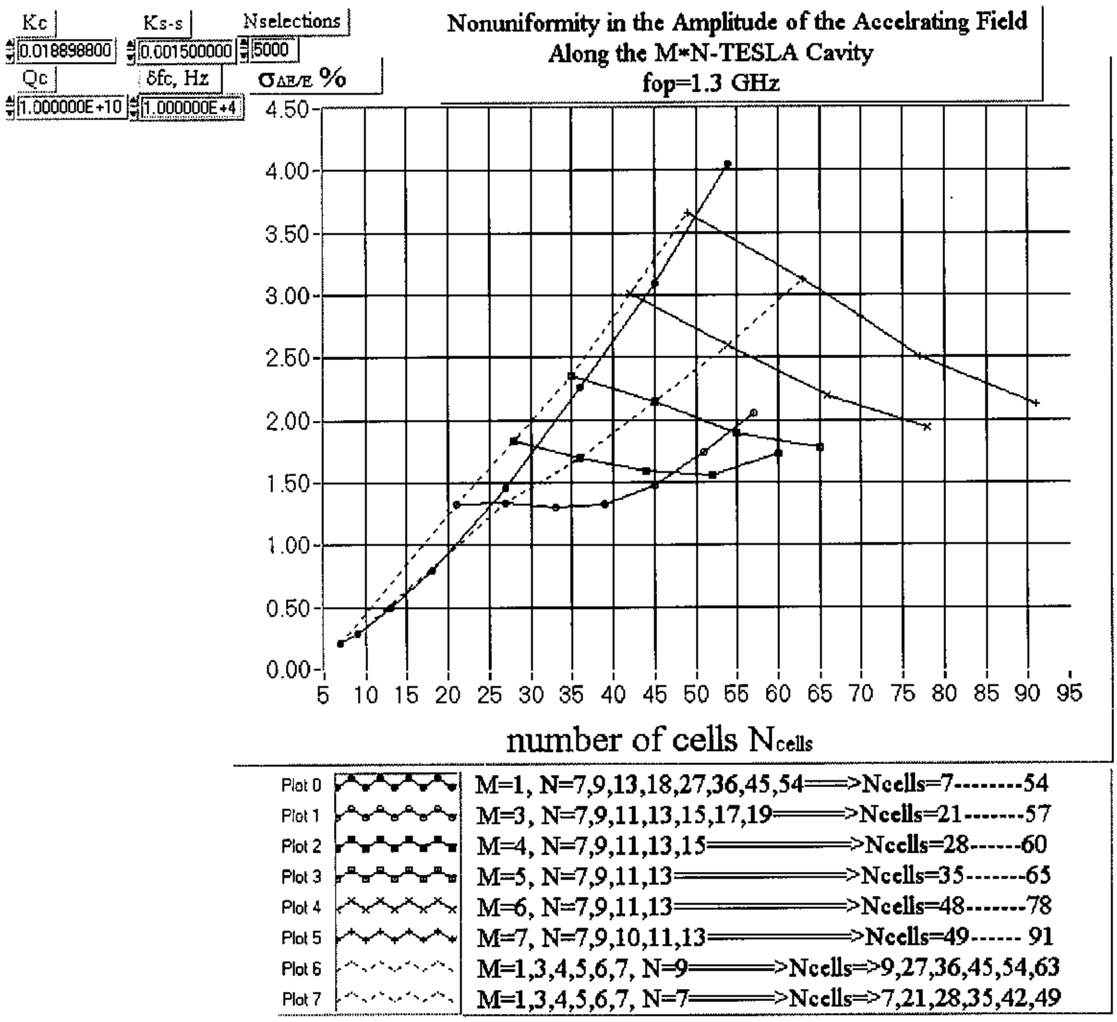

Fig.3. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=10000 Hz, $K_{s\text{-}s}$=0.0015)

Table 4

Kc: 0.018898800 | Ks-s: 0.003000000

Qc: 1.000000E+10 | dfc, Hz: 1.000000E+4

Nonuniformity in the Amplitude of the Accelrating Field
Along the M*N-TESLA Cavity
fop=1.3 GHz

Nselections: 5000

M	N	Ncells	$\sigma_{\Delta E/Eav}$	$\sigma_{\Delta E/Eav\ max}$	$\sigma_{\Delta E/Eav\ min}$	ampl-min
1	7	7	0.002062798326	6.374454182603E-3	2.025242406551E-4	9.845E-1
1	9	9	0.002925943207	9.195675061716E-3	1.899659586606E-4	9.772E-1
1	13	13	0.004933901997	1.849004772759E-2	4.084113114348E-4	9.509E-1
1	18	18	0.007952342369	2.868426099149E-2	5.691851611833E-4	9.270E-1
1	27	27	0.014480294897	5.349687265019E-2	1.014769939994E-3	8.629E-1
1	36	36	0.022117359640	7.399854807529E-2	1.294832187769E-3	8.087E-1
1	45	45	0.030803007997	1.042882453032E-1	1.976136097432E-3	7.421E-1
1	54	54	0.040245209950	1.544059436566E-1	2.160801771062E-3	6.567E-1
3	7	21	0.005274404669	1.375617638018E-2	8.597399577908E-4	9.493E-1
3	9	27	0.006574476859	1.685074964778E-2	1.219441203662E-3	9.322E-1
3	11	33	0.009490115900	2.760931326216E-2	1.348233546983E-3	9.160E-1
3	13	39	0.013290256342	4.618126770898E-2	2.083870681459E-3	8.649E-1
3	15	45	0.017858483784	6.472260607129E-2	1.901649193015E-3	8.279E-1
3	17	51	0.023095650928	8.248110220750E-2	2.653823483610E-3	7.742E-1
3	19	57	0.028480038799	1.026711608694E-1	2.543118872180E-3	7.338E-1
4	7	28	0.006158521943	1.459834601862E-2	1.211501212609E-3	9.395E-1
4	9	36	0.008178896238	2.452180581361E-2	1.822093123684E-3	9.098E-1
4	11	44	0.012870516339	4.178349501964E-2	1.697420881759E-3	8.740E-1
4	13	52	0.018953408315	6.523514847938E-2	2.549282992872E-3	8.212E-1
4	15	60	0.026451146054	8.835895110107E-2	2.826836875371E-3	7.658E-1
5	7	35	0.006936895314	1.668041795399E-2	1.529789855227E-3	9.346E-1
5	9	45	0.009807673891	3.307966910676E-2	2.571678632249E-3	8.954E-1
5	11	55	0.016429638997	5.377498077659E-2	2.111846726343E-3	8.472E-1
5	13	65	0.025448554162	8.974206636446E-2	3.182738096625E-3	7.675E-1
6	7	42	0.007649336102	1.961582593726E-2	1.952777973803E-3	9.225E-1
6	9	54	0.011553441629	3.634901108276E-2	2.430107065243E-3	8.770E-1
6	11	66	0.020605618591	8.684163757652E-2	3.081743883878E-3	7.722E-1
6	13	78	0.032810478769	1.238759100869E-1	3.744749207485E-3	6.854E-1
7	7	49	0.008206368540	2.041895284123E-2	2.172405661132E-3	9.244E-1
7	9	63	0.013357405429	4.502349906879E-2	3.055524804375E-3	8.475E-1
7	10	70	0.018981359765	7.717244427406E-2	3.451175149549E-3	7.993E-1
7	11	77	0.025575586613	9.750889566962E-2	3.676295681278E-3	7.473E-1
7	13	91	0.040628628722	1.682846293570E-1	4.480091998820E-3	6.205E-1

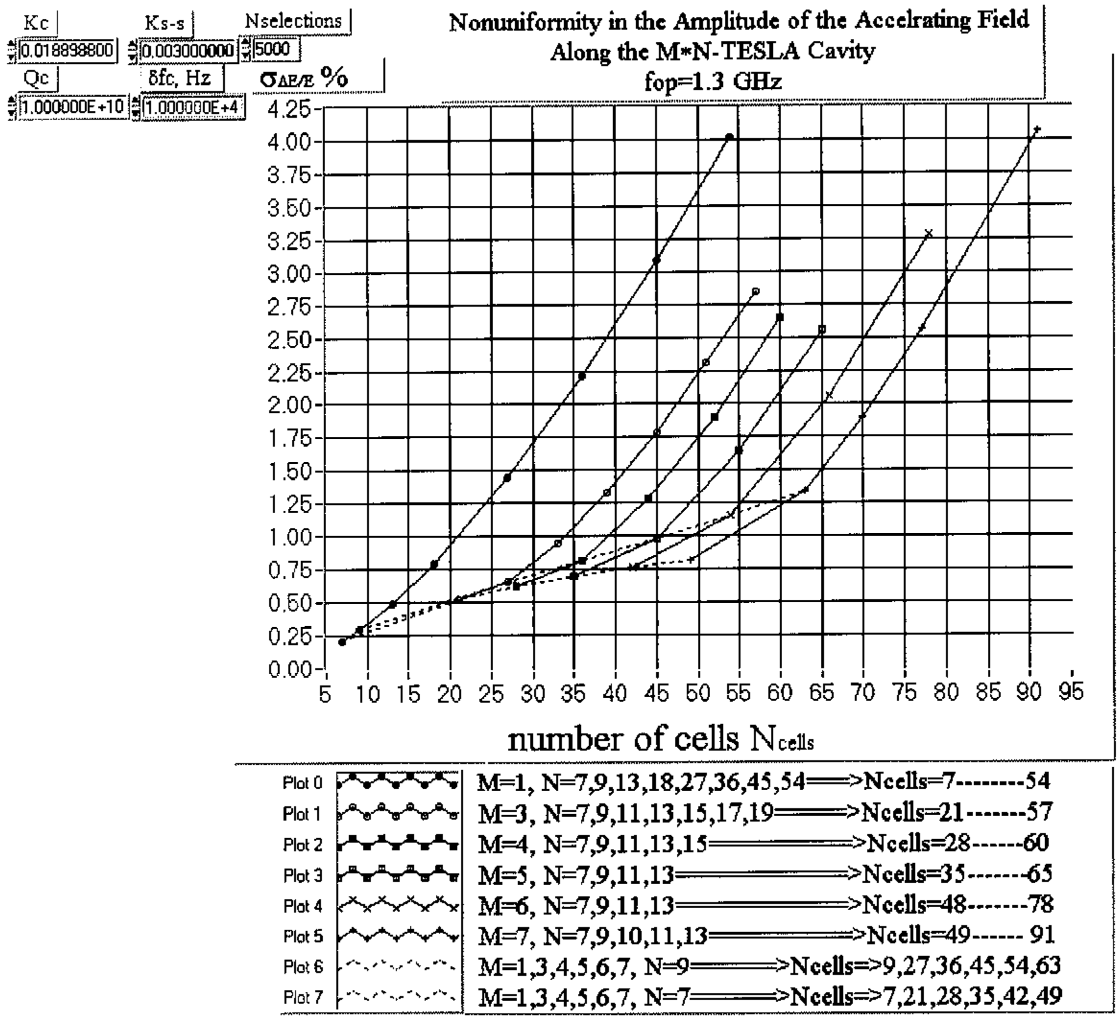

Fig.4. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=10000 Hz, $K_{s\text{-}s}$=0.003)

Table 5

Kc: 0.018898800 | Ks-s: 0.004000000

Qc: 1.000000E+10 | dfc, Hz: 1.000000E+4

Nonuniformity in the Amplitude of the Accelrating Field Along the M*N-TESLA Cavity fop=1.3 GHz

Nselections: 5000

M	N	Ncells	$\sigma_{\Delta E/E_{av}}$	$\sigma_{\Delta E/E_{av}\ max}$	$\sigma_{\Delta E/E_{av}\ min}$	ampl-min
1	7	7	0.002079958719	6.732084019409E-3	1.845487321899E-4	9.834E-1
1	9	9	0.002927379736	9.032394171811E-3	1.909511707138E-4	9.769E-1
1	13	13	0.005019134399	1.697926395938E-2	4.135270930947E-4	9.553E-1
1	18	18	0.007928519897	2.678702350263E-2	5.993422786154E-4	9.299E-1
1	27	27	0.014496609273	5.032908132627E-2	9.471909364422E-4	8.656E-1
1	36	36	0.022613546550	8.004923273269E-2	1.279135856718E-3	7.873E-1
1	45	45	0.030866339567	1.297624836269E-1	1.938108809007E-3	6.939E-1
1	54	54	0.040498917238	1.562835411848E-1	2.295197193482E-3	6.423E-1
3	7	21	0.004539135636	1.316909732481E-2	7.303793968039E-4	9.569E-1
3	9	27	0.007098510420	2.376307230056E-2	9.979588532966E-4	9.293E-1
3	11	33	0.010771071743	4.172700663211E-2	1.392873963569E-3	8.850E-1
3	13	39	0.015395969411	5.139529592150E-2	1.380157599784E-3	8.574E-1
3	15	45	0.019981856415	7.275764596870E-2	1.923136324165E-3	8.228E-1
3	17	51	0.025903831467	1.023894227764E-1	2.244224794006E-3	7.447E-1
3	19	57	0.031288505133	1.148746576163E-1	2.606462680059E-3	7.247E-1
4	7	28	0.005588859719	1.711826646204E-2	1.151612030089E-3	9.450E-1
4	9	36	0.009798652070	3.245073062646E-2	1.433535494396E-3	9.079E-1
4	11	44	0.015682756621	6.123224185886E-2	1.889495500140E-3	8.457E-1
4	13	52	0.022611244987	8.593965220644E-2	2.167553332263E-3	7.837E-1
4	15	60	0.030247573398	1.131476264406E-1	2.856433126523E-3	7.261E-1
5	7	35	0.006607374829	2.016416602613E-2	1.545881176225E-3	9.358E-1
5	9	45	0.012779621402	4.289854745612E-2	1.953176488332E-3	8.714E-1
5	11	55	0.021195981911	8.296405076891E-2	2.272239224274E-3	7.871E-1
5	13	65	0.030908182995	1.093435377999E-1	3.330080302201E-3	7.153E-1
6	7	42	0.007829245131	2.790300728055E-2	1.743733954178E-3	9.199E-1
6	9	54	0.016250275756	5.389873190940E-2	2.353510511666E-3	8.496E-1
6	11	66	0.027032223852	1.016160201097E-1	2.926133589195E-3	7.451E-1
6	13	78	0.040472713484	4.918207758414E-1	3.515597104745E-3	6.571E-3
7	7	49	0.009080261851	2.894817409857E-2	2.235496678410E-3	9.098E-1
7	9	63	0.019986664812	6.922580672436E-2	2.407837933561E-3	8.046E-1
7	10	70	0.026955314547	8.894580196854E-2	3.167532605082E-3	7.742E-1
7	11	77	0.034433088781	1.139236892060E-1	2.813500235926E-3	7.076E-1
7	13	91	0.053085019587	4.926637770121E-1	4.522850814234E-3	3.332E-3

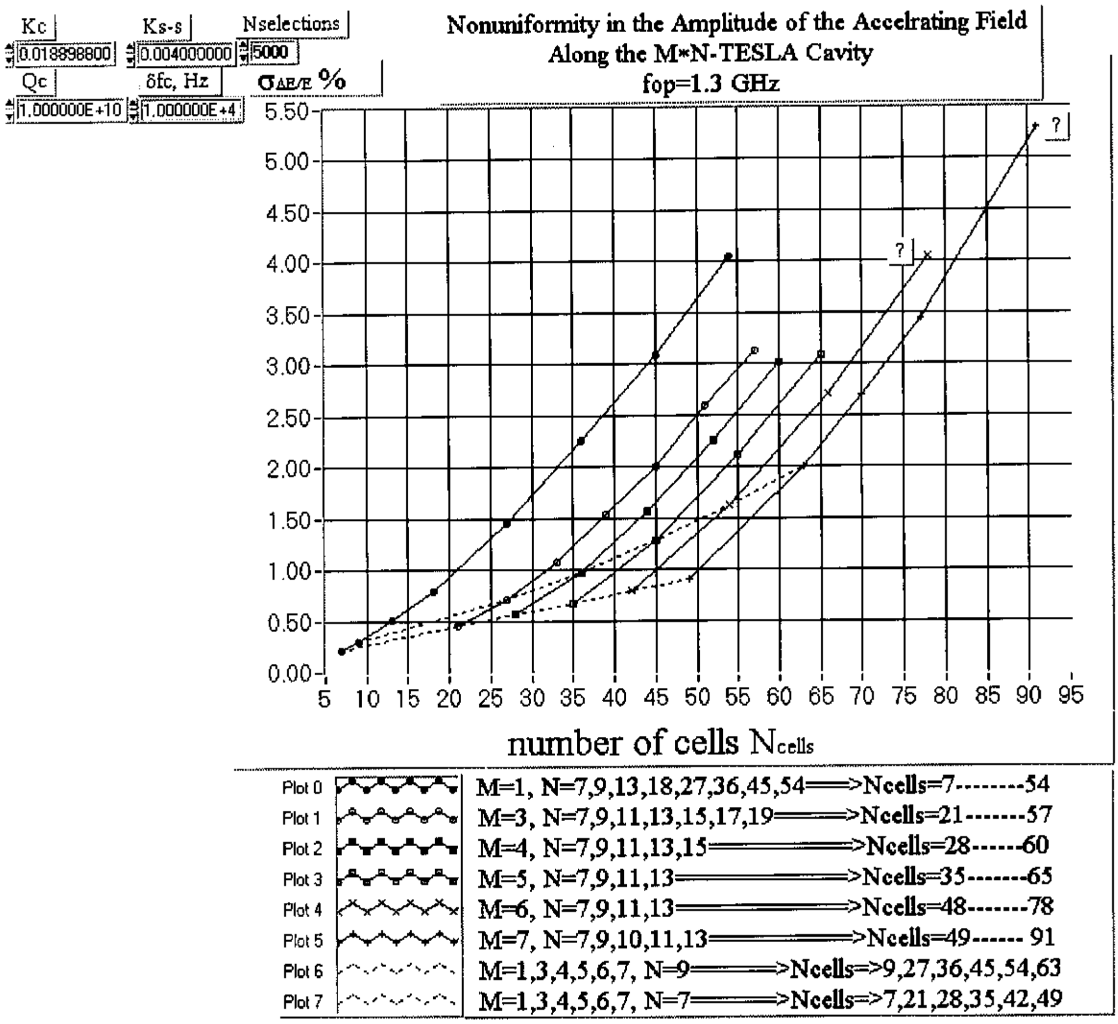

Fig.5. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=10000 Hz, $K_{s\text{-}s}$=0.004)

Table 6

Kc: 0.018898800 Ks-s: 0.005000000

Qc: 1.000000E+10 dfc, Hz: 1.000000E+4

Nonuniformity in the Amplitude of the Accelrating Field Along the M*N-TESLA Cavity

fop=1.3 GHz

Nselections: 5000

M	N	Ncells	$\sigma_{\Delta E/Eav}$	$\sigma_{\Delta E/Eav\ max}$	$\sigma_{\Delta E/Eav\ min}$	ampl-min
1	7	7	0.002062798326	6.374454182603E-3	2.025242406551E-4	9.845E-1
1	9	9	0.002925943207	9.195675061716E-3	1.899659586605E-4	9.772E-1
1	13	13	0.004933901997	1.849004772759E-2	4.084113114348E-4	9.509E-1
1	18	18	0.007952342369	2.868426099149E-2	5.691851611833E-4	9.270E-1
1	27	27	0.014480294897	5.349687265019E-2	1.014769939994E-3	8.629E-1
1	36	36	0.022117359640	7.399854807529E-2	1.294832187769E-3	8.087E-1
1	45	45	0.030803007997	1.042882453032E-1	1.976136097432E-3	7.421E-1
1	54	54	0.040245209950	1.544059436566E-1	2.160801771062E-3	6.567E-1
3	7	21	0.004756333285	1.551359423093E-2	6.373329472350E-4	9.550E-1
3	9	27	0.007906043886	2.732306705041E-2	1.214386402574E-3	9.233E-1
3	11	33	0.011797659764	4.343023380497E-2	1.303709010438E-3	8.827E-1
3	13	39	0.016678231695	5.163771055810E-2	1.474358864988E-3	8.602E-1
3	15	45	0.021679670143	7.445313051263E-2	1.864304063436E-3	7.996E-1
3	17	51	0.027691604621	9.956497860248E-2	1.880048574198E-3	7.546E-1
3	19	57	0.032963670211	1.159435109385E-1	2.170097126385E-3	7.172E-1
4	7	28	0.006347311135	2.069506084381E-2	1.074802295286E-3	9.369E-1
4	9	36	0.011446300972	3.756083111218E-2	1.396564411158E-3	8.946E-1
4	11	44	0.017749000068	6.306021412788E-2	1.910203725895E-3	8.335E-1
4	13	52	0.025042697279	8.902180855781E-2	2.336884665546E-3	7.865E-1
4	15	60	0.033251592943	1.133409901877E-1	2.883245535251E-3	7.178E-1
5	7	35	0.007960644419	2.532502835798E-2	1.264489850419E-3	9.226E-1
5	9	45	0.015228809684	5.856030614322E-2	1.417458495407E-3	8.508E-1
5	11	55	0.024381765560	8.206012324317E-2	2.553134765669E-3	7.920E-1
5	13	65	0.034460176756	1.318173502376E-1	2.885796039032E-3	6.933E-1
6	7	42	0.010123476623	3.459162223146E-2	1.511433128746E-3	8.988E-1
6	9	54	0.019890467518	6.399079529466E-2	2.036841838790E-3	8.329E-1
6	11	66	0.031267397691	1.084995971293E-1	3.036592748634E-3	7.268E-1
6	13	78	0.045765441574	4.967248034564E-1	4.032717261593E-3	6.370E-3
7	7	49	0.012520066576	4.340779185914E-2	2.015076328230E-3	8.738E-1
7	9	63	0.024549749987	9.127040212008E-2	1.917782003069E-3	7.760E-1
7	10	70	0.031563542936	1.159251480956E-1	2.667856116674E-3	7.194E-1
7	11	77	0.039313339325	1.457511840698E-1	2.927511022393E-3	6.591E-1
7	13	91	0.070325689739	4.930455422581E-1	4.225741644046E-3	1.626E-3

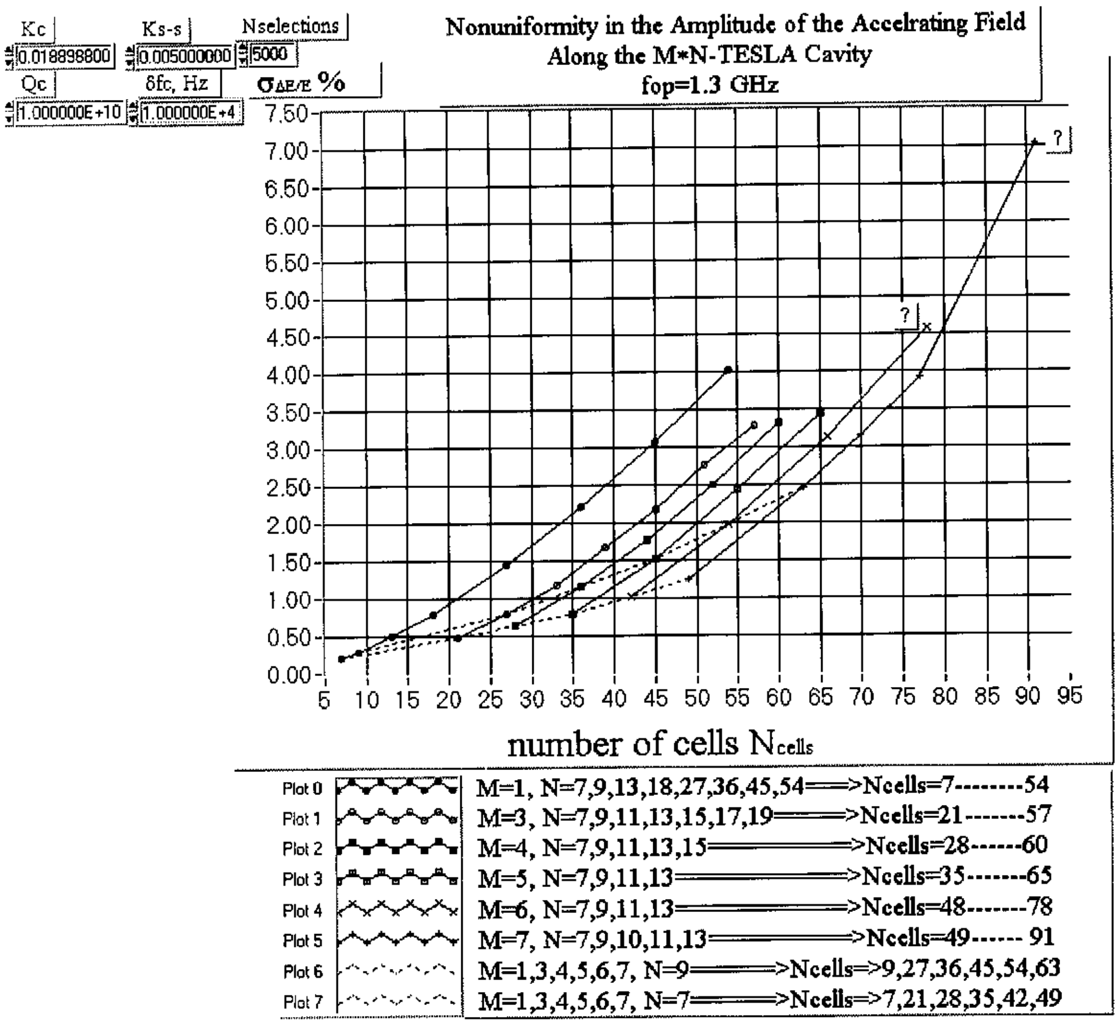

Fig.6. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=10000 Hz, K_{s-s}=0.005)

The simulations show that nonuniformity in the amplitude of the accelerating field along M×N-cell cavities increases when coupling coefficient K_{s-s} decreases and that it even can be larger then a nonuniformity along the cavity with an operational π-mode (see Fig.2 and 3).

One can see that M×9-cell cavities with K_{s-s}=0.0020171 possess a stabilization effect (see Fig.1, lower dashed line) and the more number of subcavities M the stronger stabilization effect. Of course the nonuniformity in the amplitude of the accelerating field along the M×9-cell cavities increases when M increases, but this increasing is weak.

Table 7 and Fig.7 show simulation results for uniform random cells detuning δf_c=50 kHz and $K_{s\text{-}s}$=0.0020171. One can see that $\sigma_{\Delta E/E}$ increased 5 times and the dependencies of $\sigma_{\Delta E/E}$ on the number of cells were saved.

Table 7

Nonuniformity in the Amplitude of the Accelrating Field
Along the M*N-TESLA Cavity
fop=1.3 GHz

Kc: 0.018898800 | Ks-s: 0.002017100 | Qc: 1.000000E+10 | dfc, Hz: 5.000000E+4 | Nselections: 5000

M	N	Ncells	$\sigma_{\Delta E/Eav}$	$\sigma_{\Delta E/Eav\ max}$	$\sigma_{\Delta E/Eav\ min}$	ampl-min
1	7	7	0.010258649507	3.148816844939E-2	5.892229676625E-4	9.241E-1
1	9	9	0.014667370151	4.629754037948E-2	9.440084536585E-4	8.880E-1
1	13	13	0.024875643576	7.924972212267E-2	2.069415569078E-3	8.102E-1
1	18	18	0.040003140316	1.413320178613E-1	2.691502307834E-3	6.843E-1
1	27	27	0.071931800607	2.549331379019E-1	4.756638133504E-3	4.799E-1
1	36	36	0.109963903266	4.158762147957E-1	6.345991608846E-3	3.055E-1
1	45	45	0.156837302807	5.493500894030E-1	1.132302441502E-2	2.266E-3
1	54	54	0.249515901861	6.358799686383E-1	1.250454274681E-2	4.776E-5
3	7	21	0.043338460824	1.471913497071E-1	4.584517388939E-3	6.367E-1
3	9	27	0.043353171938	1.357495861317E-1	6.876180738168E-3	6.130E-1
3	11	33	0.047120869904	1.294097669857E-1	6.512489035683E-3	6.029E-1
3	13	39	0.057199814878	1.516201225115E-1	1.070486617983E-2	5.651E-1
3	15	45	0.073058584576	2.142347583717E-1	1.080903404638E-2	4.473E-1
3	17	51	0.094605628500	3.156577116793E-1	1.208524678601E-2	3.379E-1
3	19	57	0.117690122690	3.926715619882E-1	1.617735530828E-2	2.554E-1
4	7	28	0.056641821047	2.010162364626E-1	6.630556419551E-3	5.278E-1
4	9	36	0.051130291716	1.514638000922E-1	9.765627543436E-3	5.939E-1
4	11	44	0.055098643875	1.256058196036E-1	1.350350044794E-2	5.659E-1
4	13	52	0.071320381369	2.090742265984E-1	1.362073363776E-2	4.088E-1
4	15	60	0.096439592070	3.305870503119E-1	1.643002968246E-2	3.293E-1
5	7	35	0.070401011765	2.476228226084E-1	1.162472732809E-2	4.843E-1
5	9	45	0.059758416679	1.902511459703E-1	1.327988817832E-2	4.593E-1
5	11	55	0.062401421968	1.548278628483E-1	1.692461178041E-2	4.987E-1
5	13	65	0.086701715821	2.945917943140E-1	1.652928202607E-2	3.314E-1
6	7	42	0.085401401588	2.772600673911E-1	1.437455455187E-2	4.332E-1
6	9	54	0.067336076351	2.046488694546E-1	1.599023698363E-2	4.992E-1
6	11	66	0.068651885749	1.566069663392E-1	1.790748741575E-2	4.119E-1
6	13	78	0.102332190129	3.746070219450E-1	2.303560781826E-2	2.482E-1
7	7	49	0.102912882230	5.282343803698E-1	1.634510866158E-2	9.634E-4
7	9	63	0.076339980969	2.372972787083E-1	1.792672553652E-2	4.033E-1
7	10	70	0.070122198138	1.943656879473E-1	2.125412780826E-2	4.255E-1
7	11	77	0.075550157768	1.999353943338E-1	2.433638815320E-2	3.839E-1
7	13	91	0.119123372044	5.211896057965E-1	2.162318249300E-2	5.328E-3

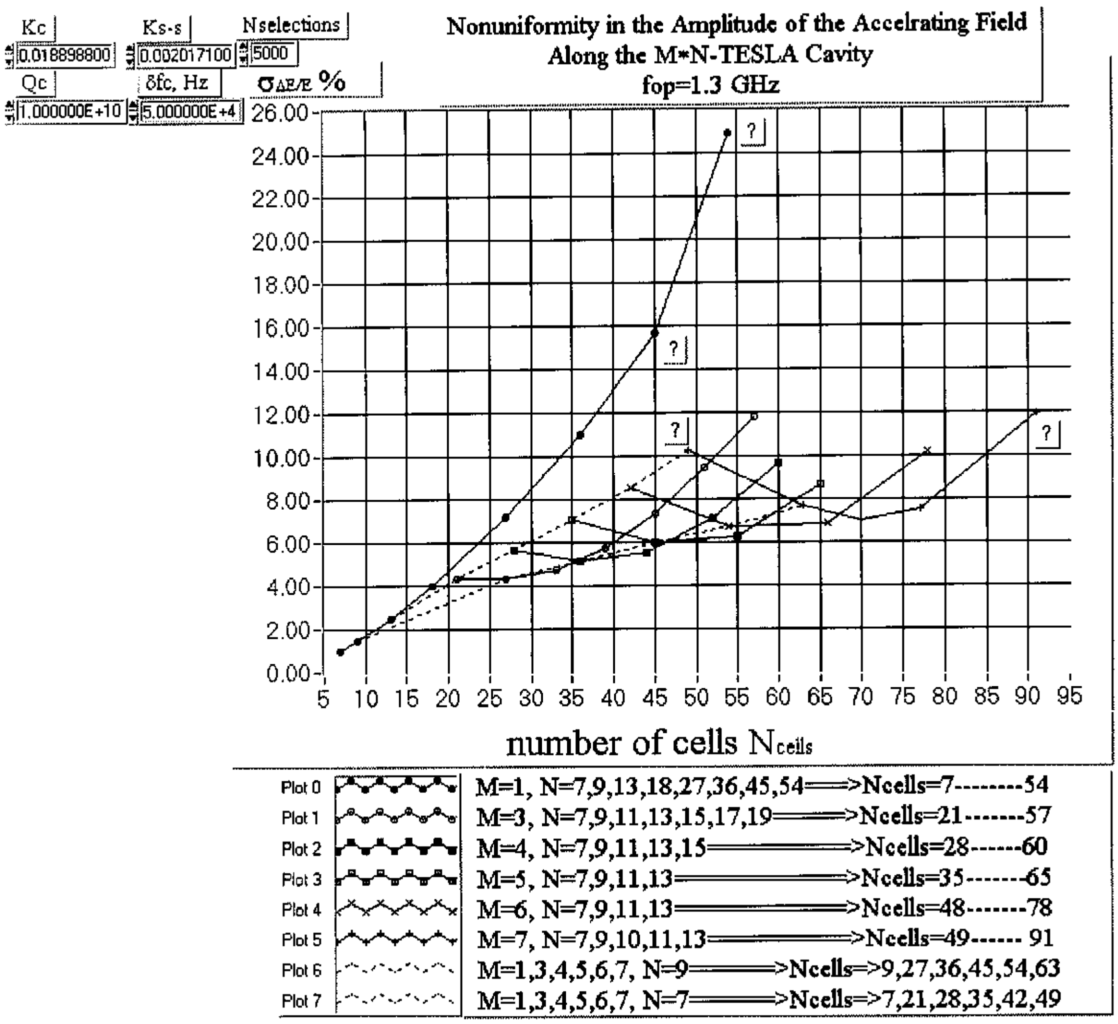

Fig.7. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity (δf_c=50000 Hz, $K_{s\text{-}s}$=0.0020171)

These data show that M×N-cell TESLA cavities possess some field amplitude stabilization. The nonuniformity in the amplitude of the accelerating field along the 4×7(9)-cell TESLA cavities is 5.51 and 3.56 times bigger then this one along the corresponding 7-cell and 9-cell cavities with an operational π-mode. The nonuniformity in the amplitude of the accelerating field along the 4×7(9)-cell TESLA cavities is 1.256 and 2.166 times less then this one along the corresponding 28-cell and 36-cell cavities with an operational π-mode.

Fig.8 shows a dependence of r.m.s. $\sigma_{\Delta E/E}$ on coupling coefficient $K_{s\text{-}s}$ for M×N-cells supercavities with K_c=0.0188988, $Q_0=10^{10}$ and random cells detuning δf_c=10000 Hz.

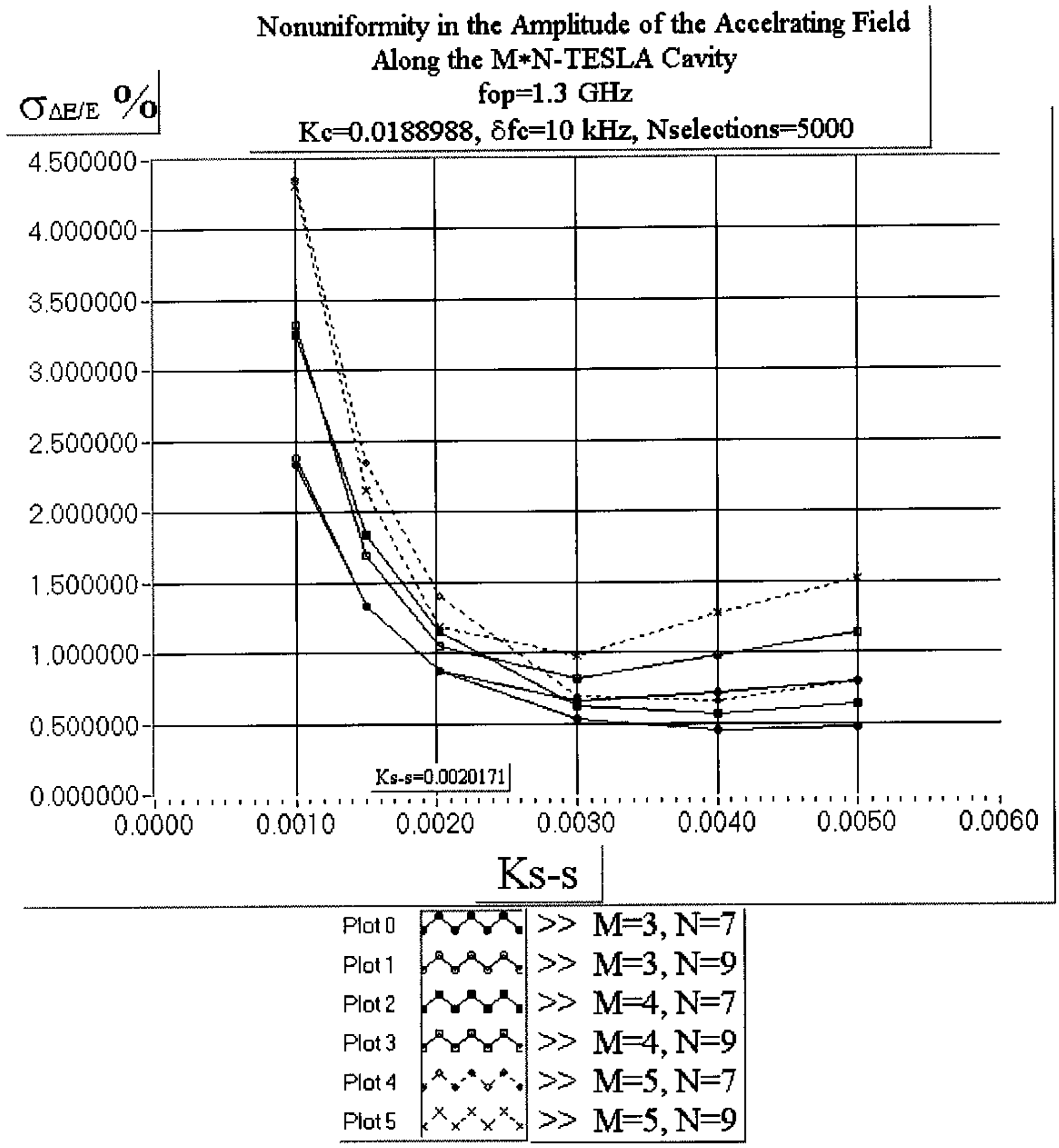

Fig.8. Relative Nonuniformity in the Amplitude of the Accelerating Field along the M×N-TESLA Cavity as function of coupling coefficient $K_{s\text{-}s}$ (δf_c=10 kHz, K_c=0.0188988)

One can see that coupling coefficient $K_{s\text{-}s}$ has an optimal value at which nonuniformity has a minimum.

III. Relative nonuniformity in the amplitude of the accelerating field along the π-mode cavity

Now we shall derive analytical expression for r.m.s. $\sigma_{\Delta E/E}$ of the relative nonuniformity in the amplitude of the accelerating field along the cavity with an operational π-mode. We shall use the system of equations (1) to simulate field amplitude distribution along the cavity in the presence of the random cells detuning.

Cells frequency in the perfectly tuned π-mode cavity must be equal to the following values

$$f_{cn} = \begin{cases} \dfrac{f_0}{\sqrt{1+\dfrac{K_c}{2}}} & \text{for } n = 1, N \\ \dfrac{f_0}{\sqrt{1+K_c}} & \text{for } n \neq 1, N \end{cases} \qquad (4)$$

Where N is a number of cells in the cavity,
n is a cell number,
f_0 is an operational π-mode frequency,
K_c is a cell-to-cell coupling coefficient.

We suppose that cells frequency has a random detuning δf_{cn}, which satisfies the following condition

$$\left(\frac{\delta f_{cn}}{f_o}\frac{\delta f_{cm}}{f_o}\right)_{av} = \begin{cases} 0 & \text{for } n \neq m \\ \sigma^2_{\delta f_c/f_0} & \text{for } n = m \end{cases} \qquad (5)$$

where $\sigma_{\delta fc/fo}$ is r.m.s. of the random relative cells detuning.

Field amplitude distribution along the perfectly tuned cavity has a view: $E_{n0}=(-1)^{n+1}E_0$. In the presence of the random cells detuning we can obtain the following expression for a relative deviation in the field amplitude in the (n+1)-st cell

$$\frac{\Delta E_{n+1}}{E_{n+1\,0}} = (-1)\left\{\frac{4}{K_c}\left(\frac{f_0}{f_{cn}}\right)^3 \frac{\delta f_{cn}}{f_0} + \frac{2}{K_c}\left(1-\frac{f_0^2}{f_{cn}^2}\right)\frac{\Delta E_n}{E_{n0}} + \frac{\Delta E_{n-1}}{E_{n-1\,0}}\right\}$$

$$\text{for} \quad n = 1, 2, 3, \dots, N-1 \qquad (6)$$

where
$$1-\frac{f_0^2}{f_{cn}^2} = \begin{cases} -\dfrac{K_c}{2} & \text{for } n = 1 \\ -K_c & \text{for } n \neq 1 \end{cases},$$

$$\left(\frac{f_0}{f_{cn}}\right)^3 = \begin{cases} \left(1+\dfrac{K_c}{2}\right)^{3/2} & \text{for } n = 1 \\ \left(1+K_c\right)^{3/2} & \text{for } n \neq 1 \end{cases},$$

$$\frac{\Delta E_{n-1}}{E_{n-1\,0}} = 0 \qquad \text{for} \quad n = 1$$

$$\frac{\Delta E_n}{E_{n0}} = 0 \qquad \text{for} \quad n = 1$$

This expression was obtained from the system of equations (1) saving the first order terms only and under the assumption that a cavity is excited through the last cell (N) at the operational frequency f_0.

The last expression can be transformed into the form

$$\frac{\Delta E_n}{E_{n0}} = (-1)\left\{\frac{4(1+K_c)^{3/2}}{K_c}\sum_{m=2}^{n-1}(n-m)\frac{\delta f_{cm}}{f_0} + \frac{4\left(1+\dfrac{K_c}{2}\right)^{3/2}}{K_c}(n-1)\frac{\delta f_{c1}}{f_0}\right\} \qquad (7)$$

$$\text{for} \quad n = 2, 3, 4, \dots, N$$

We derive the expression for r.m.s. $\sigma_{\Delta E/E}$ using the following definition of this quantity

$$\sigma_{\Delta E/E} = \sqrt{\lim_{J\to\infty}\left\{\frac{1}{J}\frac{N}{N-1}\sum_{j=1}^{J}\left[\left(\frac{1}{N}\sum_{n=1}^{N}\left(\frac{\Delta E_n}{E_{n0}}\right)_j^2\right)-\left(\frac{\Delta E}{E_0}\right)_{av\,j}^2\right]\right\}} \quad (8)$$

where

$$\left(\frac{\Delta E}{E_0}\right)_{av\,j} = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{\Delta E_n}{E_{n0}}\right)_j$$

Substituting expression (7) into (8) and taking into account condition (5) one can obtain the following expression for r.m.s. $\sigma_{\Delta E/E}$

$$\sigma_{\Delta E/E} = \frac{4}{K_c}\sigma_{\delta f_c/f_0}\sqrt{\frac{N+1}{12}\left\{N\left(1+\frac{K_c}{2}\right)^3+(1+K_c)^3\frac{2N^2-5N+2}{5}\right\}} \quad (9)$$

where $\sigma_{\delta f_c/f_0}$ is r.m.s. of the relative cells detuning.

If we can neglect K_c compared with 1 in the last expression then we obtain

$$\sigma_{\Delta E/E} = \frac{4}{K_c}\sigma_{\delta f_c/f_0}\sqrt{\frac{(N+1)(N^2+1)}{30}} \quad (10)$$

One can see that $\sigma_{\Delta E/E} \propto N^{3/2}\sigma_{\delta fc/fo}/K_c$ for a large number of cells N.

Formula (9) was derived under assumption that the cavity is excited through the last cell at the operational frequency f_0. This is a cause of some bigger nonuniformity calculated with this formula then those one represented in Fig.1-7 and Tables 1-7. The data represented in the tables and figures were calculated at the free oscillation frequency of the cavity, which is not equal to the operational frequency f_0 and it is a random quantity too.

IV. Conclusion

The undesirable nonuniformity in the amplitude of the accelerating field along the different M×N-cell TESLA cavities caused by a random cells detuning was investigated. The dependence of r.m.s. of the relative deviation of the accelerating field amplitude distribution along the cavities on the number of cells in the cavity was obtained for a different M×N-cell TESLA cavities. It was shown that M×N-cell TESLA cavities possess some stabilization effect in the amplitude of the accelerating field distribution along the cavity and coupling coefficient K_{s-s} has an optimal value (for a given value K_c=0.0188988).

It was shown that the nonuniformity in the amplitude of the accelerating field along the 4×7-cell and 4×9-cell TESLA supercavities is 5.51 and 3.56 times bigger then this one in the corresponding 7-cell and 9-cell cavities with an operational π-mode. The nonuniformity in such cavities is 1.265 and 2.166 times less then nonuniformity in the amplitude of the accelerating field along the corresponding 28-cell and 36-cell usual cavities with an operational π-mode.

An analytical expression for an estimation of the nonuniformity in the amplitude of the accelerating field along the cavities with an operational π-mode was derived.

V. References

1. E.A.Knapp, IEEE Trans. Nucl. Sci. NS-16, 329, (1969).
2. E.A.Knapp, B.C.Knapp, and J.M.Potter
   Rev. Sci. Instrum. 39, 979, (1969)
3. B.Epstain and D.T.Truns
   In: Proc. of the 1968 Proton Linear Accelerator Conf., Brookhaven, USA, BNL, (1968), pp.457-469.
4. A.Bensussan, D.T.Trans, and D.Trons
   LIM 140, 231, (1974).
5. V.F.Vikulov and V.E.Kaljuzhny
   Effect of Errors of Fabrication on the Characteristics of Standing Wave Accelerating Structures.
   Sov. Phys. Tech. Phys., 25(4), April 1980.
6. J.Sekutovich, M.Ferrario, Ch.Tang
   Superconducting Superstructure for TESLA Collider
   DESY Print, TESLA Report 96-09, August 1996.