Multipole Field Tolerances in the TESLA Final Doublet Quadrupoles

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ABSTRACT

The parameters of the TESLA linear collider have been significantly modified to produce a high luminosity [1] at both 500 GeV and also 800 GeV CM (center of mass energy). The more aggressive parameters require a review of the field quality requirements in the quadrupole magnets of the final doublet where the conditions are most demanding. In this note we calculate the tolerances for each relevant magnetic multipole, in isolation and in combination, and for both quadrupoles of one doublet, in isolation and in combination. We also account for the possibility of linear, empirical corrections to the IP (interaction point) spot size given static multipole fields.

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1 Introduction

The magnetic field multipole tolerances have been previously calculated for an earlier version of the TESLA parameters [2]. The new IP parameters used throughout this note are listed in Table 1. The smaller IP spot size for this new parameter set requires tighter field quality tolerances. The tolerances are calculated by applying magnetic field multipole components to the quadrupoles using the expansion

$$B_y + iB_x = B_0 \sum_{n\geq 2} (b_n + ia_n)(z/r_0)^{n-1} , \tag{1}$$

where $z = (x + iy)$ is the complex transverse coordinate and B_0 is the quadrupole field at the reference radius r_0. We take the quadrupole coefficient as unity, $b_2 \equiv 1$, and the skew-quadrupole as zero, $a_2 \equiv 0$. Each multipole has $2n$ magnetic poles.

Table 1. New TESLA IP parameters for both 500 and 800 GeV CM energies.

Parameter	Symbol	unit	500 GeV_{cm}	800 GeV_{cm}
Beam energy	E_b	GeV	250	400
Hor. rms norm. emittance	$\gamma\varepsilon_x$	μm	10.0	8.0
Ver. rms norm. emittance	$\gamma\varepsilon_y$	μm	0.03	0.01
Hor. IP beta function	β_x^*	mm	15.2	15.0
Ver. IP beta function	β_y^*	mm	0.41	0.31
Hor. rms IP spot size	σ_x^*	nm	558	391
Ver. rms IP spot size	σ_y^*	nm	5.00	2.00
Hor. rms IP divergence	θ_x^*	μrad	36.6	26.1
Ver. rms IP divergence	θ_y^*	μrad	12.3	6.39

In practice, the impact of such multipole components on the IP spot size and luminosity is calculated by slicing the quadrupoles, tracking particles through each slice and applying thin-lens multipole kicks to the transverse angles of each particle's trajectory in x and y (horizontally and vertically) at each slice point. The quadrupoles are sliced ten times to account for their thick-lens character. The transverse distributions are gaussian using 50000 particles. The calculations were made using a computer program written exclusively for this purpose and were also checked against tracking calculations using MAD [3].

2 Tracking Results

The advantage in using tracking is that the full 4×4-rms beam matrix (covariance matrix) can be calculated and the correlations generated among the four coordinates may be isolated. The correlations are, in principle, correctable using linear optical elements, while the scale of the determinant of the matrix is not. The 'intrinsic' horizontal and vertical emittances are calculated which represent the minimum impact on the luminosity after IP tuning has been applied (including coupling correction). In this way, the luminosity loss can be calculated both before (based on IP spot size) and after (based on intrinsic emittance) linear IP corrections have been accomplished. This provides a more realistic estimate of the static (time independent) multipole tolerances. The linear corrections implied here include:

- *Waist position using a slight variation of the doublet fields*
- *IP beta-function corrections using minor adjustments of the beta-matching section*
- *Cross-plane coupling using a weak skew-quadrupole very near the doublet*

In all cases, for multipoles at or below the tolerances listed in the tables below, these corrections mostly apply to the more sensitive vertical plane. We ignore resolution limitations of the corrections, since this is an issue which is independent of multipole effects. The cross-plane coupling induced by skew components of odd-order fields (a_n with even n) is in the $\langle x'y \rangle$ IP correlation term which is completely correctable with one skew-quadrupole placed near the doublet.

The luminosity impact for one beam is calculated in two ways; one based on the spot size increase in both planes, $\Delta L/L_0(\sigma)$, (before corrections) and one based on the intrinsic emittance increase in both planes, $\Delta L/L_0(\varepsilon)$, (after corrections). For these calculations, the other beam is assumed to pass through a perfect doublet.

$$\Delta L/L_0\,(\sigma) = 1 - 2\Big/\sqrt{(1+\sigma_x^2/\sigma_{x0}^2)(1+\sigma_y^2/\sigma_{y0}^2)} \tag{2}$$

$$\Delta L/L_0\,(\varepsilon) = 1 - 2\Big/\sqrt{(1+\varepsilon_x/\varepsilon_{x0})(1+\varepsilon_y/\varepsilon_{y0})} \tag{3}$$

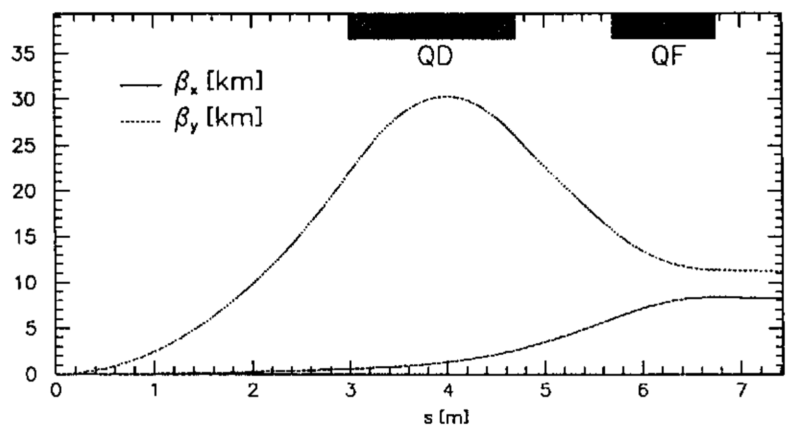

Figure 1. Beta function through the final doublet. The IP is at $s = 0$, at the far left.

For small perturbations to the spot size and emittance, the luminosity loss is well represented by Eqs. (2) and (3) since large, non-gaussian tails are not generated by weak multipoles at tolerance levels (1% luminosity loss). This has been verified by integrating the overlap of two different particle distributions with one affected by an n^{th}-order multipole. At 1% loss, there is no distinguishable difference between the integrated overlap and Eq. (2). Larger levels of luminosity loss (>10%) calculated with Eqs. (2) and (3) tend to overestimate the loss, especially for large n. The arrangement of the final doublet and the beta functions through the system is shown in Figure 1.

2.1 Multipoles Applied to Single Quadrupoles

Table 2 lists the multipole tolerances, each one at a time, for one QD quadrupole (closest to IP) for a 1% luminosity loss both before and after IP correction. For example, from the table, the luminosity loss before (after) IP corrections will be 1% if the QD magnet on one side of the IP has a relative skew-sextupole field of $a_3 = 1.6\times10^{-4}$ (2.2×10^{-4}), at a radius of r_0 = 10 mm. The second set of tolerances listed for $\Delta L/L_0(\varepsilon)$ = 1% are likely the most relevant. Table 3 lists the multipole tolerances of one QF quadrupole (2nd from IP). In all cases above, the vertical beam size and intrinsic emittance is most significantly affected. The relative horizontal spot size increase is always much less than the vertical, especially for the QD quadrupole where β_x is relatively small.

Table 2. Multipole tolerances for 1% luminosity loss at 500 GeV CM for one <u>QD</u> quadrupole (closest to IP) with each multipole tolerance calculated in isolation.

	$n =$	3	4	5	6
$\Delta L/L_0(\sigma) = 1\%$	Normal ($b_n/10^{-3}$)	0.42	10	200	3300
	Skew ($a_n/10^{-3}$)	0.16	3.4	70	1200
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	0.61	18	280	4600
	Skew ($a_n/10^{-3}$)	0.22	7.4	100	2000

Table 3. Multipole tolerances for 1% luminosity loss at 500 GeV CM for one <u>QF</u> quadrupole (2nd from IP) with each multipole tolerance calculated in isolation.

	$n =$	3	4	5	6
$\Delta L/L_0(\sigma) = 1\%$	Normal ($b_n/10^{-3}$)	0.60	6.0	52	400
	Skew ($a_n/10^{-3}$)	0.062	0.60	6.1	52
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	0.85	11	72	600
	Skew ($a_n/10^{-3}$)	0.090	1.3	8.6	80

There is also a slight asymmetry with respect to the sign of the multipole, due to the thick lens character of the doublet quadrupoles. The asymmetry is, however, insignificant and is ignored here. Figure 2 shows the IP spot size increase, the emittance increase, and the relative luminosity as a function of the relative skew and normal-sextupole strength for the QF quadrupole alone.

The effects of multipole fields of order higher than 12-pole ($n > 6$) are not significant if held anywhere near reasonable levels (<25%). For each increase in n, the tolerances roughly increase by an order of magnitude such that 14-pole tolerances and above are extremely loose.

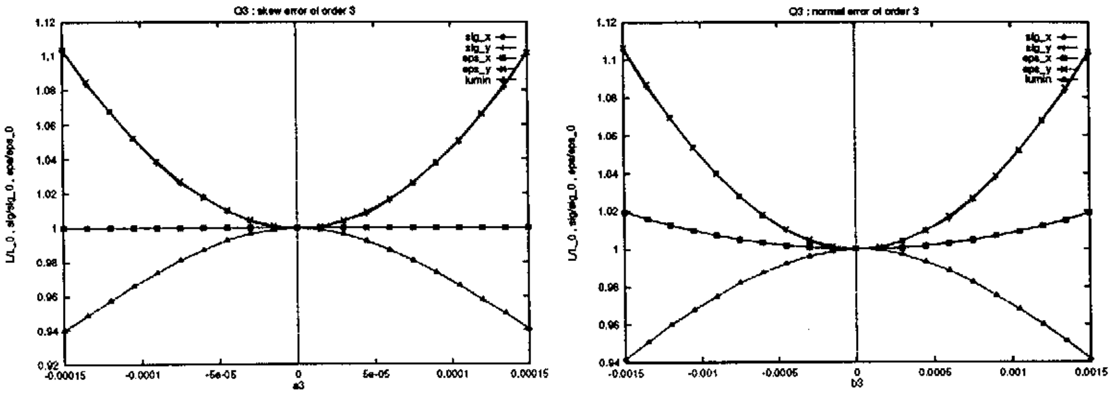

Figure 2. Horizontal (◊) and vertical (+) IP spot size increase, horizontal ( ) and vertical (×) emittance increase, and relative luminosity (Δ) as a function of the relative skew-sextupole strength (a_3, at left) and the normal-sextupole strength (b_3, at right), each at a reference radius, r_0, of 10 mm for the QF quadrupole alone.

2.2 Multipoles Applied to Both Quadrupoles

It is also useful to check the luminosity loss incurred when both quadrupoles of a single doublet have multipole fields arranged to add or cancel with respect to each other. Since the two quadrupoles are adjacent (insignificant phase advance between quadrupoles) there can be very effective cancellations and/or additions of the multipole effects from each magnet. With this simplification in mind, a simple expression can be used to approximate the 'sum mode' and 'difference mode' multipole tolerances for the doublet.

$$\Delta L/L_0 \approx 1\% \cdot \left(\frac{b_{n\mathrm{F}}}{b_{n\mathrm{F}t}} - \frac{b_{n\mathrm{D}}}{b_{n\mathrm{D}t}} \right)^2 \tag{4}$$

Here $b_{n\mathrm{F}}$ is the n^{th}-order multipole, and $b_{n\mathrm{F}t}$ is its tolerance in the QF quadrupole and $b_{n\mathrm{D}}$ is the n^{th}-order multipole in the QD quadrupole. An identical expression can be written for the skew, a_n, coefficients. The tolerance for the n^{th}-order multipole applied equally to each quadrupole, with a potential sign reversal from one quadrupole to the other, is then given by

$$b_{n\pm} \approx \frac{1}{\left| 1/b_{n\mathrm{F}t} \pm 1/b_{n\mathrm{D}t} \right|} \,. \tag{5}$$

The "–" sign represents the 'systematic' case where the two quadrupoles, when excited in the same polarity (e.g. both focussing), have the same sign and amplitude multipoles. In this case a cancellation is generated and the net tolerances are looser than either single magnet tolerance. The "+" sign represents the 'random' case where the two quadrupoles, when excited in the same polarity, have opposite sign multipoles. In this case an addition is generated and the net tolerances are tighter than either single magnet tolerance. The simplification of Eq. (4) has been verified by running the combined multipoles in the tracking calculations. Eq. (4) is consistent with the tracking results to within 5-25%. The agreement is poorest when the two tolerances, b_{nFt} and b_{nDt}, are similar, as in the case of b_3 (~25% discrepancy where the true, tracking tolerance is tighter). The 'systematic' combined tracking tolerances are given in Table 4 and the 'random' combined tracking tolerances are given in Table 5.

Table 4. Multipole tolerances for 1% luminosity loss at 500 GeV CM for the case where both QD and QF have the same 'systematic' multipoles (each applied individually).

	$n =$	3	4	5	6
$\Delta L/L_0(\sigma) = 1\%$	Normal ($b_n/10^{-3}$)	1.0	12	70	500
	Skew ($a_n/10^{-3}$)	0.11	0.74	6.6	52
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	1.5	24	100	750
	Skew ($a_n/10^{-3}$)	0.16	1.6	9.6	90

Table 5. Multipole tolerances for 1% luminosity loss at 500 GeV CM for the case where QD and QF have the same multipoles but of opposite sign (each applied individually).

	$n =$	3	4	5	6
$\Delta L/L_0(\sigma) = 1\%$	Normal ($b_n/10^{-3}$)	0.25	3.7	42	360
	Skew ($a_n/10^{-3}$)	0.047	0.52	6.2	52
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	0.36	6.7	60	580
	Skew ($a_n/10^{-3}$)	0.067	1.3	8.6	82

2.3 Tolerances at 800 GeV CM

At 800 GeV CM the multipole tolerances, in most cases, become looser due to the reduced beam size in the quadrupoles. The skew-sextupole tolerance is, however, slightly tighter than that at 500 GeV CM. Table 6 and Table 7 list the multipole tolerances for the same two quadrupoles, each independently, at 800 GeV CM, where the IP parameters have been adjusted according to Table 1. For brevity, only the tolerances based on emittance growth and for single quadrupoles are listed (compare with Table 2 and Table 3).

Table 6. Multipole tolerances for 1% luminosity loss at 800 GeV CM for one QD quadrupole with each multipole tolerance calculated in isolation.

	$n =$	3	4	5	6
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	0.65	27	600	14000
	Skew ($a_n/10^{-3}$)	0.18	8.8	150	5000

Table 7. Multipole tolerances for 1% luminosity loss at 800 GeV CM for one QF quadrupole with each multipole tolerance calculated in isolation.

	$n =$	3	4	5	6
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	0.95	17	160	2000
	Skew ($a_n/10^{-3}$)	0.072	1.4	12	160

Of course, the installation of a weak skew-sextupole magnet very near the final doublets could also be used to mitigate the effects of the tightest tolerance listed here. Furthermore, after field measurements are made, magnets can be intentionally combined in such a way as to produce a cancellation, which can significantly alleviate the multipole effects.

2.4 Worst Case Combined Multipoles and Quadrupoles

Finally we list a multipole budget based on the worst case tolerances which occur at 500 GeV CM where the QD and QF magnets have the same multipoles but are opposite in sign (see Table 5) and all multipoles from $n = 2$ through $n = 6$ are applied simultaneously. This budget is somewhat arbitrarily assembled to nearly maintain the more challenging tolerances (e.g. a_3) and to tighten the trivial ones (e.g. $n > 4$).

Table 8. Simultaneous multipole tolerances for 1% luminosity loss in the worst case: at 500 GeV CM where QD and QF have the same multipoles but are opposite in sign.

	$n =$	3	4	5	6
$\Delta L/L_0(\varepsilon) = 1\%$	Normal ($b_n/10^{-3}$)	0.10	1.0	5.0	50
	Skew ($a_n/10^{-3}$)	0.05	0.2	1.0	10

The tolerances listed here are expected to be achievable based on field measurements of the HERA quadrupoles scaled to the LHC magnets [4]. The only multipole that is expected to come close to these tolerances is a_3 (skew-sextupole). The tolerance may be met by a careful arrangement of magnets so as not to generate the worst case.

3 Conclusions

The tolerances for the multipole fields of single quadrupoles of the TESLA-500 final doublets are listed in Table 2 and Table 3. The tolerances for the multipole fields of the TESLA-500 combined final doublets are listed in Table 4 and Table 5. The tightest single quadrupole tolerance at 500 GeV CM is the skew-sextupole component, a_3, in the QF magnet (2nd from IP with large β_x) which should be held below 9.0×10^{-5} (Table 3) at a reference radius of 10 mm for a corrected luminosity loss of <1%. This reduces to 6.7×10^{-5} (Table 5) in the unlikely case that the two quadrupoles have equal but opposite sign skew-sextupole component. At 800 GeV CM all tolerances are looser except for the skew-sextupole component in QF (see Table 7). Finally, if all tolerances meet those of Table 8, the simultaneous, worst case effects should produce ≤1% luminosity loss after linear correction.

4 References

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[4] "The Large Hadron Collider", P. Lefèvre, T. Pettersson editors, CERN/AC/9505(LHC), p. 99, October 1995.