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Compton Polarimeter Studies for TESLA

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ABSTRACT

In this note, we present calculations about Compton longitudinal polarimetry for the TESLA accelerator project. We first study Compton scattering kinematic, cross section and asymetry for $250 GeV$ electrons and several photon energies. Then we compare three methods to extract the polarisation from the scattered particles measurement. Finaly, we compute the expected luminosity and counting rate considering two basic options: zero crossing angle and a matching between electron and photon beam shapes, a small crossing angle and a laser beam focalised on the electron beam.

Contents

1 Compton Polarimetry

In Compton polarimetry, the polarisation P_e of the beam is extracted from the measurement of the experimental asymetry A_{exp} in the scattering of a circulary polarized photon beam (from a laser) on the electron beam:

$$A_{exp} = \frac{n^+ - n^-}{n^+ + n^-} \qquad (1)$$

where n^+ (resp. n^-) is the number of Compton scattering events before (resp. after) a laser or an electron beam polarization reversal. This experimental asymmetry A_{exp} is related to the known theoretical longitudinal asymmetry A_l for Compton scattering for electron and photon with spin parallel $\sigma^{\rightarrow}_{\Rightarrow}$ and anti-parallel $\sigma^{\leftarrow}_{\Rightarrow}$,

$$A_l = \frac{\sigma^{\rightarrow}_{\Rightarrow} - \sigma^{\leftarrow}_{\Rightarrow}}{\sigma^{\rightarrow}_{\Rightarrow} + \sigma^{\leftarrow}_{\Rightarrow}}, \qquad (2)$$

through the relation

$$A_{exp} = \frac{n^+ - n^-}{n^+ + n^-} = P_e P_\gamma A_l. \qquad (3)$$

A_{exp} and the photon beam polarisation P_γ are measured quantities and A_l is calculated in the framework of the standard model [1] (see Eq. 16 and fig. 3.2). So the only unknown quantity is the electron beam longitudinal polarization P_e.

Similar technique is also used to measure the transverse asymetry. This method is well established [2][3] and is currently used at several high energy machines[4][5].

2 The TESLA beam

The TESLA beam characteristics are summarise in the table 2 [6]. In the framework of this studies

Beam energy	[GeV]	250.000
Gamma = E/mc^2 (for electrons)		4.892E+05
Horizontal emittance	[m]	2.862E-11
Vertical emittance	[m]	5.110E-13
Horizontal normalized emittance	[μ m]	14.000
Vertical normalized emittance	[μ m]	.250
Bunch length	[mm]	.700
Bunch Population		3.6E+10
Number of bunches		1130
Bunch separation	[ns]	708.
Repetition Rate	[Hz]	5.0
Averaged curent	[μA]	32.6

Table 1: TESLA beam characteristics

for Compton polarimetry, we are interested in beam energy, average curent (if a continuous photon source is used) and beam geometrical characteristic at the Compton interaction point.

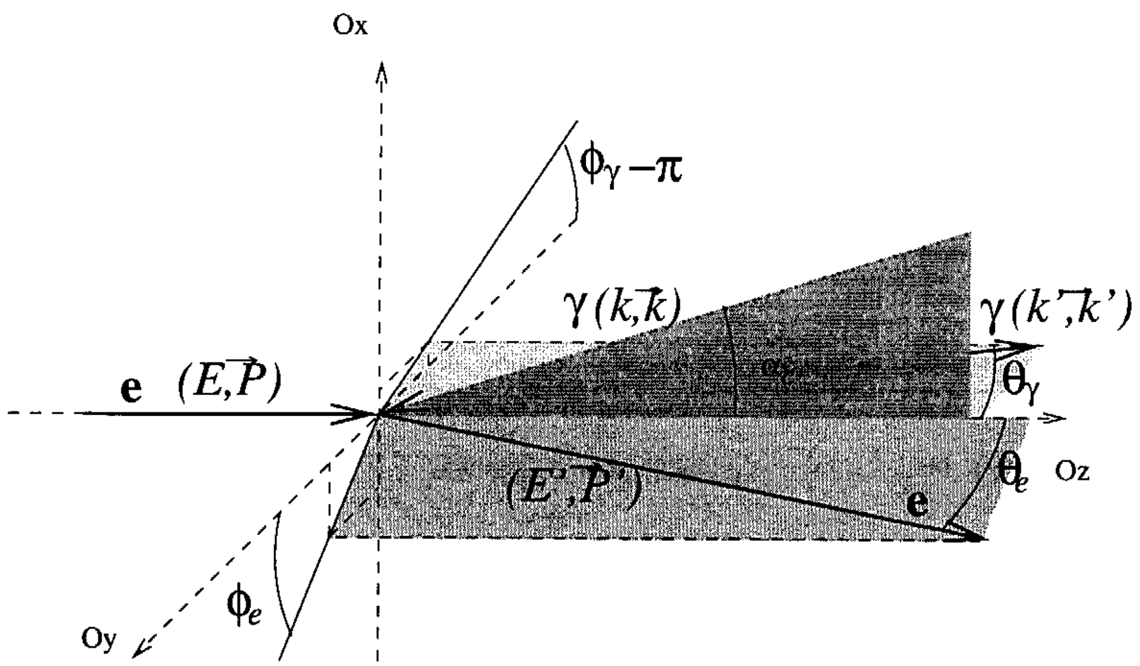

Figure 1: ***Compton Scattering***

3 Compton Scattering

In this section we first recall the kinematic of the Compton scattering. After a brief summary of the dynamic (polarized and unpolarized cross section, longitudinal asymmetry), the statistical accuracy on the polarization measurement will be derived as a function of the luminosity.

3.1 Kinematic of Compton scattering $e\,\gamma \rightarrow e\,\gamma$

We note (Fig. 1)

- Incident electron e with energy E and momentum $\vec{p} = (0, 0, p)$ along (Oz) axis,
- Incident photon γ with energy k , incident angle α_c with respect to (Oz) and momentum $\vec{k} = (0, -k \sin\alpha_c, -k\cos\alpha_c)$,
- Scattered electron e' with energy E', scattering angle θ_e with respect to (Oz) and momentum $\vec{p'} = (p' \sin\theta_e \cos\phi_e, p' \sin\theta_e \sin\phi_e, p' \cos\theta_e)$,
- Scattered photon γ' with energy k', scattering angle θ_γ with respect to (Oz) and momentum $\vec{k'} = (k' \sin\theta_\gamma \cos\phi_\gamma, k' \sin\theta_\gamma \sin\phi_\gamma, k' \cos\theta_\gamma)$.

The scattered photon energy k' is related to the scattered photon angle θ_γ by :

$$k' = k\frac{E + p\cos\alpha_c}{E + k - p\cos\theta_\gamma + k(\cos\alpha_c\cos\theta_\gamma + \sin\alpha_c\sin\theta_\gamma\cos\phi_\gamma)}. \tag{4}$$

If the particules are scattered in the incident plane ($\phi_\gamma = 0$), we get :

$$k' = k\frac{E + p\cos\alpha_c}{E + k - p\cos\theta_\gamma + k\cos(\alpha_c - \theta_\gamma)}. \tag{5}$$

For photon incident angle $\alpha_c = 0$, this equation can be simplified, using $\gamma = E/m$, leading to

$$\frac{k'}{k} \simeq \frac{4a\gamma^2}{1 + a\theta_\gamma^2\gamma^2}, \tag{6}$$

where

$$a = \frac{1}{1 + \frac{4k\gamma}{m}} = \frac{1}{1 + \frac{4kE}{m^2}}. \tag{7}$$

The maximum scattered photon energy k'_{max}, corresponding to the minimum scattered electron energy E'_{min}, is reached for $\theta_\gamma = 0.$,

$$k'_{max} = 4ak\gamma^2 = 4ak\frac{E^2}{m^2}, \tag{8}$$

$$E'_{min} = E - k'_{max} + k = E - 4ak\frac{E^2}{m^2} + k \simeq E - 4ak\frac{E^2}{m^2}, \tag{9}$$

while the minimum scattered photon energy k'_{min}, corresponding to the maximum scattered electron energy E'_{max}, is for $\theta_\gamma = \pi$,

$$k'_{min} = k, \tag{10}$$

$$E'_{max} = E - k'_{min} + k = E. \tag{11}$$

The photon scattering angle at which $k' = k'_{max}/2$ is

$$\theta_{\gamma 1/2} = \frac{m}{E\sqrt{a}} = \frac{1}{\gamma\sqrt{a}}. \tag{12}$$

The scattered electron momentum p' is related to the scattered electron angle θ_e by a second order equation:

$$p'^2(C^2 - B^2) - 2ABp' + m^2C^2 - A^2 = 0,$$
$$p' = \frac{AB \pm C\sqrt{(A^2 - m^2(C^2 - B^2))}}{C^2 - B^2},$$

where

$$\begin{aligned} A &= m^2 + Ek + kp\cos\alpha_c, \\ B &= p\cos\theta_e - k(\cos\alpha_c\cos\theta_e + \sin\alpha_c\sin\theta_e\cos\phi_e), \\ C &= E + k. \end{aligned}$$

The maximum electron angle is obtained for $A^2 = m^2(C^2 - B^2)$. For small photon incident angle and energy, one gets :

$$\theta_e^{max} \simeq 2\frac{k}{m}. \tag{13}$$

E=250 GeV $\alpha_c = 10mrad$

k = 1.165 eV λ = 1064.2 nm	
a	0.183
$k'_{max}(GeV)$	204.2
$E'_{min}(GeV)$	45.777
$\theta_{\gamma 1/2}(\mu rad)$	4.78
$\theta_e^{max}(\mu rad)$	4.56

k = 2.330 eV λ =532.1 nm	
a	0.101
$k'_{max}(GeV)$	224.8
$E'_{min}(GeV)$	25.194
$\theta_{\gamma 1/2}(\mu rad)$	6.44
$\theta_e^{max}(\mu rad)$	9.12

k = 5.000 eV λ = 248.0 nm	
a	0.050
$k'_{max}(GeV)$	237.6
$E'_{min}(GeV)$	12.409
$\theta_{\gamma 1/2}(\mu rad)$	9.17
$\theta_e^{max}(\mu rad)$	19.57

Table 2: ***Kinematic Parameters.*** *For electron beam energy $E = 250GeV$ and for LASER energies of $k = 1.16\ eV$ (IR NdYAG), $k = 2.33\ eV$ (Green Argon) and $k = 5.0\ eV$ (UV KrF).*

These kinematic parameters are listed in table (2) for a LASER with energy $k = 1.16\ eV$ (resp. $k = 2.33\ eV$, $k = 5.0\ eV$) or wavelength $\lambda = 1064\ nm$ (resp. $\lambda = 532\ nm$, $\lambda = 248\ nm$). The relationships between the energies (k' or E') and the angles (θ_γ or θ_e) of the scattered γ or e^- are shown on figure (2).

One can see (Fig. 2) that **scattered electrons and photons (with $k'/k'_{max} > 0.1$) have a very small opening angle**. This means that to allow scattered photon or electron detection one needs to separate the scattered electron, the scattered photon and the incident electron. A solution is the use of a magnétic chicane [7][8]. If enouth space is avaible, another way is to install the polarimeter after a dipole of the beam line.

3.2 Compton Scattering Cross Section and Asymmetry

If the crossing angle α_c is equal to 0., **the differential unpolarized cross section** is [9],[10](Fig. 3)

$$\boxed{\frac{d\sigma}{d\rho} = 2\pi r_0^2 a \left[\frac{\rho^2(1-a)^2}{1-\rho(1-a)} + 1 + \left(\frac{1-\rho(1+a)}{1-\rho(1-a)}\right)^2\right],} \tag{14}$$

where

- the classical electron radius r_0 is given by $r_0 = \alpha\hbar c/mc^2 = 2.817\ 10^{-13}\ cm$,
- $\rho = k'/k'_{max}$ is the scattered γ energy normalized to the maximal energy (Eq. 8),
- a a kinematic parameter given by Eq. 7 : $a = 1/(1 + \frac{4kE}{m^2})$.

For **the total cross section**, we get after integration of the previous equation (Eq. 14)

$$\boxed{\sigma = \pi\ {r_0}^2 a \frac{(-1 - 14\ a + 16\ a^2 - 2\ a^3 + a^4 + 2\ \ln(a) - 12\ \ln(a)a - 6\ \ln(a)a^2)}{(-1+a)^3}}, \tag{15}$$

which only depends of a i.e. of kE (Eq. 7).

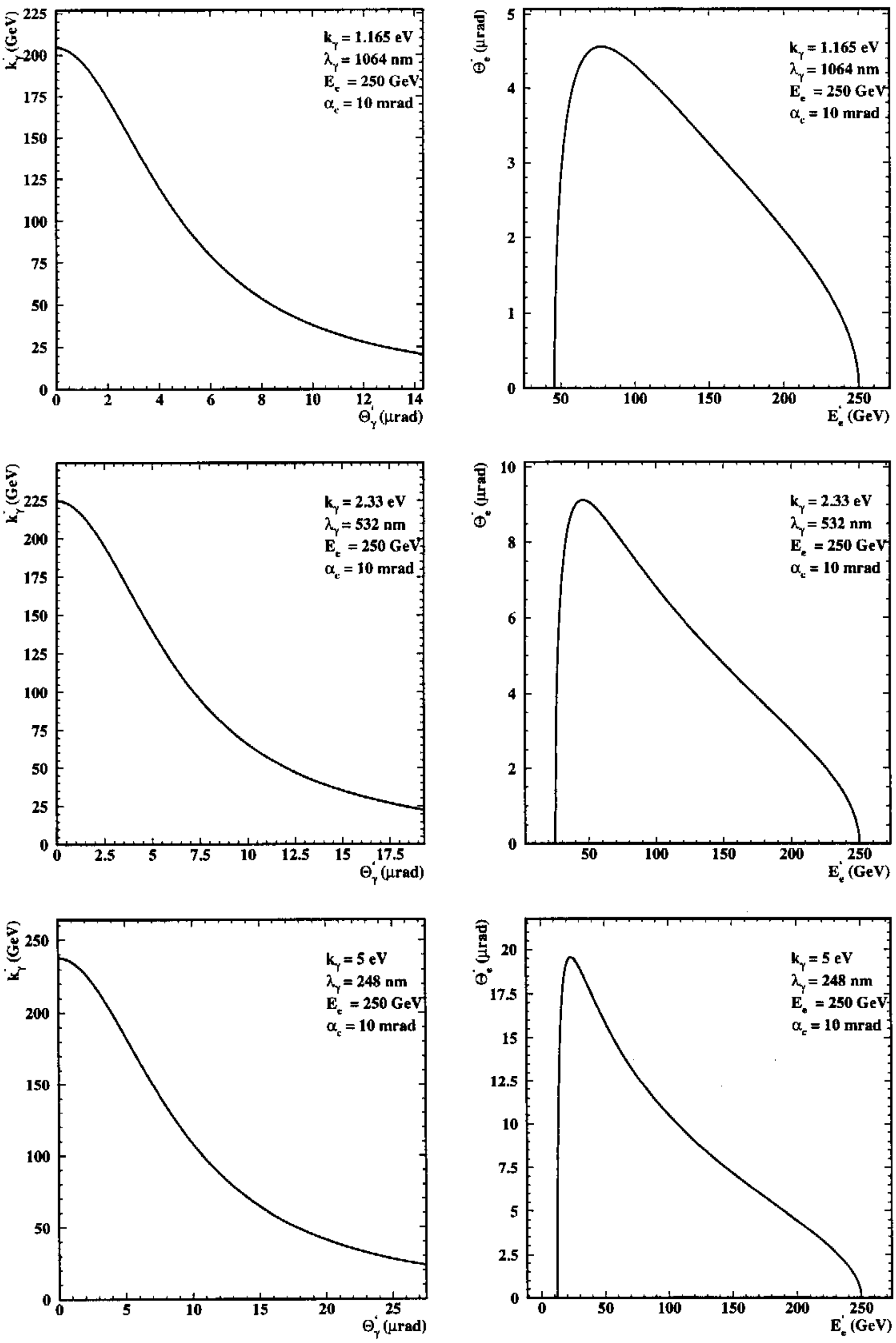

Figure 2: ***Kinematic of the Compton scattering.*** *Scattered γ energy k'_γ as a function of scattered γ angle θ_γ (left). Scattered e^- energy E'_e as a function of scattered e^- angle θ_e (right). For an electron beam energy of* 250 *GeV and a laser of 1064 nm, 532 nm, and 248 nm, with a crossing angle α_c=10 mrad.*

The longitudinal differential asymmetry (Fig. 3) is given by

$$A_l = \frac{\sigma_{\Rightarrow}^{\rightarrow} - \sigma_{\Rightarrow}^{\leftarrow}}{\sigma_{\Rightarrow}^{\rightarrow} + \sigma_{\Rightarrow}^{\leftarrow}} = \frac{2\pi r_0^2 a}{\frac{d\sigma}{d\rho}} (1 - \rho(1+a)) \left[1 - \frac{1}{(1-\rho(1-a))^2}\right],$$

$$A_l = \frac{1}{\left[\frac{\rho^2(1-a)^2}{1-\rho(1-a)} + 1 + \left(\frac{1-\rho(1+a)}{1-\rho(1-a)}\right)^2\right]} (1 - \rho(1+a)) \left[1 - \frac{1}{(1-\rho(1-a))^2}\right]. \tag{16}$$

The longitudinal asymmetry is maximum for $\rho = 1$, i.e $k' = k'_{max}$ (high energy scattered photon) or $E' = E'_{min}$ (low energy scattered electron) :

$$\boxed{A_l^{max} = \frac{(1-a)(1+a)}{(1+a^2)}.} \tag{17}$$

We note that the asymmetry A_l (Fig. 3) is negative at low scattered photon energy, positive at higher energy and vanishes for $\rho_0 = 1/(1+a)$, i.e for

$$k'_0 = \frac{2k\gamma^2}{1 + 2\frac{k\gamma}{m}}. \tag{18}$$

3.3 Consideration on the experimental measurement of the electron polarization

The longitudinal polarization P_e of the electron beam is extracted from the asymmetry between 2 measurements of Compton scattering with parallel (+) or anti parallel (-) polarization of the electron and Laser beams. Each measurement, performed with a luminosity $\mathcal{L}_+$ ($\mathcal{L}_-$) during a time T_+ (T_-), will be later normalized to the same integrated luminosity. We consider now three possibilities.

- **Differential Polarization Measurement**
  Event by event, the scattered photon or electron energy can be determined. So the numbers of Compton scattering events n_+^i and n_-^i are measured as a function of the scattered γ or e energy in N_b bins. From the asymmetry of these numbers, a measurement of the polarization P_e^i is performed for each energy bin. The weighted mean of P_e^i gives the electron polarization.

- **Integrated Polarization Measurement**
  Without energy measurement for the scattered particles, only the numbers of compton scattering events integrated over the energy range N_+ and N_- can be measured. From the asymmetry of these numbers, the electron polarization is deduced if we know the detection efficiency and the energy threshold for the scattered particles detection.

- **Energy Polarization weighted Measurement**
  If we can only measure the energy integrated over the energy range, the polarization will be deduced from the asymmetry between the integrated energies E_+ and E_-. In this case, we have also to know the detection efficiency and the energy threshold. Furthermore the error also depends on these 2 parameters.

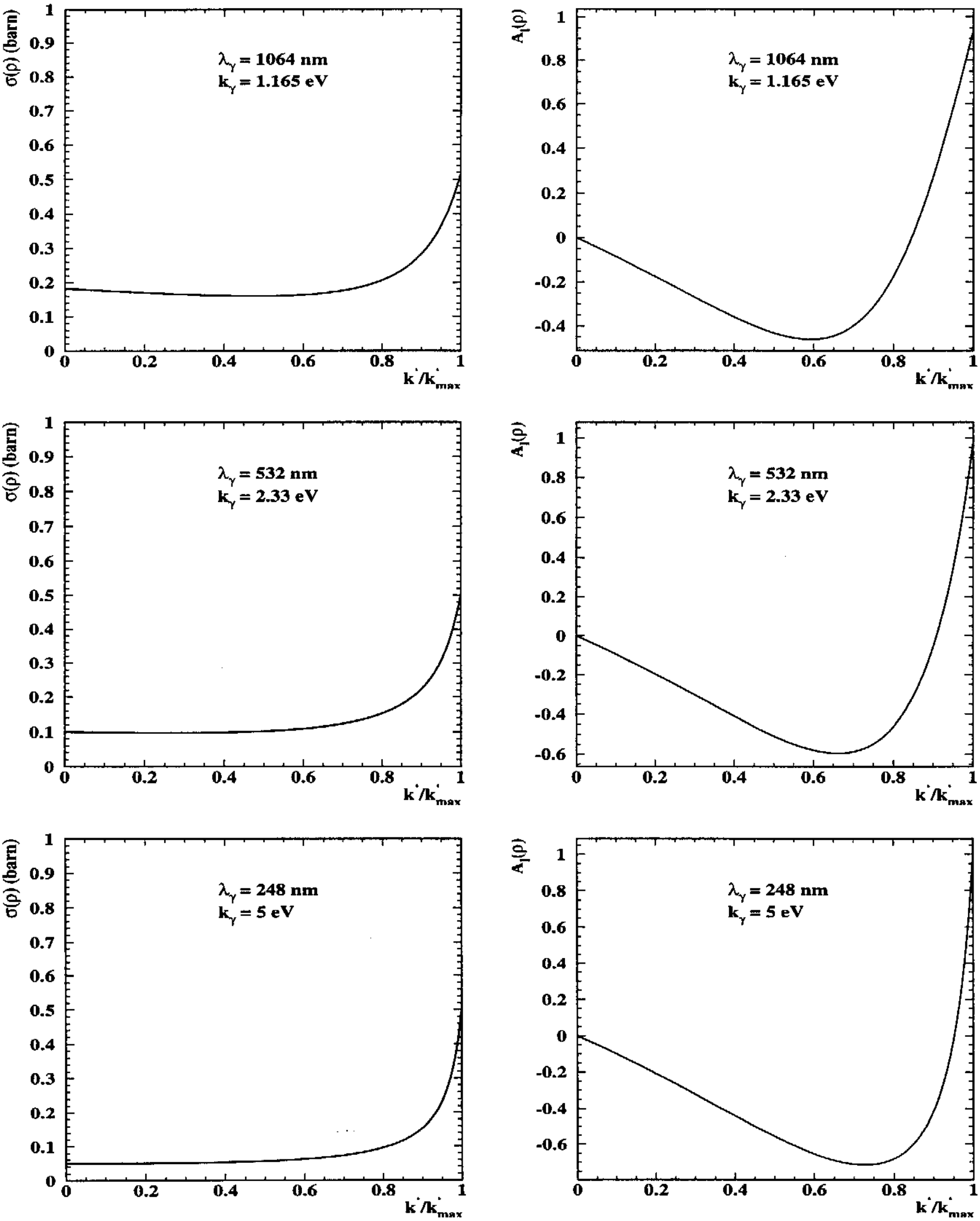

Figure 3: ***Compton unpolarized cross section and longitudinal asymmetry as a function of the scattered photon energy*** k'_γ. *For an electron beam energy of* $250\ GeV$ *and a* $1064\ nm$ *(Up),* $532\ nm$ *(Middle) and* $248\ nm$ *(Down) laser wavelength, with a crossing angle of* $\alpha_c = 10.\ mrad$.

For the 2 measurements with parallel (anti parallel) polarization of the electron and Laser beams, we assume now in the next sections, where we will examine for these 3 methods their efficiency to reach a given statistical accuracy, the same integrated luminosity $\mathcal{L}_+T_+ = \mathcal{L}_-T_- = \mathcal{L}\ T/2$ (where $\mathcal{L}$ is the mean luminosity and T is the total time of the measurements) and the same differential efficiency $\epsilon_+(\rho) = \epsilon_-(\rho) = \epsilon(\rho)$.

3.3.1 Differential Polarization Measurement

The numbers of compton scattering events as a function of the scattered photon energy for each of the N_b energy bins are

$$n^i_+ \quad = \quad \mathcal{L}_+\ T_+ \int_{\rho_i}^{\rho_{i+1}} d\rho\ \ \epsilon_+(\rho)\ \frac{d\sigma}{d\rho}(\rho)\ \ (1 + P_e P_\gamma A_l(\rho)), \tag{19}$$

$$n^i_- \quad = \quad \mathcal{L}_-\ T_- \int_{\rho_i}^{\rho_{i+1}} d\rho\ \ \epsilon_-(\rho)\ \frac{d\sigma}{d\rho}(\rho)\ \ (1 - P_e P_\gamma A_l(\rho)), \tag{20}$$

where $\frac{d\sigma}{d\rho}(\rho)$ is the unpolarized differential Compton cross section (Eq. 14) and $A_l(\rho)$ the differential asymmetry (Eq. 16).

The experimental asymmetry for each bin is related to the electron polarization by

$$A^i_{exp} \quad = \quad \frac{n^i_+ - n^i_-}{n^i_+ + n^i_-} \quad = \quad P_e P_\gamma \frac{\int d\rho\epsilon\ \frac{d\sigma}{d\rho}\ A_l}{\int d\rho\epsilon\ \frac{d\sigma}{d\rho}} \quad = \quad P_e P_\gamma\ < A_l >_i \quad \simeq \quad P_e P_\gamma\ \ A^i_l, \tag{21}$$

where A^i_l is the longitudinal polarization at the center of the bin.

The electron polarization measured for each bin P^i_e given by

$$P^i_e \quad = \quad \frac{A^i_{exp}}{P_\gamma\ < A_l >_i} \quad \simeq \quad \frac{A^i_{exp}}{P_\gamma\ \ A^i_l} \tag{22}$$

is then almost independent of the detection efficiency and have an absolute error dP^i_e given by

$$\frac{dP^{i2}_e}{P^{i2}_e} \quad = \quad \frac{{dA^i_{exp}}^2}{{A^i_{exp}}^2} \quad = \quad 4\frac{n^i_+ n^i_-}{n^{i3}_t}\ \frac{1}{(P_e P_\gamma A^i_l)^2} \quad = \quad \frac{1 - (P_e P_\gamma A^i_l)^2}{2n^i_t}\ \frac{1}{(P_e P_\gamma A^i_l)^2},$$

$$dP^{i2}_e \quad = \quad \frac{1 - (P_e P_\gamma A^i_l)^2}{2n^i_t}\ \frac{1}{(P_\gamma A^i_l)^2} \quad = \quad \frac{1}{\mathcal{L}TP^2_\gamma}\ \frac{1 - (P_e P_\gamma A^i_l)^2}{\sigma^i A^{i2}_l},$$

where n^i_t is the total number of events for the bin i

$$n^i_t = n^i_+ + n^i_- = \mathcal{L}T \int_{\rho_i}^{\rho_{i+1}} d\rho\ \ \epsilon\ \frac{d\sigma}{d\rho}(\rho) = \mathcal{L}T\ \sigma^i.$$

The final electron polarization, obtained as the weighted mean of these polarization measurements,

$$P_e \quad = \quad \frac{\sum_{i=1}^{N_b} \frac{P_e^i}{dP_e^{i\,2}}}{\sum_{i=1}^{N_b} \frac{1}{dP_e^{i\,2}}}, \tag{23}$$

is also almost independent of the detection efficiency and does not depend on the detection threshold.

The error achieved with a total number of events N_t for an energy threshold ρ_{min}

$$N_t = \sum_{i=1}^{N_b} n_t^i = \mathcal{L}T \int_{\rho_{min}}^{1} d\rho \;\; \epsilon \;\; \frac{d\sigma}{d\rho}(\rho) = \mathcal{L}T \; \sigma_t \qquad with \qquad \sigma_t = \int_{\rho_{min}}^{1} d\rho \;\; \epsilon \;\; \frac{d\sigma}{d\rho}(\rho) \tag{24}$$

is given by

$$\frac{1}{dP_e{}^2} \;=\; \sum_{i=1}^{N_b} \frac{1}{dP_e^{i2}} \;=\; \mathcal{L}T \;\; P_\gamma^2 \; \sum_{i=1}^{N_b} \frac{\sigma^i A_l^{i2}}{1-(P_e P_\gamma A_l^i)^2} \;=\; \mathcal{L}T \;\; P_\gamma^2 \; \int_{\rho_{min}}^{1} \frac{\epsilon \;\sigma A_l{}^2}{1-(P_e P_\gamma A_l)^2},$$

$$\boxed{\left(\frac{dP_e}{P_e}\right)^{-2} \;=\; \mathcal{L}T \;\; P_e^2 P_\gamma^2 \;\; \sigma_t \; < \frac{A_l^2}{1-P_e^2 P_\gamma^2 A_l^2} > \;\simeq\; \mathcal{L}T \;\; P_e^2 P_\gamma^2 \;\; \sigma_t \;\; < A_l^2 >,} \tag{25}$$

where the mean value $< A_l^2 >$ stands for

$$< A_l^2 > \;\; = \;\; \frac{\int_{\rho_{min}}^{1} d\rho \;\; \epsilon(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; A_l(\rho)^2}{\int_{\rho_{min}}^{1} d\rho \;\; \epsilon(\rho) \;\; \frac{d\sigma}{d\rho}(\rho)}. \tag{26}$$

The needed time t_D to achieve an accuracy $\Delta P_e/P_e$ is then

$$\boxed{t_D^{-1} \;\; = \;\; \mathcal{L} \;\; \left(\frac{\Delta P_e}{P_e}\right)^2 \;\; P_e^2 P_\gamma^2 \;\; \sigma_t \; < \frac{A_l^2}{1-P_e^2 P_\gamma^2 A_l^2} > \simeq \mathcal{L} \;\; \left(\frac{\Delta P_e}{P_e}\right)^2 \;\; P_e^2 P_\gamma^2 \;\; \sigma_t \;\; < A_l^2 >} \tag{27}$$

and corresponds to a total number of events

$$N_t^D \;\; = \;\; \mathcal{L} t_D \sigma_t \;\; \simeq \;\; \frac{1}{\left(\frac{\Delta P_e}{P_e}\right)^2 \;\; P_e^2 P_\gamma^2 \;\; < A_l^2 >}. \tag{28}$$

We note that the needed time t_D as well as the square of the error are proportional to the inverse of $< A_l^2 >$.

3.3.2 Integrated Polarization Measurement

The numbers of compton scattering events integrated over the energy range N_+ and N_- are

$$N_+ \;\; = \;\; \mathcal{L}_+ \;\; T_+ \int_{\rho_{min}}^{1} d\rho \;\; \epsilon_+(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; (1 + P_e P_\gamma A_l(\rho)), \tag{29}$$

$$N_- = \mathcal{L}_- \; T_- \int_{\rho_{min}}^{1} d\rho \;\; \epsilon_-(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; (1 - P_e P_\gamma A_l(\rho)). \tag{30}$$

The experimental integrated asymmetry is related to the electron polarization by

$$A_{exp} = \frac{N_+ \; - \; N_-}{N_+ \; + \; N_-} = P_e P_\gamma \; \frac{\int \epsilon \;\; \sigma A_l}{\int \epsilon \;\; \sigma} \;\; = \;\; P_e P_\gamma \;\; < A_l > \tag{31}$$

where the mean value $< A_l >$ stands for

$$< A_l > \; = \; \frac{\int_{\rho_{min}}^{1} d\rho \;\; \epsilon(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; A_l(\rho)}{\int_{\rho_{min}}^{1} d\rho \;\; \epsilon(\rho) \;\; \frac{d\sigma}{d\rho}(\rho)}. \tag{32}$$

Thus the measured electron polarization

$$P_e \; = \; \frac{A_{exp}}{P_\gamma \;\; < A_l >} \tag{33}$$

is proportional to the inverse of the mean longitudinal asymmetry and so depends on the detection efficiency and on the detection threshold ρ_{min}.

The absolute and relative errors on the experimental integrated asymmetry are

$$dA_{exp}{}^2 \; = \; 4 \frac{N_+ N_-}{N_t^3} = \frac{1}{\mathcal{L}T} \frac{1}{\sigma_t} \left(1 - \left(P_e P_\gamma \frac{\int \epsilon \sigma A_l}{\sigma_t} \right)^2 \right) \; = \; \frac{1}{\mathcal{L}T} \frac{1}{\sigma_t} \left(1 - (P_e P_\gamma < A_l >)^2 \right),$$

$$\boxed{\left(\frac{dA_{exp}}{A_{exp}} \right)^{-2} \; = \; \left(\frac{dP_e}{P_e} \right)^{-2} \; = \; \mathcal{L}T \;\; P_e^2 P_\gamma^2 \;\; \sigma_t \;\; \frac{< A_l >^2}{1 - P_e^2 P_\gamma^2 < A_l >^2},} \tag{34}$$

where σ_t and N_t are given by equation (24).

The needed time t_I to achieve an accuracy $\Delta P_e / P_e$ is then

$$\boxed{t_I^{-1} \; = \; \mathcal{L} \; \left(\frac{\Delta P_e}{P_e} \right)^2 \;\; P_e^2 P_\gamma^2 \; \sigma_t \; \frac{< A_l >^2}{1 - P_e^2 P_\gamma^2 < A_l >^2} \; \simeq \; \mathcal{L} \; \left(\frac{\Delta P_e}{P_e} \right)^2 \;\; P_e^2 P_\gamma^2 \; \sigma_t < A_l >^2,} \tag{35}$$

corresponding to a total number of events

$$N_t^I \; = \; \mathcal{L} t_I \sigma_t \; \simeq \; \frac{1}{\left(\frac{\Delta P_e}{P_e} \right)^2 \;\; P_e^2 P_\gamma^2 \;\; < A_l >^2}. \tag{36}$$

We note that the needed time t_I as well as the square of the error are proportional to the inverse of $< A_l >^2$. We have seen (Fig. 3) that the asymmetry A_l is negative at low scattered photon energy and positive at higher energy. So in an integrated asymmetry measurement, it will be helpful to impose an hardware threshold to cut the lower energy part of that spectrum and increase the mean longitudinal asymmetry.

For a 100% efficiency $\epsilon(\rho)$ of the detection and a threshold ρ_{min}=0, the mean longitudinal (weighted by the cross section) asymmetry is :

$$< A_l > \; = \; \frac{\int_0^1 d\rho \;\; A_l \frac{d\sigma}{d\rho}}{\int_0^1 d\rho \;\; \frac{d\sigma}{d\rho}} \; = \; \frac{\int d\rho \;\; 2\pi r_0^2 \;\; (1-\rho(1+a)) \;\; \left[1 - \frac{1}{(1-\rho(1-a))^2}\right]}{\int \frac{d\sigma}{d\rho}},$$

$$< A_l > \; = \; -\frac{(-5+7\,a-3\,a^2+a^3-2\,\ln(a)-2\,\ln(a)a) \;\; (-1+a)}{(-1-14\,a+16\,a^2-2\,a^3+a^4+2\,\ln(a)-12\,\ln(a)a-6\,\ln(a)a^2)}. \tag{37}$$

3.3.3 Energy weighted Polarization Measurement

The integrated energy E_+ and E_- over the energy range and over the time t are given by :

$$E_+ \;=\; \mathcal{L}_+ T_+ \int_{\rho_{min}}^1 d\rho \;\; E \;\; \epsilon_+(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; (1 + P_e P_\gamma A_l(\rho)), \tag{38}$$

$$E_- \;=\; \mathcal{L}_- T_- \int_{\rho_{min}}^1 d\rho \;\; E \;\; \epsilon_-(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; (1 - P_e P_\gamma A_l(\rho)), \tag{39}$$

with a statistical error $dE_\pm$ due to the fluctuation of the unmeasured number of events $\frac{dN_\pm}{d\rho}$

$$\frac{dN_\pm}{d\rho} = \mathcal{L}_\pm T_\pm \epsilon_\pm \frac{d\sigma}{d\rho}(1 \pm P_e P_\gamma A_l)$$

$$dE_\pm^2 \;=\; \mathcal{L}\frac{T}{2} \int_{\rho_{min}}^1 d\rho \;\; E^2 \;\; \epsilon_\pm(\rho) \;\; \frac{d\sigma}{d\rho}(\rho) \;\; (1 \pm P_e P_\gamma A_l(\rho)). \tag{40}$$

The experimental integrated energy asymmetry is related to the electron polarization by

$$A_{exp} \;=\; \frac{E_+ \;-\; E_-}{E_+ \;+\; E_-} \;=\; P_e P_\gamma \;\frac{\int \epsilon\sigma E A_l}{\int \epsilon\sigma E} \;=\; P_e P_\gamma \;\frac{< E A_l >}{< E >}. \tag{41}$$

Thus the measured electron polarization

$$P_e \;=\; \frac{A_{exp}}{P_\gamma \; \frac{<EA_l>}{<E>}} \tag{42}$$

is proportional to the inverse of the mean longitudinal asymmetry (weighted by the energy) and so depends on the detection efficiency and on the detection threshold ρ_{min}.

The absolute and relative errors on the experimental integrated asymmetry are

$$dA_{exp}{}^2 = 4 \;\; \frac{E_+^2 \;\, dE_-^2 \;\; + \;\; E_-^2 \;\, dE_+^2}{(E_+ \;\; + \;\; E_-)^4}$$

$$= \frac{1}{\mathcal{L}T} \; \frac{\left(\int \epsilon\sigma E^2\right)\left(\int \epsilon\sigma E\right)^2 \;+\; P_e^2 P_\gamma^2 \;\left[\, \left(\int \epsilon\sigma E^2\right)\left(\int \epsilon\sigma E A_l\right)^2 \;-2\;\left(\int \epsilon\sigma E\right)\left(\int \epsilon\sigma E A_l\right)\left(\int \epsilon\sigma E^2 A_l\right)\,\right]}{\left(\int \epsilon\sigma E\right)^4},$$

$$\boxed{\left(\frac{dA_{exp}}{A_{exp}}\right)^{-2} = \left(\frac{dP_e}{P_e}\right)^{-2} = \mathcal{L}TP_e^2P_\gamma^2\sigma_t\frac{<EA_l>^2}{<E^2>}\frac{1}{1+P_e^2P_\gamma^2\left(\frac{<EA_l>^2}{<E>^2}-2\frac{<EA_l><E^2A_l>}{<E><E^2>}\right)},} \tag{43}$$

where σ_t is given by equation (24) and the mean value $\frac{<EA_l>^2}{<E^2>}$ by

$$\frac{<EA_l>^2}{<E^2>} = \frac{\left(\int_{\rho_{min}}^1 d\rho\ \epsilon(\rho)\ \frac{d\sigma}{d\rho}(\rho)\ EA_l(\rho)\right)^2}{\int_{\rho_{min}}^1 d\rho\ \epsilon(\rho)\ \frac{d\sigma}{d\rho}(\rho)\ E^2}. \tag{44}$$

These errors will have to be estimated since we don't measure the number of scattered events.

The needed time t_E to achieve an accuracy $\Delta P_e/P_e$ is then

$$t_E^{-1} = \mathcal{L}\left(\frac{\Delta P_e}{P_e}\right)^2 P_e^2P_\gamma^2\ \sigma_t\ \frac{<EA_l>^2}{<E^2>}\ \frac{1}{1+P_e^2P_\gamma^2\left(\frac{<EA_l>^2}{<E>^2}-2\frac{<EA_l><E^2A_l>}{<E><E^2>}\right)}$$

$$\boxed{t_E^{-1} \simeq \mathcal{L}\left(\frac{\Delta P_e}{P_e}\right)^2 P_e^2P_\gamma^2\ \sigma_t\ \frac{<EA_l>^2}{<E^2>},} \tag{45}$$

corresponding to a total number of events

$$N_t^E = \mathcal{L}t_E\sigma_t \simeq \frac{1}{\left(\frac{\Delta P_e}{P_e}\right)^2\ P_e^2P_\gamma^2\ \frac{<EA_l>^2}{<E^2>}}. \tag{46}$$

3.3.4 Conclusions

The needed time or the total number of events to achieve an accuracy $\Delta P_e/P_e$ is given by

$$\boxed{t_{meth} = \frac{1}{\mathcal{L}\left(\frac{\Delta P_e}{P_e}\right)^2\ P_e^2P_\gamma^2}\ \frac{1}{A_{meth}^2}\ \frac{1}{\sigma_t},} \tag{47}$$

where, according to the method (differential, integrated, energy weighted), A_{meth}^2 stands for (See Eq. 26,32 and 44)

$$<A^2>\ :\ <A_l>^2\ :\ \frac{<EA_l>^2}{<E^2>}$$

with the relation

$$<A_l>^2\ <\ \frac{<EA_l>^2}{<E^2>}\ <\ <A^2>.$$

The integrated unpolarized cross section (Eq. 24), the mean asymetries (Eq. 26,32 and 44) and the corresponding needed time are shown on figures (4),(5) and (6) as a function of the energy threshold. The needed time for an accuracy $\Delta P_e/P_e = 1\%$ is given for a luminosity $\mathcal{L} = 1\ \mu barn^{-1}s^{-1} = 10^6 barn^{-1}s^{-1}$, a detection efficiency $\epsilon = 100\%$, an electron polarization $P_e = 80\%$ and a Laser polarization $P_\gamma = 100\%$. We see the advantage for the integrated and energy weighted methods to put a hardware threshold $\rho_{min} \simeq 0.85$. Above this threshold, the 3 methods are equivalent. There is a

Laser : k = 1.165 eV λ = 1064.2 nm											
E (GeV)	a	σ (barn)	k_{max} (GeV)	k_0 (GeV)	A_l^{max} (%)	$< A_l >$ (%)	$\sqrt{< A_l^2 >}$ (%)	$< A_l >_e$ (%)	t_I (s)	t_D (s)	t_E (s)
250.	0.183	0.198	204.2	172.6	93.51	-7.07	39.13	4.16	15.74	0.37	59.05

Laser : k = 2.330 eV λ = 532.1 nm											
E (GeV)	a	σ (barn)	k_{max} (GeV)	k_0 (GeV)	A_l^{max} (%)	$< A_l >$ (%)	$\sqrt{< A_l^2 >}$ (%)	$< A_l >_e$ (%)	t_I (s)	t_D (s)	t_E (s)
250.	0.101	0.136	224.8	204.2	97.99	-16.68	44.40	-8.69	4.06	0.42	19.25

Laser : k = 5.000 eV λ = 248.0 nm											
E (GeV)	a	σ (barn)	k_{max} (GeV)	k_0 (GeV)	A_l^{max} (%)	$< A_l >$ (%)	$\sqrt{< A_l^2 >}$ (%)	$< A_l >_e$ (%)	t_I (s)	t_D (s)	t_E (s)
250.	0.050	0.084	237.6	226.4	99.51	-26.81	50.57	-22.04	2.47	0.51	4.57

Table 3: ***Cross section, kinematic parameters, maximal asymmetry, mean asymmetries*** ($< A_l >$, $< A_l^2 >^{0.5}$, $< EA_l >/< E >$ ***) and needed time to obtain an accuracy*** $\Delta P_e/P_e = 1\%$ ***on the electron polarization.*** *For a luminosity* $\mathcal{L} = 1.\ \mu barn^{-1}s^{-1}$, *an electron polarization* $P_e = 80\%$, *a Laser polarization* $P_\gamma = 100\%$ *and with an energy threshold* $\rho_{min} = 0$. *For electron beam energy* $E = 250GeV$ *and for a LASER of* $k = 1.16\ eV$ *(up, IR NdYAG),* $k = 2.33\ eV$ *(middle, Green Argon) and* $k = 5.0\ eV$ *(down, UV KrF)*

factor 2 with the optimum obtained in the differential method with an energy threshold =0. We recall that for the energy weighted method, the error on the electron polarization will have to be estimated.

Nevertheless, for the integrated and energy weighted methods, the measured polarization (Eq. 33 and 42) depends on the detection efficiency and on the energy threshold. The figures (4), (5) and (6) show the variation of the measured asymmetry ($\simeq < A_l >$ or $< EA_l >/< E >$) with the energy threshold for these 2 methods (as well as $\sqrt{< A_l^2 >}$). Around $\rho_{min} \simeq 0.85$ a variation of 1 % on ρ gives a 2 % variation on the measured asymmetry and then on the electron polarization.

Finally, table (3) gives more complete results for some available lasers. We can see that to perform a 1 % measurement in 1 hour (at 250 GeV with $P_e = 80\%$), we need a luminosity of order of 10^2 to $1.510^2\ barn^{-1}s^{-1}$ according to the laser energy.

4 Expected Luminosity and Counting Rates

The needed time (Eq. 27,35,45) as well as the total number of events (Eq. 28,36,46) to achieve an accuracy $\Delta P_e/Pe$ on the electron polarization depends, for a given incident e^- energy, on

- the unpolarized Compton cross section, i.e. the energy of the Laser;
- the mean longitudinal asymmetry, i.e. the energy of the Laser;
- the luminosity, i.e. the shapes of the electron and Laser beams and of the crossing angle α_c of the 2 beams.

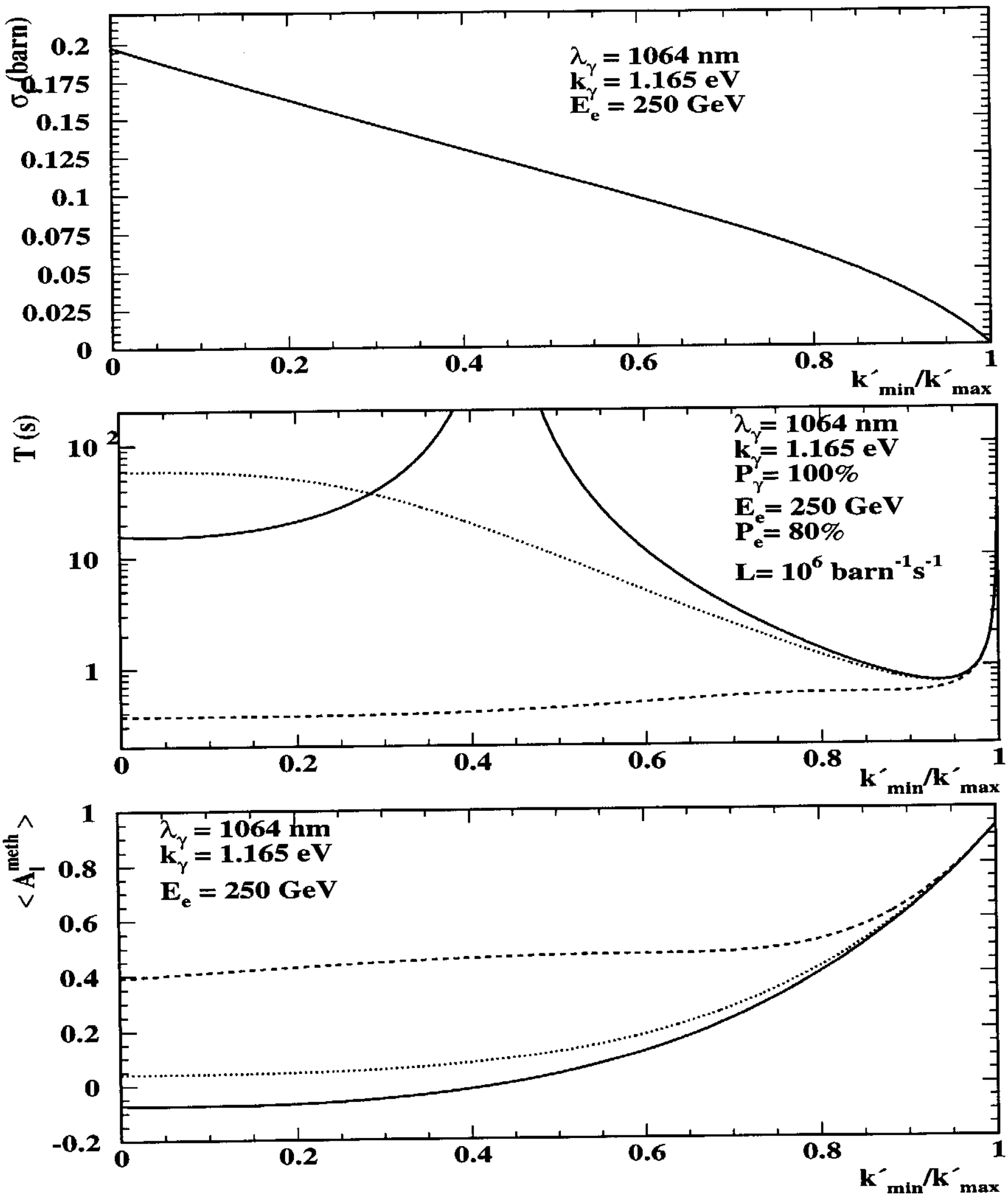

Figure 4: ***Unpolarized integrated cross section, needed time for an accuracy $\Delta P_e/P_e$ = 1 % on the electron polarization, mean asymmetry A_l^{meth} as a function of energy threshold.*** *For the 3 methods: integrated method (solid line $A_l^{meth} = < A_l >$), differential method (dashed line $A_l^{meth} = < A_l^2 >^{0.5}$) and energy weighted (dotted line $A_l^{meth} = < EA_l > / < E >$). For a laser of 1064 nm and for an electron beam energy of 250 GeV. The needed time is for a luminosity $\mathcal{L} = 1\ \mu barn^{-1}s^{-1}$, beam polarizations $P_e = 80\%$ and $P_\gamma = 100\%$.*

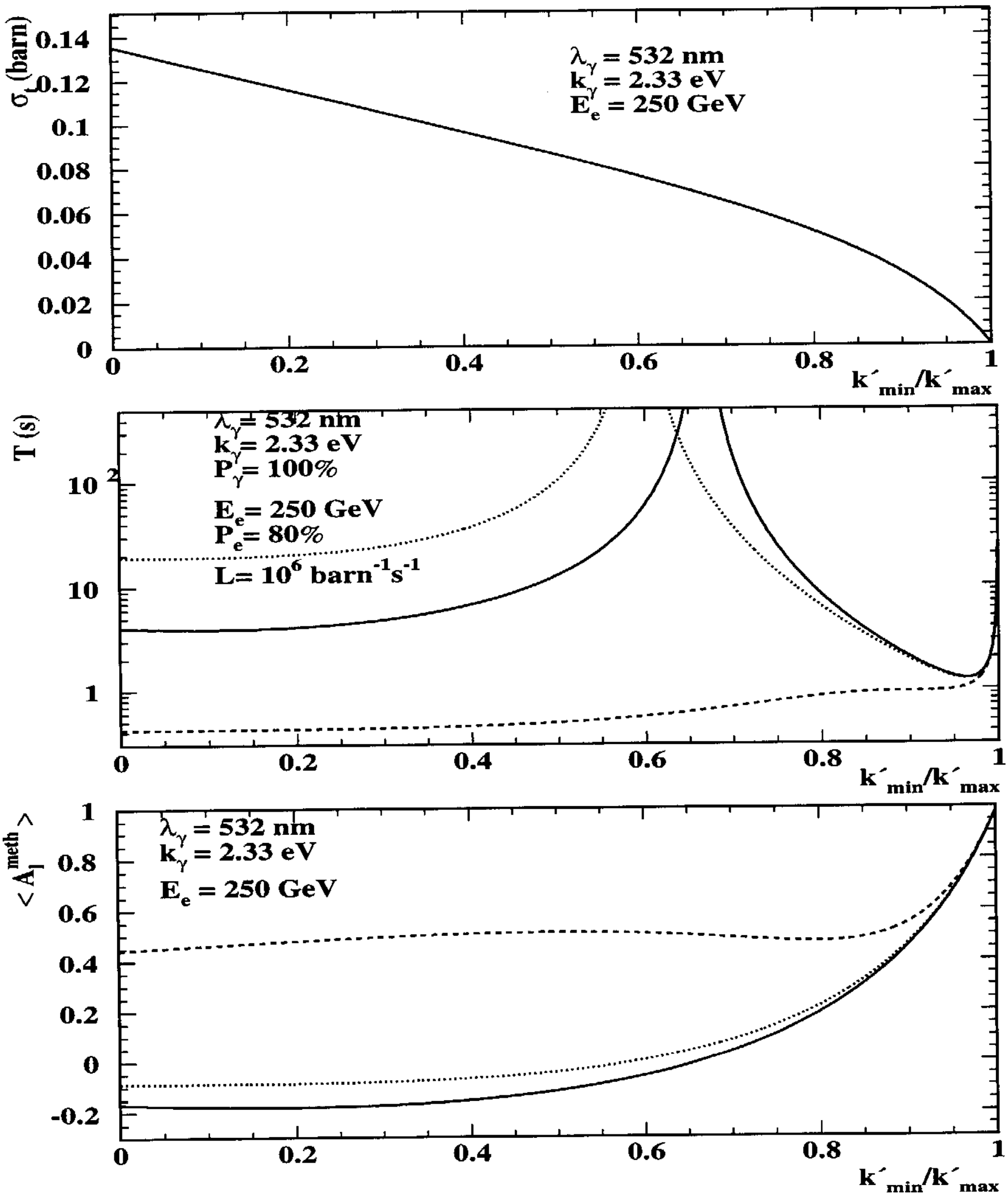

Figure 5: ***Unpolarized integrated cross section, needed time for an accuracy $\Delta P_e/P_e$ = 1 % on the electron polarization, mean asymmetry A_l^{meth} as a function of energy threshold.*** *For the 3 methods: integrated method (solid line $A_l^{meth} = < A_l >$), differential method (dashed line $A_l^{meth} = < A_l^2 >^{0.5}$) and energy weighted (dotted line $A_l^{meth} = < EA_l > / < E >$). For a laser of 532 nm and for an electron beam energy of 250 GeV. The needed time is for a luminosity $\mathcal{L} = 1\ \mu barn^{-1}s^{-1}$, beam polarizations $P_e = 80\%$ and $P_\gamma = 100\%$.*

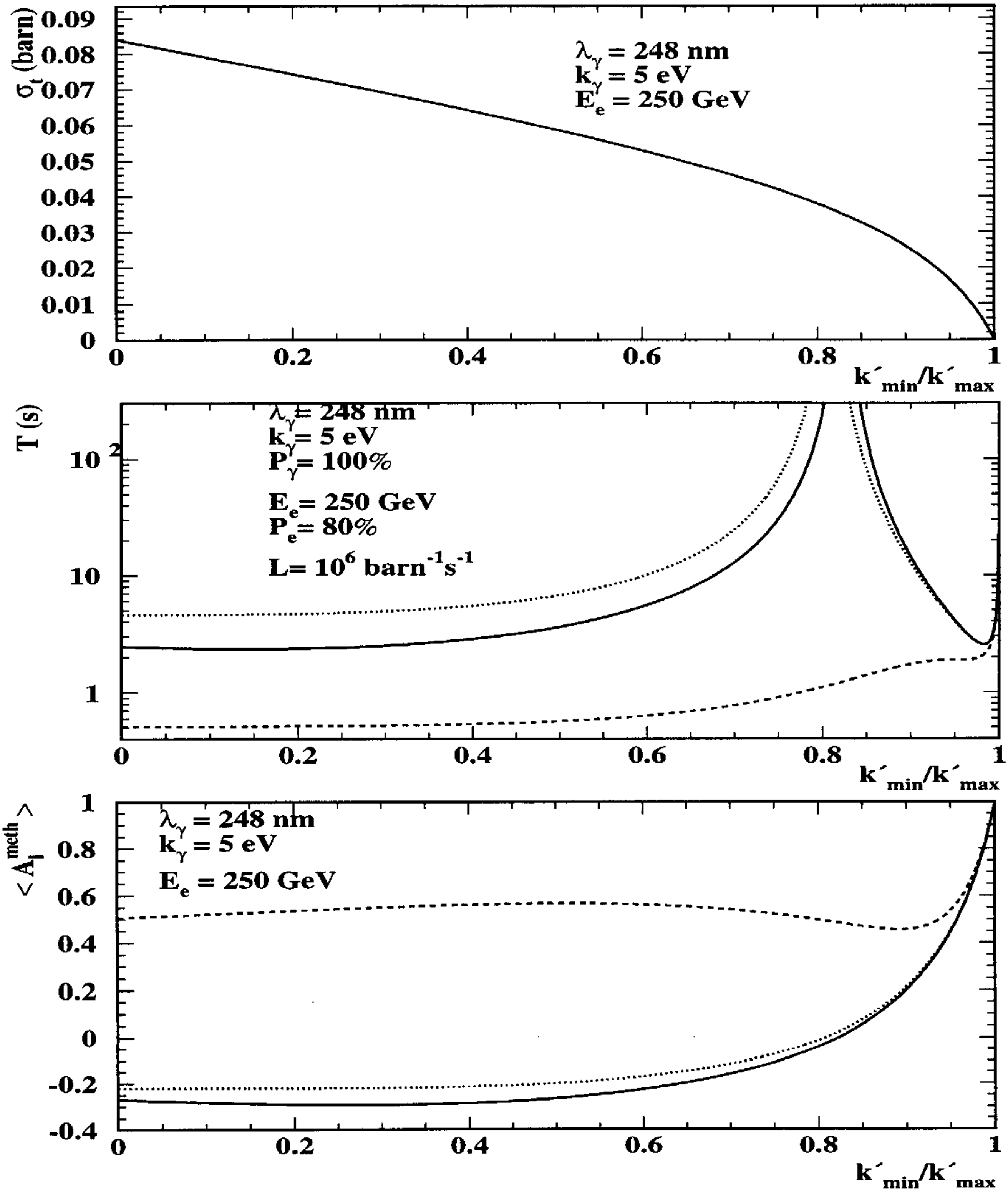

Figure 6: ***Unpolarized integrated cross section, needed time for an accuracy*** $\Delta P_e/P_e$ ***= 1*** $\%$ ***on the electron polarization, mean asymmetry*** A_l^{meth} ***as a function of energy threshold.*** *For the 3 methods: integrated method (solid line* $A_l^{meth} =< A_l >$*), differential method (dashed line* $A_l^{meth} =< A_l^2 >^{0.5}$*) and energy weighted (dotted line* $A_l^{meth} = < EA_l >/< E >$*). For a laser of 248 nm and for an electron beam energy of 250 GeV. The needed time is for a luminosity* $\mathcal{L} = 1$ $\mu barn^{-1}s^{-1}$*, beam polarizations* $P_e = 80\%$ *and* $P_\gamma = 100\%$.

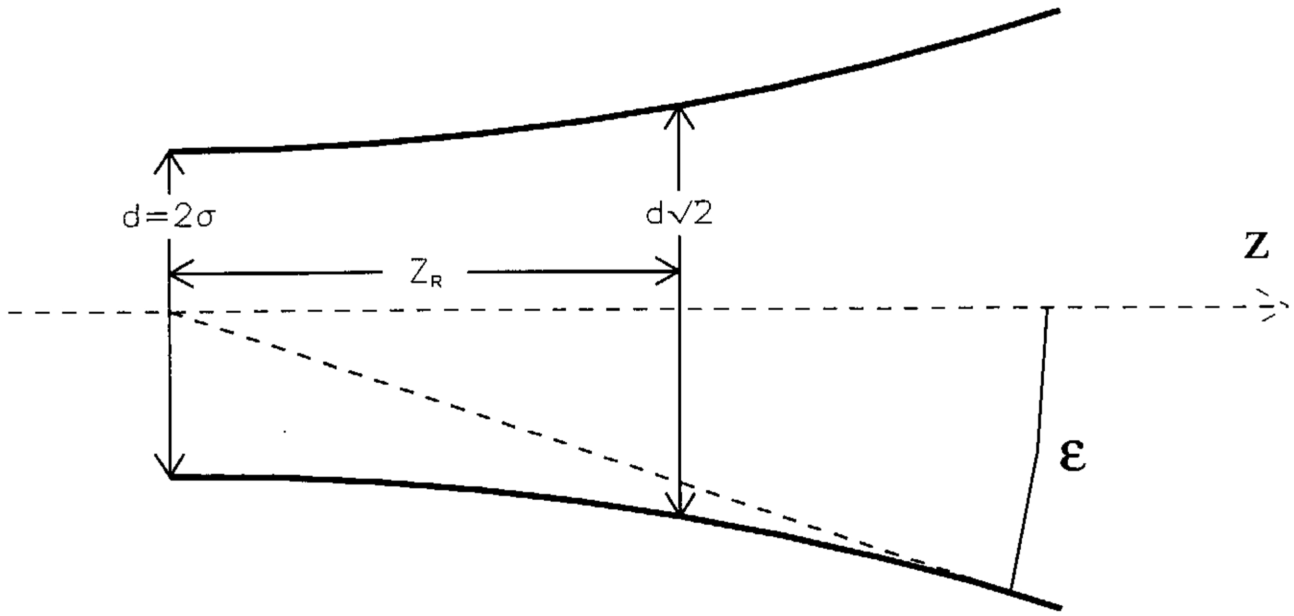

Figure 7: ***Beam shape.*** *Waist d, angular divergence ϵ and Rayleigh range Z_R .*

In the previous section, we have studied the cross section and asymmetry evolution with the beam parameters. We will now look at the luminosity for the interaction of the 2 beams with a crossing angle α_c.

4.1 General beam shape

The beam density in the beam frame (x, y, z where z is along the beam axis) is taken as the product of two normalized gaussians in x and y with a normalisation factor N_0

$$\rho(x,y,z) = N_0 \ \left(\frac{1}{\sqrt{2\pi}\sigma_x(z)} \ exp\left(-\frac{x^2}{2{\sigma_x}^2(z)}\right)\right) \ \left(\frac{1}{\sqrt{2\pi}\sigma_y(z)} \ exp\left(-\frac{y^2}{2{\sigma_y}^2(z)}\right)\right) \tag{48}$$

where $\sigma_x(z)$ and $\sigma_y(z)$ are the x and y beam sizes at z.

The evolution along z of these beam extensions is characterized (Fig. 7) by the waist beam size and the angular divergence ϵ (parametrized by the Rayleigh range Z)

$${\sigma_{x,y}}^2(z) = {\sigma_{x,y}}^2(0) \quad \left(1. + \frac{z^2}{{Z_{x,y}}^2}\right). \tag{49}$$

So that the waist size and the asymptotic angular divergence are

$$\begin{aligned} d(0) &= 2\sigma(0), \\ tan(\epsilon) &= \sigma(0)/Z. \end{aligned} \tag{50}$$

The normalization factor N_0 for the Laser beam (power P_L and wavelength λ) is deduced from :

$$\frac{dN_\gamma}{dt} = \frac{P_L}{h\,\nu} = \frac{N_{0\gamma} \int\int\int \rho_\gamma \, dz \, dx \, dy}{dt} = \frac{N_{0\gamma} \int \, dz}{dt} = \frac{N_{0\gamma} \, c \, dt}{dt} = N_{0\gamma} \, c,$$

so we have :

$$N_{0\gamma} = \frac{P_L}{h\,\nu\,c} = \frac{P_L\,\lambda}{h\,c^2}. \tag{51}$$

The normalization factor N_0 for the electron beam (intensity I_e) is deduced from :

$$\frac{dN_e}{dt} = \frac{I_e}{e} = \frac{N_{0e}\ \int\int\int \rho_e\, dz\, dx\, dy}{dt} = \frac{N_{0e}\ \int\ dz}{dt} = \frac{N_{0e}\, c\, dt}{dt} = N_{0e}\, c,$$

so we have:

$$N_{0e} = \frac{I_e}{e\, c}. \tag{52}$$

We will assume now that

- the electron and Laser beam are focussed on the same point taken as the origin of the reference system,
- the electron and Laser beam are symmetric in x and y i.e.

$$\sigma_x(z) = \sigma_y(z) = \sigma\sqrt{\left(1. + \frac{z^2}{Z^2}\right)}.$$

4.2 Luminosity and interaction length

The total luminosity for the interaction of the Laser and the electron beams is

$$\mathcal{L} = \int\int\int\ v_{rel}\ \rho_e\ \rho_\gamma\ dz\ dx\ dy \tag{53}$$

where v_{rel}, the relative velocity of the two beams, is $v_{rel}\ =\ c\,(1 + cos(\alpha_c))$ with α_c crossing angle and ρ_e, ρ_γ are the electron and Laser beam's densities.

4.2.1 Luminosity for zero crossing angle

In the case of zero crossing angle, we have the following characteristics :

1. **Differential luminosity** given by

$$\frac{d\mathcal{L}_0}{dz} = \int\int\ v_{rel}\ \rho_e\ \rho_\gamma\ dx\ dy = \frac{I_e}{e}\ \frac{P_L\ \lambda}{h\, c^2}\ \frac{1}{\pi}\ \frac{1}{\left(\sigma_e^2 + \sigma_\gamma^2\right) + z^2\left(\left(\frac{\sigma_\gamma}{Z_\gamma}\right)^2 + \left(\frac{\sigma_e}{Z_e}\right)^2\right)},$$

$$\frac{d\mathcal{L}_0}{dz} = \frac{I_e}{e}\ \frac{P_L\ \lambda}{h\, c^2}\ \frac{1}{\pi}\ \frac{1}{\left(\sigma_e^2 + \sigma_\gamma^2\right) + z^2\left({\epsilon_\gamma}^2 + {\epsilon_e}^2\right)}. \tag{54}$$

The differential luminosity depends on 4 parameters $(\sigma_e, \epsilon_e, \sigma_\gamma, \epsilon_\gamma)$. For constant beam emittance $\mathcal{E}\ =\ \sigma\epsilon$, the width of this distribution (related to the interaction length which will be defined later) decreases when increasing the angular divergences.

2. **Total luminosity** given by

$$\mathcal{L}_0 = \int\int\int\ v_{rel}\ \rho_e\ \rho_\gamma\ dz\ dx\ dz =\ \frac{I_e}{e}\ \frac{P_L\ \lambda}{h\, c^2}\ \frac{1}{\sqrt{\left(\frac{\sigma_e^2}{Z_e}\right)^2 + \left(\frac{\sigma_\gamma^2}{Z_\gamma}\right)^2 + \left(\frac{\sigma_e^2}{Z_e}\right)\left(\frac{\sigma_\gamma^2}{Z_\gamma}\right)\left(\frac{Z_\gamma}{Z_e} + \frac{Z_e}{Z_\gamma}\right)}},$$

$$\boxed{\mathcal{L}_0 = \frac{I_e}{e}\frac{P_L\,\lambda}{h\,c^2}\frac{1}{\sqrt{(\sigma_e\epsilon_e)^2+(\sigma_\gamma\epsilon_\gamma)^2+(\sigma_e\epsilon_e)(\sigma_\gamma\epsilon_\gamma)\left(\frac{Z_\gamma}{Z_e}+\frac{Z_e}{Z_\gamma}\right)}}.} \tag{55}$$

The total luminosity depends only on 3 parameters : the emittance of the beams ($\mathcal{E}_e = \sigma_e\epsilon_e$, $\mathcal{E}_\gamma = \sigma_\gamma\epsilon_\gamma$) and the beam shape matching (Z_e/Z_γ).

3. **Interaction length**
   Another interesting feature is the interaction length defined as the length containing a fraction κ of the total luminosity $\mathcal{L}_0$ and given by

$$\int_{-L/2}^{L/2}\frac{d\mathcal{L}_0}{dz}dz = \kappa\ \mathcal{L}_0,$$

$$\boxed{L_0^{int}(\kappa) = 2\ tan\left(\frac{\kappa\pi}{2}\right)\sqrt{\frac{\sigma_e^2+\sigma_\gamma^2}{\epsilon_e^2+\epsilon_\gamma^2}}.} \tag{56}$$

   As already mentioned for the differential luminosity, the interaction length decreases when increasing the angular divergences and depends on 4 parameters.

4. **Beam optimal matching**
   The maximum of luminosity $\mathcal{L}_0^{max}$ is reached for $Z_e = Z_\gamma$, i.e. by matching the electron and Laser beam shapes.

$$\mathcal{L}_0^{max} = \frac{I_e}{e}\frac{P_L\,\lambda}{h\,c^2}\frac{1}{(\sigma_e\epsilon_e+\sigma_\gamma\epsilon_\gamma)} \qquad with \qquad Z_e = Z_\gamma. \tag{57}$$

   Thus, **the maximal luminosity for zero crossing angle is only limited by the emittance of the Laser and electron beams**. In this case the interaction length, which depends only on 2 parameters related either to the electron beam (σ_e , ϵ_e) or to the Laser beam (σ_γ , ϵ_γ), is fixed by the relations

$$L_0^{int}(\kappa) = 2\,tan\left(\frac{\kappa\pi}{2}\right)\frac{\sigma_e}{\epsilon_e} = 2\,tan\left(\frac{\kappa\pi}{2}\right)\frac{\sigma_\gamma}{\epsilon_\gamma} = 2\,tan\left(\frac{\kappa\pi}{2}\right)Z_e = 2\,tan\left(\frac{\kappa\pi}{2}\right)Z_\gamma. \tag{58}$$

4.2.2 Luminosity for non zero crossing angle

When the angular divergences of the electron and Laser beams are small wrt the crossing angle α_c, the differential, total luminosity and interaction length can be approximated by

- **Differential luminosity, Interaction length**

$$\frac{d\mathcal{L}}{dz} \simeq c\ (1.+cos(\alpha_c))\ \frac{N_{0e}N_{0\gamma}}{2\pi}\ \frac{1}{\sigma_e^2+\sigma_\gamma^2}\ exp\left(-\frac{2z^2sin^2(\alpha_c/2)}{\sigma_e^2+\sigma_\gamma^2}\right). \tag{59}$$

  The differential luminosity distribution is gaussian, so the interaction length for $\kappa = 0.68$ corresponding to $\pm 1\sigma$ is

$$\boxed{L_{\alpha_c}^{int} \simeq 2\frac{\sqrt{\sigma_e^2+\sigma_\gamma^2}}{sin(\alpha_c)}.} \tag{60}$$

  Thus the interaction length is reduced by smaller beam spot sizes or by bigger crossing angle.

- **Total luminosity**

$$\mathcal{L} \simeq c\ (1. + cos(\alpha_c))\ \frac{N_{0e}N_{0\gamma}}{\sqrt{2\pi}}\ \frac{1}{\sqrt{\sigma_e^2+\sigma_\gamma^2}}\ \frac{1}{sin(\alpha_c)},$$

$$\boxed{\mathcal{L} \simeq\ \frac{(1. + cos(\alpha_c))}{\sqrt{2\pi}}\ \frac{I_e}{e}\ \frac{P_L\ \lambda}{h\, c^2}\ \frac{1}{\sqrt{\sigma_e^2+\sigma_\gamma^2}}\ \frac{1}{sin(\alpha_c)}.} \qquad (61)$$

The total luminosity is reduced by bigger beam spot sizes or by bigger crossing angle. **Thus a high luminosity for non zero crossing angle depends on the ability to obtain small beam spot sizes and small crossing angle.**

4.3 Results

4.3.1 Zero crossing angle

The maximal luminosity in this case is given by equation (57):

$$\mathcal{L}_0^{max} = \frac{I_e}{e}\ \frac{P_L\ \lambda}{h\, c^2}\ \frac{1}{(\sigma_e\epsilon_e + \sigma_\gamma\epsilon_\gamma)} \qquad with \qquad Z_e = Z_\gamma.$$

The emittance of the electron beam of TESLA is negligible ($\simeq 10^{-11}\ m\ rad$) in comparison with the Laser emittance which is limited to $\lambda/4/\pi$ for a perfect Laser. Thus in this case the maximal emittance is independent of the Laser wavelength and is given by

$$\begin{aligned}\mathcal{L}_0^{max} = 4\,\pi\ \frac{I_e}{e}\ \frac{P_L}{h\, c^2} &= 131.7\ \ 10^{24} cm^{-2}s^{-1}\ \ (I_e(\mu A)\ P_L(W)) \\ &= 131.7\ \ barn^{-1}s^{-1}\ \ (I_e(\mu A)\ P_L(W))\,. \end{aligned} \qquad (62)$$

For an electron intensity of 100 μA and a Laser power of 1 W, we obtain a luminosity $\mathcal{L} = 1.3\ 10^{28}\ cm^{-2}s^{-1}$. The equivalent electron intensity for TESLA (1130 bunches of $3.610^{10}\ e^-$ with a repetition rate of 5 Hz) is 32.6 μA. For a continuous Laser of power 1. W with emittance $\lambda/4/\pi$ (or $\lambda/2/\pi$), the maximal luminosity for optimal beam matching is given in tables (4),(5) and (6). Using the table (3) for the others parameters (cross section and mean asymetry), the time needed for a 1% measurement of the electron polarization at 250 GeV is also given in these tables for the differential method without energy threshold. These times varies from 1.44 mn for a 1064 nm Laser of perfect emittance to 3.97 mn for a 248 nm Laser with 2 times the perfect emittance.

An other difficult point is the matching between the electron and Laser beams in order to reach the maximal luminosity. We recall here that the interaction length for the maximal luminosity depends only on the shape of one beam (electron or Laser) while the shape of the other beam must be adapted to fulfill the condition $Z_e = Z_\gamma$.
As seen in table (4), for an electron beam with a spot size of 50. μm and with an emittance of $10^{-11}\ m\ rad$, the interaction length for a perfect Laser and for the maximal luminosity is huge ($\simeq$ 300 m). By decreasing the Laser beam spot i.e. by increasing its angular divergence, we can reduce this interaction length to a more realistic geometrical length ($\simeq 1\ m$), but then the beam matching is not optimal, the luminosity decreases according to equation (55) and the needed time increases to 1.5

mn. To have an idea on the effect of the Laser emittance, we note that if the IR Laser had a worst emittance of $2\lambda/4/\pi$, then the time to get 1% error on P_e with an interaction length of $1\ m$ would increase to 3.0 mn.
To decrease the interaction length in the beam optimal matching conditions, we have to decrease the electron beam spot size. For a given emittance, the interaction length varies as $L_0^{int} \simeq \sigma_e^2$. If we would have a focalisation of the electron beam to have an electron spot size of 20. μm, the interaction length for the maximal luminosity becomes 49. m. For an interaction length of $\simeq 1\ m$, the needed time is close to the minimal one ($1.44\ mn$) for a Laser with a perfect emittance and becomes 2.9 mn for a Laser with an emittance 2 times greater.
In conclusion, the interaction length for the maximal luminosity can be big if the electron beam size is bigger than $\sigma_e \geq 20\mu m$ and one has to expect a loss of luminosity due to the limited experimental interaction length or due to the non optimal matching of the 2 beams.

4.3.2 Non zero crossing angle

It is not obvious that a zero crossing angle can be easily achieved. Some optical elements must be in the electron beam, so we have to study synchrotron radiation damage, protection against accidental electron beam damage. If a small crossing angle is necessary, the needed time will be increased. We will study here a solution with a crossing angle $\alpha_c = 10\ mrad$.

- **Dependence of the maximal expected luminosity as a function of the crossing angle.** An approximation for the luminosity in the case of non zero crossing angle of the 2 beams is given by equation (61) :

$$\mathcal{L} = \frac{(1. + cos(\alpha_c))}{\sqrt{2\pi}} \frac{I_e}{e} \frac{P_L\,\lambda}{h\,c^2} \frac{1}{\sqrt{\sigma_e^2+\sigma_\gamma^2}} \frac{1}{sin(\alpha_c)} .$$

If we can neglect the Laser spot size ($\sigma_\gamma << \sigma_e$), we obtain the maximal luminosity at non zero crossing angle

$$\mathcal{L}_\alpha^{max} \simeq \frac{2}{\sqrt{2\pi}} \frac{I_e}{e} \frac{P_L}{h\,c^2} \frac{\lambda}{\sigma_e} \frac{1}{sin(\alpha_c)} ,$$

$$\mathcal{L}_\alpha^{max} \simeq 8.36\ \ 10^{24} cm^{-2} s^{-1} \ \left(\frac{\lambda}{\sigma_e} \ \frac{I_e(\mu A)\ P_L(W)}{\alpha_c(rad)} \right) , \tag{63}$$

to be compared with the case at zero crossing angle for a perfect Laser (Eq. 62)

$$\mathcal{L}_0^{max} = 131.7 \quad 10^{24} cm^{-2} s^{-1} \quad (I_e(\mu A)\ P_L(W)) \ .$$

For a TESLA beam spot size of $\sigma_e = 50\ \mu m$, a 1 % measurement of the electron polarization at I_e = 32.6 μA requires 13 mn for a 1 W NdYAG Laser. This time holds for the differential method with a zero energy threshold and an electron polarization of 80 %. The interaction length (Eq. 60) for negligible Laser spot size,

$$L_{\alpha_c}^{int} \simeq 2\frac{\sqrt{\sigma_e^2+\sigma_\gamma^2}}{sin(\alpha_c)} \simeq 2\frac{\sigma_e}{sin(\alpha_c)} ,$$

is small ($1.\ cm$) in these conditions (electron spot size $\sigma_e = 50\ \mu m$ and $\alpha_c = 10\ mrad$).

$\lambda = 1064.2nm \quad I_e = 32.60\mu A \quad P_L = 1.00W \quad \alpha_c = 0.0\ mrad$

$\mathcal{E}_\gamma = \lambda/4/\pi$ = 0.8469E-07 m rad , σ_e = 20.0 μm

$\sigma_\gamma\ \mu m$	1063.	119.	169.	239.	339.	480.	678.	960.	1357.	1920.
$\mathcal{L}\ barn^{-1}s^{-1}$	4292.	4234.	4264.	4279.	4286.	4290.	4291.	4292.	4292.	4291.
$L^{int}\ cm$	4905.	63.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	1.44	1.46	1.45	1.45	1.45	1.44	1.44	1.44	1.44	1.44

$\mathcal{E}_\gamma = \lambda/4/\pi$ = 0.8469E-07 m rad , σ_e = 50.0 μm

$\sigma_\gamma\ \mu m$	2657.	115.	166.	237.	337.	478.	678.	959.	1357.	1919.
$\mathcal{L}\ barn^{-1}s^{-1}$	4292.	3937.	4111.	4201.	4247.	4270.	4282.	4288.	4290.	4292.
$L^{int}\ cm$	30658.	63.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	1.44	1.57	1.51	1.47	1.46	1.45	1.45	1.44	1.44	1.44

$\mathcal{E}_\gamma = 2\ \lambda/4/\pi$ = 0.1694E-06 m rad , σ_e = 20.0 μm

$\sigma_\gamma\ \mu m$	1503.	169.	239.	339.	480.	678.	959.	1357.	1919.	2714.
$\mathcal{L}\ barn^{-1}s^{-1}$	2146.	2132.	2139.	2143.	2145.	2146.	2146.	2146.	2146.	2146.
$L^{int}\ cm$	4905.	62.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	2.89	2.91	2.90	2.89	2.89	2.89	2.89	2.89	2.89	2.89

$\mathcal{E}_\gamma = 2\ \lambda/4/\pi$ = 0.1694E-06 m rad , σ_e = 50.0 μm

$\sigma_\gamma\ \mu m$	3757.	166.	237.	337.	478.	678.	959.	1357.	1919.	2714.
$\mathcal{L}\ barn^{-1}s^{-1}$	2146.	2056.	2101.	2124.	2135.	2141.	2144.	2145.	2146.	2146.
$L^{int}\ cm$	30658.	62.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	2.89	3.01	2.95	2.92	2.90	2.89	2.89	2.89	2.89	2.89

Table 4: ***Luminosity, interaction length and needed time for a 1 % measurement of the electron polarization with IR NdYAG continuous Laser*** (k = 1.16 eV) ***and for various Laser beam sizes.*** *Luminosities are given for an electron beam of intensity 32.6* μA *and emittance* 10^{-11} $m\ rad$ *and for a Laser beam of power 1. W and emittance* $\lambda/4/\pi$ *or* $2\lambda/4/\pi$. *Interaction length is for* κ*=0.683. Needed time are given using differential method with a zero energy threshold and assuming an electron polarization of 50 %. Each table corresponds to a given Laser emittance and electron beam spot size. The second column gives then results for an optimal beam matching while the next ones are for some values of the Laser beam spot size (corresponding to interaction length around 10 m).*

$\lambda = 532.1nm \quad I_e = 32.60\mu A \quad P_L = 1.00W \quad \alpha_c = 0.0\ mrad$

$\mathcal{E}_\gamma = \lambda/4/\pi$ = 0.4234E-07 m rad , σ_e = 20.0 μm

$\sigma_\gamma\ \mu m$	751.	84.	119.	169.	239.	339.	480.	678.	960.	1358.
$\mathcal{L}\ barn^{-1}s^{-1}$	4291.	4176.	4234.	4264.	4279.	4286.	4289.	4290.	4290.	4288.
$L^{int}\ cm$	4905.	62.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	1.64	1.69	1.66	1.65	1.65	1.64	1.64	1.64	1.64	1.64

$\mathcal{E}_\gamma = \lambda/4/\pi$ = 0.4234E-07 m rad , σ_e = 50.0 μm

$\sigma_\gamma\ \mu m$	1878.	78.	115.	166.	237.	337.	478.	678.	959.	1357.
$\mathcal{L}\ barn^{-1}s^{-1}$	4291.	3612.	3937.	4111.	4201.	4247.	4270.	4282.	4287.	4290.
$L^{int}\ cm$	30658.	63.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	1.64	1.95	1.79	1.71	1.68	1.66	1.65	1.64	1.64	1.64

$\mathcal{E}_\gamma = 2\ \lambda/4/\pi$ = 0.8469E-07 m rad , σ_e = 20.0 μm

$\sigma_\gamma\ \mu m$	1063.	119.	169.	239.	339.	480.	678.	960.	1357.	1920.
$\mathcal{L}\ barn^{-1}s^{-1}$	2146.	2117.	2132.	2139.	2143.	2145.	2146.	2146.	2146.	2145.
$L^{int}\ cm$	4905.	63.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	3.28	3.32	3.30	3.29	3.28	3.28	3.28	3.28	3.28	3.28

$\mathcal{E}_\gamma = 2\ \lambda/4/\pi$ = 0.8469E-07 m rad , σ_e = 50.0 μm

$\sigma_\gamma\ \mu m$	2657.	115.	166.	237.	337.	478.	678.	959.	1357.	1919.
$\mathcal{L}\ barn^{-1}s^{-1}$	2146.	1968.	2056.	2101.	2124.	2135.	2141.	2144.	2145.	2146.
$L^{int}\ cm$	30658.	63.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	3.28	3.58	3.42	3.35	3.31	3.30	3.29	3.28	3.28	3.28

Table 5: ***Luminosity, interaction length and needed time for a 1 % measurement of the electron polarization with Green Argon continuous Laser** ($k = 2.33\ eV$) **and for various Laser beam sizes.*** *Luminosities are given for an electron beam of intensity 32.6 μA and emittance 10^{-11} $m\ rad$ and for a Laser beam of power 1. W and emittance $\lambda/4/\pi$ or $2\lambda/4/\pi$. Interaction length is for κ=0.683. Needed time are given using differential method with a zero energy threshold and assuming an electron polarization of 50 %. Each table corresponds to a given laser emittance and electron beam spot size. The second column gives then results for an optimal beam matching while the next ones are for some values of the Laser beam spot size (corresponding to interaction length around 10 m).*

$\lambda = 248.0nm \quad I_e = 32.60\mu A \quad P_L = 1.00W \quad \alpha_c = 0.0\ mrad$

$\mathcal{E}_\gamma = \lambda/4/\pi$ = 0.1973E-07 m rad , $\sigma_e = 20.0\ \mu$m

$\sigma_\gamma\ \mu m$	513.	56.	81.	115.	163.	231.	327.	463.	655.	927.
$\mathcal{L}\ barn^{-1}s^{-1}$	4287.	4045.	4167.	4230.	4261.	4277.	4284.	4287.	4286.	4282.
$L^{int}\ cm$	4905.	62.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	1.99	2.11	2.04	2.01	2.00	1.99	1.99	1.99	1.99	1.99

$\mathcal{E}_\gamma = \lambda/4/\pi$ = 0.1973E-07 m rad , $\sigma_e = 50.0\ \mu$m

$\sigma_\gamma\ \mu m$	1282.	48.	75.	111.	160.	229.	326.	462.	654.	926.
$\mathcal{L}\ barn^{-1}s^{-1}$	4287.	2981.	3567.	3912.	4098.	4195.	4244.	4268.	4280.	4286.
$L^{int}\ cm$	30658.	62.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	1.99	2.86	2.39	2.18	2.08	2.03	2.01	2.00	1.99	1.99

$\mathcal{E}_\gamma = 2\ \lambda/4/\pi$ = 0.3947E-07 m rad , $\sigma_e = 20.0\ \mu$m

$\sigma_\gamma\ \mu m$	725.	81.	115.	163.	231.	327.	463.	655.	927.	1311.
$\mathcal{L}\ barn^{-1}s^{-1}$	2145.	2084.	2115.	2131.	2139.	2143.	2144.	2145.	2145.	2144.
$L^{int}\ cm$	4905.	63.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	3.97	4.09	4.03	4.00	3.98	3.98	3.97	3.97	3.97	3.98

$\mathcal{E}_\gamma = 2\ \lambda/4/\pi$ = 0.3947E-07 m rad , $\sigma_e = 50.0\ \mu$m

$\sigma_\gamma\ \mu m$	1814.	75.	111.	160.	229.	326.	462.	654.	926.	1310.
$\mathcal{L}\ barn^{-1}s^{-1}$	2145.	1784.	1956.	2049.	2097.	2122.	2134.	2140.	2143.	2145.
$L^{int}\ cm$	30658.	62.	125.	250.	500.	1000.	2000.	4000.	8000.	16000.
$Time\ mn$	3.97	4.78	4.36	4.16	4.06	4.02	3.99	3.98	3.98	3.97

Table 6: ***Luminosity, interaction length and needed time for a 1 % measurement of the electron polarization with UV KrF high power pulsed Laser** ($k = 5.0\ eV$) **and for various Laser beam sizes.** Luminosities are given for an electron beam of intensity 32.6 μA and emittance $10^{-11}\ m\ rad$ and for a Laser beam of power 1 W and emittance $\lambda/4/\pi$ or $2\lambda/4/\pi$. Interaction length is for κ=0.683. Needed time are given using differential method with a zero energy threshold and assuming an electron polarization of 50 %. Each table corresponds to a given Laser emittance and electron beam spot size. The second column gives then results for an optimal beam matching while the next ones are for some values of the Laser beam spot size (corresponding to interaction length around 10 m).*

- **Luminosity and Needed time.**
  Table (7) gives the luminosity and the time needed for a 1 % measurement of the electron polarization for various values of the Laser spot size. These numbers confirm the first estimation where the Laser beam spot size had been neglected.

4.3.3 Expected Counting Rates

The electron beam of intensity $I_e = 32.6\mu A$ (corresponding to 1130 bunches of 3.6 10^{10} electrons with a repetition rate of 5 Hz) and polarization P_e = 80 % crosses the Laser beam with a crossing angle α_c = 10 $mrad$ and a spot size σ_e = 50 μm. The Laser beam of power P_L = 1 W and polarization P_γ = 100 % has a spot size is σ_γ = 50 μm.

The differential counting rate, depending on the luminosity $\mathcal{L}$ and on the differential unpolarized cross section $d\sigma/d\rho$ (Eq. 14 and Fig. 3) is given by

$$\frac{dN}{d\rho\ dt} = \mathcal{L}\ \frac{d\sigma}{d\rho}\ , \tag{64}$$

where $\rho = k'/k'_{max}$ is the scattered γ energy normalized to the maximal energy.

For a zero energy threshold, we obtain the **total counting rate**

$$\frac{dN}{dt} = \mathcal{L}\ \sigma\ , \tag{65}$$

where σ is the total unpolarized cross section (Eq. 15).

We recall the approximated formula for **the luminosity** $\mathcal{L}$ to see the variation with the experimental parameters. In this equation, the angular divergences of the Laser and electron beams have been neglected. In the results presented here, the luminosity $\mathcal{L}$ has been exactly integrated

$$\mathcal{L} \simeq \frac{2}{\sqrt{2\pi}}\ \frac{I_e}{e}\ \frac{P_L\, G}{h\, c^2}\ \frac{\lambda}{\sqrt{\sigma_e^2 + \sigma_\gamma^2}}\ \frac{1}{sin(\alpha_c)}\ .$$

The needed time for a 1 % measurement of the electron polarization has the general form

$$t_{meth} = \frac{1}{\mathcal{L}\ \left(\frac{\Delta P_e}{P_e}\right)^2\ P_e^2 P_\gamma^2}\ \frac{1}{A^2_{meth}}\ \frac{1}{\sigma}\ , \tag{66}$$

where, according to the method (differential,integrated,energy weighted), the mean longitudinal asymmetry A^2_{meth} stands for

$$< A_l^2 > \ :\ < A_l >^2\ :\ \frac{< E A_l >^2}{< E^2 >}\ .$$

The number of events needed for a 1 % measurement of the electron polarization is obtained from

$\lambda = 1064.2nm \quad I_e = 32.60\mu A \quad P_L = 1.00W \quad \alpha_c = 10.0mrad$									
$\sigma_e = 20.0\ \mu$m									
$\sigma_\gamma\ \mu m$	30.8	38.7	46.0	51.0	54.8	57.9	60.7	63.1	65.2
$\mathcal{L}\ barn^{-1}s^{-1}$	790.	666.	578.	530.	497.	473.	454.	438.	425.
$L^{int}\ cm$	0.73	0.87	1.00	1.10	1.17	1.23	1.28	1.32	1.36
$Time\ mn$	8.	9.	11.	12.	12.	13.	14.	14.	15.
$\sigma_e = 50.0\ \mu$m									
$\sigma_\gamma\ \mu m$	30.8	38.7	46.0	51.0	54.8	57.9	60.7	63.1	65.2
$\mathcal{L}\ barn^{-1}s^{-1}$	494.	459.	427.	406.	391.	379.	369.	360.	353.
$L^{int}\ cm$	1.17	1.26	1.36	1.43	1.48	1.53	1.57	1.61	1.64
$Time\ mn$	13.	14.	15.	15.	16.	16.	17.	17.	18.

$\lambda = 532.1nm \quad I_e = 32.60\mu A \quad P_L = 1.00W \quad \alpha_c = 10.0mrad$									
$\sigma_e = 20.0\ \mu$m									
$\sigma_\gamma\ \mu m$	21.8	27.4	32.6	36.0	38.7	41.0	42.9	44.6	46.1
$\mathcal{L}\ barn^{-1}s^{-1}$	491.	428.	380.	352.	333.	318.	306.	297.	289.
$L^{int}\ cm$	0.59	0.68	0.76	0.82	0.87	0.91	0.95	0.98	1.01
$Time\ mn$	14.	16.	19.	20.	21.	22.	23.	24.	24.
$\sigma_e = 50.0\ \mu$m									
$\sigma_\gamma\ \mu m$	21.8	27.4	32.6	36.0	38.7	41.0	42.9	44.6	46.1
$\mathcal{L}\ barn^{-1}s^{-1}$	266.	255.	243.	235.	229.	224.	220.	217.	213.
$L^{int}\ cm$	1.09	1.14	1.19	1.23	1.27	1.29	1.32	1.34	1.36
$Time\ mn$	26.	28.	29.	30.	31.	31.	32.	33.	33.

Table 7: ***Luminosity, interaction length and needed time for a 1 % measurement of the electron polarization with standard continuous Laser $k = 1.16\ eV$ (IR NdYAG) and $k = 2.33\ eV$ (Green Argon).*** *Luminosities are given for an electron beam of intensity 32.6 μA, for a Laser beam of power 1 W . Interaction length is for κ=0.683. Needed time are given using differential method with a zero energy threshold and assuming an electron polarization of 80 %. Each table corresponds to a given electron beam spot size. The columns give results for some values of the Laser beam spot size.*

$\lambda = 248.0nm$ $I_e = 32.60\mu A$ $P_L = 1.00W$ $\alpha_c = 10.0mrad$									
$\sigma_e = 20.0\ \mu$m									
$\sigma_\gamma\ \mu m$	14.9	18.7	22.2	24.6	26.4	28.0	29.3	30.4	31.5
$\mathcal{L}\ barn^{-1}s^{-1}$	271.	247.	226.	213.	204.	197.	191.	186.	181.
$L^{int}\ cm$	0.50	0.55	0.60	0.63	0.66	0.69	0.71	0.73	0.75
$Time\ mn$	31.	35.	38.	40.	42.	43.	45.	46.	47.
$\sigma_e = 50.0\ \mu$m									
$\sigma_\gamma\ \mu m$	14.9	18.7	22.2	24.6	26.4	28.0	29.3	30.4	31.5
$\mathcal{L}\ barn^{-1}s^{-1}$	130.	127.	124.	121.	120.	118.	117.	115.	114.
$L^{int}\ cm$	1.04	1.07	1.09	1.11	1.13	1.15	1.16	1.17	1.18
$Time\ mn$	66.	67.	69.	70.	71.	72.	73.	74.	74.

Table 8: ***Luminosity, interaction length and needed time for a 1 % measurement of the electron polarization with continuous Laser*** $k = 5.0\ eV$ ***(UV KrF)****. Luminosities are given for an electron beam of intensity 32.6 μA, for a Laser beam of power 1 W. Interaction length is for κ=0.683. Needed time are given using differential method with a zero energy threshold and assuming an electron polarization of 80 %. Each table corresponds to a given electron beam spot size. The columns give results for some values of the Laser beam spot size.*

$$N = \mathcal{L}\ t_{meth}\ \sigma\ . \tag{67}$$

We present in table (9) the expected counting rate and needed time for a 1 % measurement of the electron polarization as a function of the Laser and electron beam energies. These numbers hold **for the differential method with a zero energy threshold**. Counting rates are typically 80 Hz for a IR Laser. Needed time varies from 15 mn at 1064 nm to 70 mn at 248 nm.

With a non zero energy threshold, the counting rates are reduced in the ratio σ_t/σ where σ_t is the differential unpolarized cross section integrated from the energy threshold to the maximum energy (Fig. 4,5 and 6). Table (10) gives values of this ratio for various energy threshold.
The number of events needed increase with the energy threshold like the square of the mean longitudinal asymmetry (Fig. 4,5 and 6). Table (11) gives the variation of the mean longitudinal asymmetry for various energy threshold.
The variation of the needed time with energy threshold depends on the 2 previous variations (cross section and mean longitudinal asymmetry).

$\lambda = 1064.2nm$ $P_L = 1.0W$ $\sigma_\gamma = 50.1\mu m$ $P_\gamma = 100.0\%$	
$I_e = 32.60\mu A$ $\sigma_e = 50.0\mu m$ $P_e = 80.0\%$ $\alpha_c = 10.0mrad$	
E GeV	250.0
$\mathcal{L}$ $barn^{-1}s^{-1}$	416.193
σ $barn$	0.198
$rate$ Hz	82.4
$\sqrt{< A_l^2 >}$ %	39.131
$Time$ s	893.1
$Events$ 10^3	73.6

$\lambda = 532.1nm$ $P_L = 1.0W$ $\sigma_\gamma = 35.4\mu m$ $P_\gamma = 100.0\%$	
$I_e = 32.60\mu A$ $\sigma_e = 50.0\mu m$ $P_e = 80.0\%$ $\alpha_c = 10.0mrad$	
E GeV	250.0
$\mathcal{L}$ $barn^{-1}s^{-1}$	238.479
σ $barn$	0.136
$rate$ Hz	32.4
$\sqrt{< A_l^2 >}$ %	44.400
$Time$ s	1771.1
$Events$ 10^3	57.3

$\lambda = 248.0nm$ $P_L = 1.0W$ $\sigma_\gamma = 24.2\mu m$ $P_\gamma = 100.0\%$	
$I_e = 32.60\mu A$ $\sigma_e = 50.0\mu m$ $P_e = 80.0\%$ $\alpha_c = 10.0mrad$	
E GeV	250.0
$\mathcal{L}$ $barn^{-1}s^{-1}$	122.121
σ $barn$	0.084
$rate$ Hz	10.3
$\sqrt{< A_l^2 >}$ %	50.570
$Time$ s	4187.0
$Events$ 10^3	43.0

Table 9: ***Luminosity, total cross section, counting rate, mean asymmetry, needed time and number of events for a 1 % measurement of the electron polarization with standard continuous Laser*** $k = 1.16$ eV ***(IR)***, $k = 2.33$ eV ***(Green) and*** $k = 5.0$ eV ***(UV)****. Luminosities and counting rate are given for an electron beam of intensity 32.6* μA *and of beam spot size* 50 μm *and for a Laser beam of power 1.0 W and of beam spot size* 50 μm*. Needed time and number of events are given for the differential method with a zero energy threshold and for electron polarization of* 80% *and a photon polarizations of* 100%

$\lambda = 1064.2nm \quad \alpha_c = 10.0mrad$	
E GeV	250.0
σ $barn$	0.198
σ_t/σ ρ_{min} =0.00	1.000
σ_t/σ ρ_{min} =0.10	0.909
σ_t/σ ρ_{min} =0.20	0.822
σ_t/σ ρ_{min} =0.30	0.737
σ_t/σ ρ_{min} =0.40	0.654
σ_t/σ ρ_{min} =0.50	0.573
σ_t/σ ρ_{min} =0.60	0.491
σ_t/σ ρ_{min} =0.70	0.406
σ_t/σ ρ_{min} =0.80	0.311
σ_t/σ ρ_{min} =0.90	0.190

$\lambda = 532.1nm \quad \alpha_c = 10.0mrad$	
E GeV	250.0
σ $barn$	0.136
σ_t/σ ρ_{min} =0.00	1.000
σ_t/σ ρ_{min} =0.10	0.927
σ_t/σ ρ_{min} =0.20	0.854
σ_t/σ ρ_{min} =0.30	0.782
σ_t/σ ρ_{min} =0.40	0.710
σ_t/σ ρ_{min} =0.50	0.636
σ_t/σ ρ_{min} =0.60	0.558
σ_t/σ ρ_{min} =0.70	0.473
σ_t/σ ρ_{min} =0.80	0.373
σ_t/σ ρ_{min} =0.90	0.239

$\lambda = 248.0nm \quad \alpha_c = 10.0mrad$	
E GeV	250.0
σ $barn$	0.084
σ_t/σ ρ_{min} =0.00	1.000
σ_t/σ ρ_{min} =0.10	0.941
σ_t/σ ρ_{min} =0.20	0.883
σ_t/σ ρ_{min} =0.30	0.823
σ_t/σ ρ_{min} =0.40	0.763
σ_t/σ ρ_{min} =0.50	0.699
σ_t/σ ρ_{min} =0.60	0.629
σ_t/σ ρ_{min} =0.70	0.549
σ_t/σ ρ_{min} =0.80	0.451
σ_t/σ ρ_{min} =0.90	0.309

Table 10: ***Variation of the integrated cross section with the energy threshold*** $\rho_{min} = k'_{min}/k'_{max}$. *$\sigma$ is the total unpolarized cross section while σ_t is the differential unpolarized cross section integrated from the energy threshold to the maximum energy.*

$\lambda = 1064.2nm \quad \alpha_c = 10.0mrad$	
E GeV	250.0
$< A_l >= \sqrt{< A_l^2 >}$ %	39.131
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.00	1.000
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.10	1.048
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.20	1.097
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.30	1.142
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.40	1.177
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.50	1.198
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.60	1.206
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.70	1.223
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.80	1.329
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.90	1.675

$\lambda = 532.1nm \quad \alpha_c = 10.0mrad$	
E GeV	250.0
$< A_l >= \sqrt{< A_l^2 >}$ %	44.400
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.00	1.000
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.10	1.038
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.20	1.077
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.30	1.112
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.40	1.139
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.50	1.150
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.60	1.138
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.70	1.099
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.80	1.066
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.90	1.230

$\lambda = 248.0nm \quad \alpha_c = 10.0mrad$	
E GeV	250.0
$< A_l >= \sqrt{< A_l^2 >}$ %	50.570
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.00	1.000
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.10	1.030
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.20	1.061
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.30	1.089
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.40	1.112
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.50	1.122
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.60	1.113
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.70	1.072
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.80	0.986
$< A_l >_t / < A_l > \quad \rho_{min}$ =0.90	0.907

Table 11: ***Variation of the mean asymmetry with the energy threshold*** $\rho_{min} = k'_{min} / k'_{max}$. $< A_l >$ *is the mean asymmetry for a zero energy threshold (* $< A_l^2 >^{0.5}$ *for the differential method) while* $< A_l >_t$ *is the mean asymmetry from the energy threshold to the maximum energy.*

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