

# Wake-fields Induced by the Electron Beam Passing through the TESLA Accelerating System

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## Abstract

The construction of the future high energy electron colliders have very important considerations given to the R&D of superconducting electron linacs. For that purpose, in the frame of wide international collaboration the TESLA Test Facility is being constructed at DESY, Hamburg.

A particular problem is the generation of parasitic RF fields (wakefields) caused by beam bunches passing through RF accelerating cavities and all cavity-like objects in the beam line. These induced fields give rise to various beam instabilities which degrade the phase volume of the beam at the interaction point of the collider. This report will verify of the magnitude of these wake fields.

## I GENERAL CONSIDERATIONS

The feasibility study of TESLA 500 GeV (CM) electron-positron linear collider was undertaken in 1990 [1]. The solution of an accelerating system is based up on superconducting RF structures operating in the L band at the 1.3 GHz fundamental frequency. The basic parameters of single accelerating cavity and parameters of electron beam to be accelerated are summarised in the Table 1.

Table 1. RF cavity parameters/electron beam parameters

<b>Cavity:</b>		
Frequency		1300 MHz
Number of cells in one cavity		9
Accelerating field		30 MV/m
Cavity length (including end beam tubes)		1.276 m
Iris diameter between cells		0.07 m
HOM loss factor	-longitudinal	8.5 V/pC(for $s = 1$ mm)
	-transverse	18 V/pC/m
<b>Beam:</b>		
Bunch length		$s = 1$ mm, 0.5 mm
Bunch charge		8 nC
Energy spread		0.2%
Macropulse length		800 msec
Number of bunches in macropulse		800
Macropulse repetition rate		10 Hz

The experimental set-up at TTF (TESLA Test Facility)[2] consisted of 32 9-optimized RF cell niobium cavities [3] designed and constructed at DESY, Hamburg. It also included all necessary RF supply and cryogenic systems, RF-type electron gun and control systems. TTF will enable the verification and modification of both design parameters and cost estimates before the decision on construction of collider is taken.

The main characteristics of TTF are given in the Table 2.

Table 2

Number of cryomodules	4
Number of RF cavities in 1 cryomodule	8
Number of interconnecting flexible bellows	9 per cryomodule
Bath temperature	2 K

Throughout all the steps of TESLA project progress, the generation of wakefields and related higher order modes (HOM) damping and/or absorption is given particular attention [3,4]. The resonant accelerating cavities are designed so as to obtain the lowest (R/Q) of HOM even at the cost of lowering (R/Q) of the fundamental accelerating mode. All interconnecting flexible beam tubes (bellows) are designed to be as smooth as possible.

The shape of the cavity and of joining bellows which were used here for wakefield calculations are given in Fig.1 and Fig. 12.

## II DEFINITIONS

A charged particle passing any cavity-like accelerator component generates electromagnetic wakefields. These wakefields interact, in turn, with following particles which leads to variation of their longitudinal momentum. If exciting particle pass off the axis, also the transverse deflections are excited.

The longitudinal delta-wake potential  $W_L^d$  is defined in the terms of energy loss of witness charge Q which follows the exciting point charge q with the same speed at the distance s

$$\Delta U = QqW_L^d(s).1 \quad [1]$$

It can also be obtained by integration of the work of excited field over the witness charge along the whole structure[5]

$$W_L^d = -\frac{1}{L \cdot Q_0} \int dz E_z(z, t=(z+s)/bc).2 \quad [2]$$

The dimensions of  $W_L^d$  is [V/Coul].

A transverse-kick obtained by the witness charge Q is given by

$$\vec{p}_\perp(s) = \frac{Q \cdot q}{bc} \vec{W}_\perp^d(s, \vec{r}_q, \vec{r}_Q), \quad [3]$$

where  $\vec{W}_\perp^d$  is the transverse delta-wake potential and  $\vec{r}$  is the radial off-set of q and Q.

The  $\vec{W}_\perp^d$  which is a transverse plane vector is defined by

$$\vec{W}_\perp^d(s, \vec{r}_q, \vec{r}_Q) = \frac{1}{L \cdot Q} \int_0^L dz [\vec{E}_T + (\vec{v} \times \vec{B})_\perp](t = (z-s)/bc), \quad [4]$$

where  $\vec{E}_T$  and  $\vec{B}$  are the fields excited by charge q at a time  $t = (z-s)/bc$ . For relativistic case  $\beta \cong 1$ .

Delta-wake potentials are caused by the geometry of the accelerator. Knowing the charge distribution allows a convenient calculation of the longitudinal wake (given by Eq. 5) at the position s behind the bunch head

$$W_L(s) = \frac{1}{Q} \int_0^\infty I(s-s') W_L^d(s') ds', \quad [5]$$

where  $I$  is a bunch charge distribution.

The longitudinal loss factor, or energy-loss per particle of single bunch, is then

$$k_l = \frac{1}{Q} \int_{-\infty}^{\infty} ds I(s) W_L(s). \quad [6]$$

For a Gaussian bunch with an rms length s

$$k_l(\mathbf{s}) = \frac{1}{\sqrt{2\pi} s} \int_0^\infty ds \exp(-s^2/2s^2) W_L(s, \mathbf{s}). \quad [7]$$

Similarly, the transverse loss factor for a transverse (dipole) mode is defined as  $bc/Q^2$  multiplied by the total transverse momentum given to the bunch by its own wake

$$k_{\perp} = \frac{c}{Q^2} \int ds \bar{W}_{\perp}(s) I(s) ,12 \quad [8]$$

where 
$$\bar{W}_{\perp}(s) = \frac{I}{Q} \int_0^{\infty} ds' I(s-s') W_{\perp}^d(s') .13 \quad [9]$$

Here again  $k_{\perp}$  and  $\bar{W}_{\perp}(s)$  are functions of bunch charge distribution and structure geometry.

### III METHODS OF CALCULATION

The wake potentials, related energy loss factor, and transverse loss-factor are the figures-of-merit in the design of a high-energy high-intensity accelerator. Efforts are made to minimize these factors for modes whose frequencies are higher than the fundamental in accelerating cavities and to minimize for all modes elsewhere along the beam line.

The wake potentials and loss factors are calculated in terms of structure normal-modes in the frequency domain or by direct numerical solution of integral Maxwell equations in the time domain.

The frequency domain method is based on an identification of all resonant modes of the structure with their  $\omega$  and (R/Q) values. The point charge loss factor  $k_n$  of mode n is [6]

$$k_n = \frac{w_n}{2} \left( \frac{R}{Q} \right)_n .15 \quad [10]$$

The delta-wake potentials are then

$$W_{\perp}^d = \sum_{n=1}^{\infty} k_{on} \cos(\omega_n t) \quad [11]$$

$$W_{\perp}^d = \sum_{n=1}^{\infty} \frac{k_{1n} \cdot C}{\omega_{1n} a^2} \sin(\omega_{1n} t) \quad [12]$$

with  $t = (z-s)/\beta c$ .

Since the mode fields are orthogonal, the mode loss factors may be summed up giving the total loss factor. Since the computations are limited to resonant modes, a part is missing for losses above the cut-off frequency of the beam pipe. Estimating these losses is made analytically using the "optical resonator model" [7] leading to  $\sim s^{-3/2}$  asymptotic dependence of loss factor, or using the Lawson diffraction model [8], this dependence scales as  $\sim s^{-1/2}$ . The  $\sim s^{-1/2}$  behaviour is prevalent for single cell cavities, whereas for infinite periodic structures the loss factor scales as  $\sim s^{-3/2}$  [9].

Using the time domain methods, the Maxwell's equations are first written in the integral form and are replaced by a finite difference set of equations using the FIT algorithm [10,11,12]. Time

domain codes TBCI[13], ABCI[14] and DBCI[15] calculate directly the bunch wake-potentials and loss factors for any structure.

## IV RESULTS

### IV.1 Loss factors vs number of cells.

The calculations of longitudinal and transverse wakes and loss-parameters were performed using ABCI code (version No 8.2 updated in 1994).

The wakes and loss factors were computed for  $s = 1$  mm and 0.5 mm for a Gaussian relativistic bunch passing through a RF TESLA cavity.

The geometry of the cavity shown in Fig. 1 was taken from actual design drawing No 9-93-2214/0.000. The dependence of loss-factors on the number of cells in the cavity is summarized in Table 3.

Table 3. The loss-factors as a function of number of cells and cell separation for the TESLA 1.3 GHz structure. The values were computed for  $\sigma = 1$  mm of Gaussian bunch.

Number of cells	cell separation [cm]	monopole loss factor [V/pC]	transverse loss factor [V/pC/m]
1	-	1.558	3.511
2	0.0	2.870	6.344
2	4.0	2.917	6.488
2	6.0	2.935	6.539
2	8.0	2.951	6.581
2	12.0	2.977	6.650
2	22.0	3.025	6.776
2	40.0	3.081	6.873
2	52.0	3.103	6.911
2	80.0	3.124	6.965
3	0.0	4.073	8.880
4	0.0	5.222	11.25
5	0.0	6.347	13.510
7	0.0	8.608	17.87

9*	0.0	10.620	21.99
9**	0.0	11.05	20.6

\*)End cells non compensated; \*\*) end cells compensated

Figs. 2 and 3 depict the change of loss factors/cell taken from this Table. Neither the summation of loss factors is allowed in an array of strongly coupled cells nor scaling  $k_{array} \approx k_{cell} \sqrt{n}$  found in the literature [16].

The parameter of interest is a distribution of deposited energy as a function of frequency. Since the ABCI code is equipped with FFT functions the longitudinal coupling impedance can be obtained, enabling thus the evaluation of above distribution. This requires the tracking of wake fields for a relatively long time ( $\sim$  distance  $s$  behind the bunch head). The frequency spectrum of longitudinal loss factor and integral of loss factor vs frequency were computed for single cell and 9-cell cavity. The results, which are illustrated in Figs. 4-7 show that the total longitudinal loss-factor reduction vs number of cells is mainly due to HOM loss factor reduction. The  $k_L$  per cell of fundamental mode is reduced by  $\sim 2.3\%$ , while the total  $k_L$ /cell reduction is more than 24% when compared from one cell to 9-cell cavity.

The end cell compensation is also clearly seen in Figs.2 and 3 (both loss factors are substantially changed).

#### IV.2 Wake potential for $s = 1$ mm and $s = 0.5$ mm Gaussian bunch in 9-cell cavity

The computed longitudinal monopole wakes are illustrated in Fig. 8 for  $s = 1$  mm and in Fig. 9 for  $s = 0.5$  mm. Transverse dipole wakes are given in Figs. 10 and 11. The loss factors are given in Table 4 as a function of bunch length.

Table 4. Loss parameters of 9-cell cavity

s bunch (mm)	$k_{L \text{ monop.}}(V/pC)$	$k_{\perp \text{ dipole}}(V/pC/m)$
2	7.63	25.31
1	11.05	20.60
0.5	15.58	12.96

The longitudinal monopole loss factor of HOM in 9-cell cavity is 8.97 V/pC for  $s = 1$  mm and 13.52 V/pC for  $s = 0.5$  mm.

### IV. 3 Wake potentials in TESLA flexible bellows

The outline of bellow foreseen to connect the beam tubes of RF cavities in one cryomodule is shown in Fig. 12. The geometrical dimensions taken to computations correspond to Design Draw No 393 4419/B.

The monopole wakes excited by  $s = 1$  mm and  $s = 0.5$  mm bunches are illustrated in Figs. 13 - 14 and transverse dipole wakes in Figs.15 - 16.

The loss factors of the bellow are summarized in Table 5.

Table 5. Loss parameters of TESLA bellow

s bunch (mm)	$k_{L \text{ monop.}}(\text{V/pC})$	$k_{\perp \text{ dipole}}(\text{V/pC/m})$
2	0.81	3.04
1	1.29	2.36
0.5	1.96	1.77

### V CONCLUSIONS

The obtained results are to be treated as a verification of the design values for loss parameters. They were obtained using ABCI code version 8.2.

The runs were made on SUN-Sparc 10 (TESLAHB) and on CONVEX C-3210 machines to compare the execution time and accuracy. The CPU times in all cases were similar to within 2%, but the real execution time on CONVEX is up to 10 times longer with large values of UBT parameter (the distance behind the bunch front to which the wake in cavity is computed). For the case of  $UBT = 1$  m the typical CPU time for  $s = 0.5$  mm is around 70 h.

Efforts are being made to trace the wake-fields over prolonged time following the passage of the bunch through studied structures. At the values of UBT exceeding 1.5 m the computational instabilities start to occur (erratic behaviour of  $W_L(s)$  on Fig. 17) which have to be identified and solved.

The frequency distributions of longitudinal loss factors vs frequency will be given under the separate cover (in preparation).

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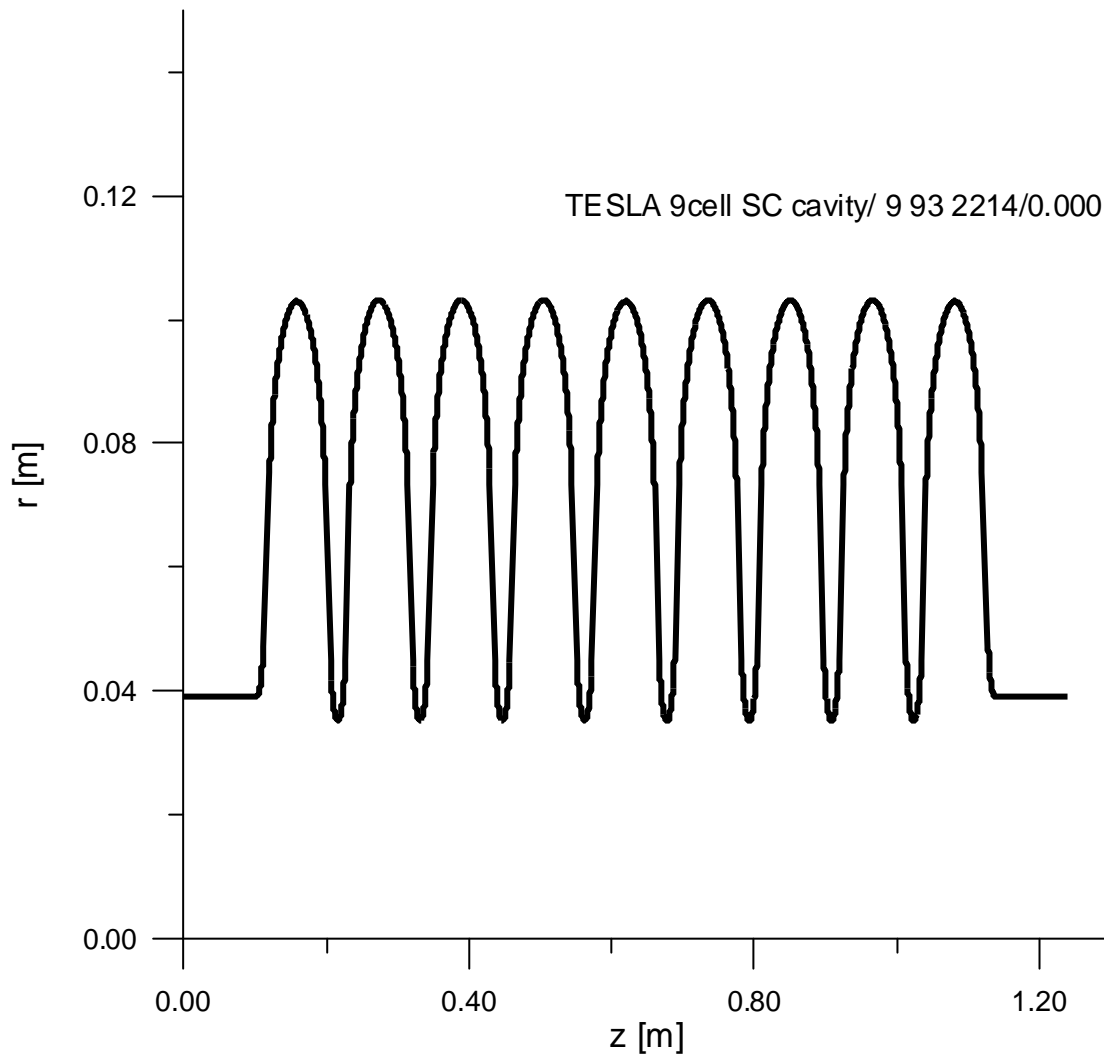


Fig.1 The outline of 9-cell TESLA RF cavity geometry. The dimensions for the computations were taken from Design Draw. 9-93-2214/0.0000

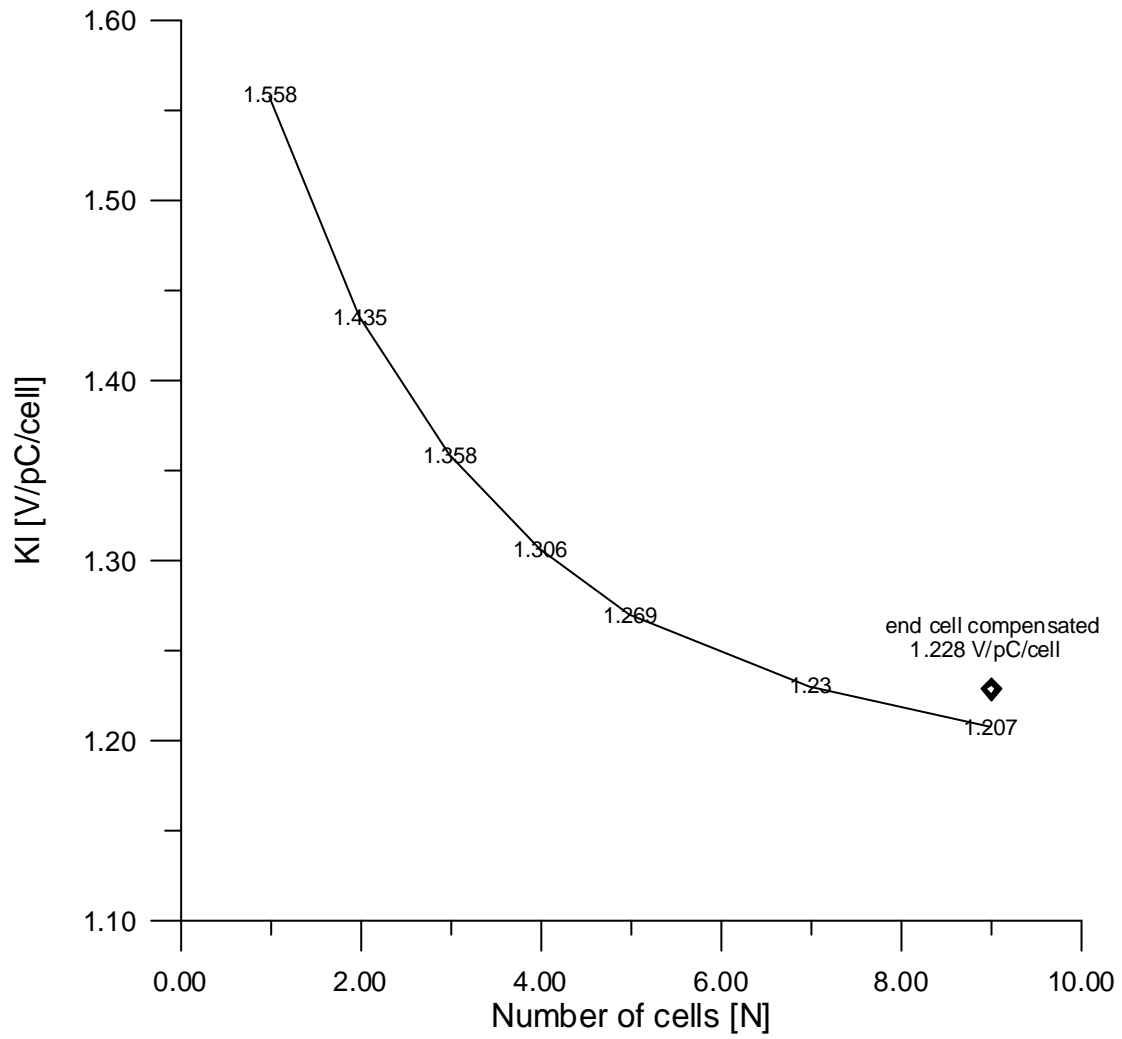


Fig.2 Longitudinal loss factor dependence on the number of cells.  
Gaussian Sig= 1mm bunch in TESLA RF cavity geometry.

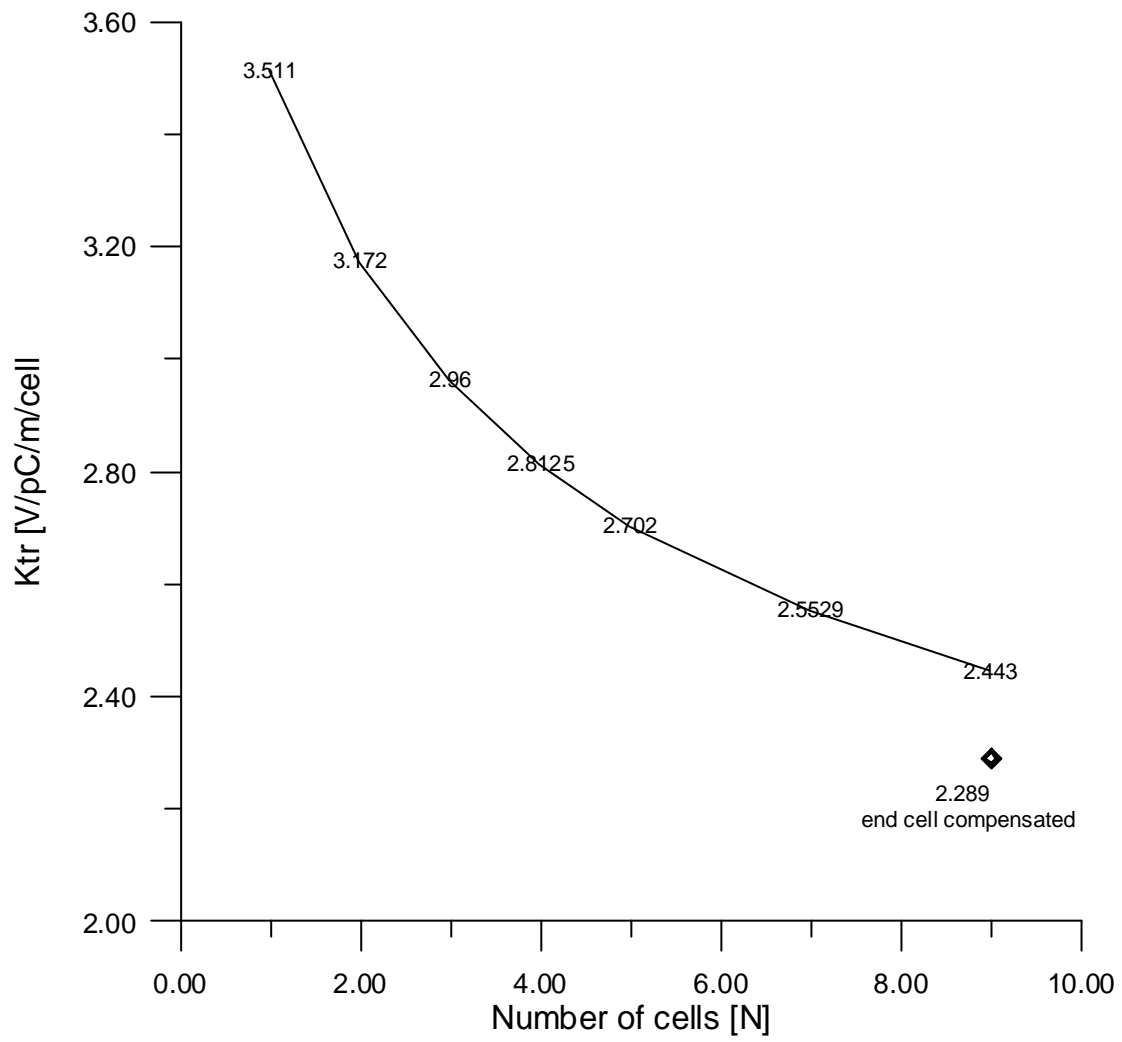


Fig.3 Transverse (dipole) loss factor as a function of the number of cells. Gaussian Sig= 1 mm bunch in TESLA RF geometry.

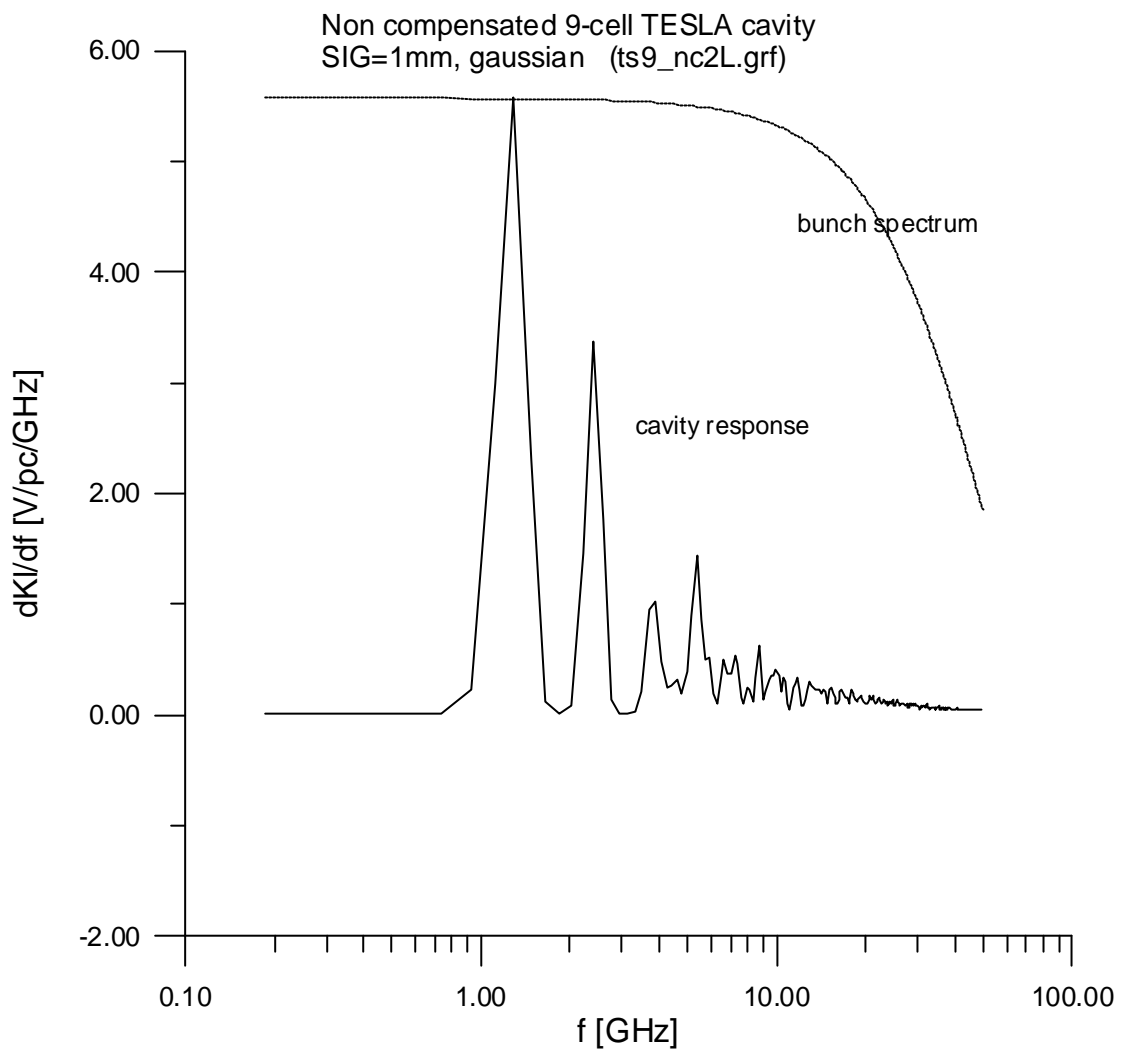


Fig.4 Frequency spectrum of longitudinal loss factor. Sig=1 Gaussian bunch in TESLA 9-cell non-compensated RF cavity geometry.

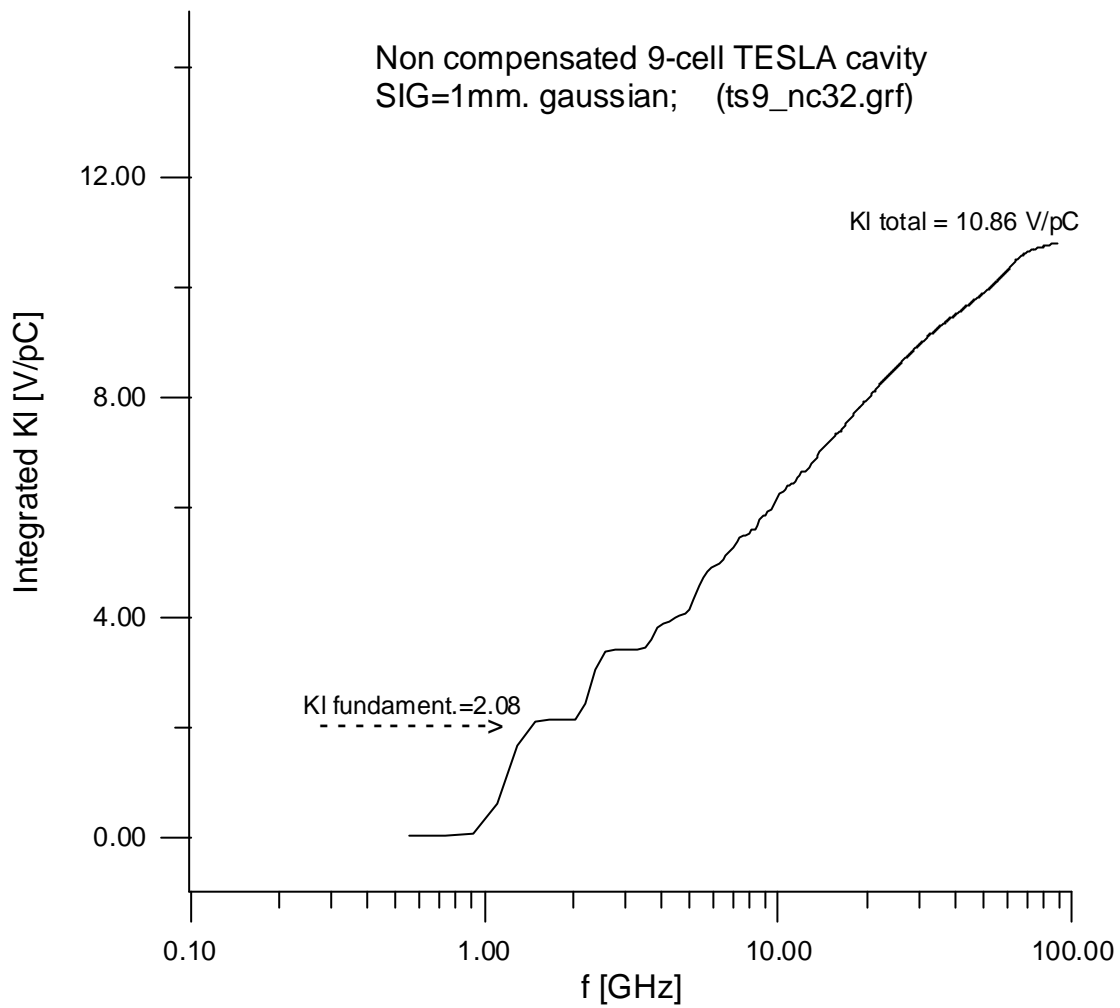


Fig.5. Integrated longitudinal loss factor as a function of frequency.  
Sig=1 mm bunch in 9-cell non-compensated RF cavity.

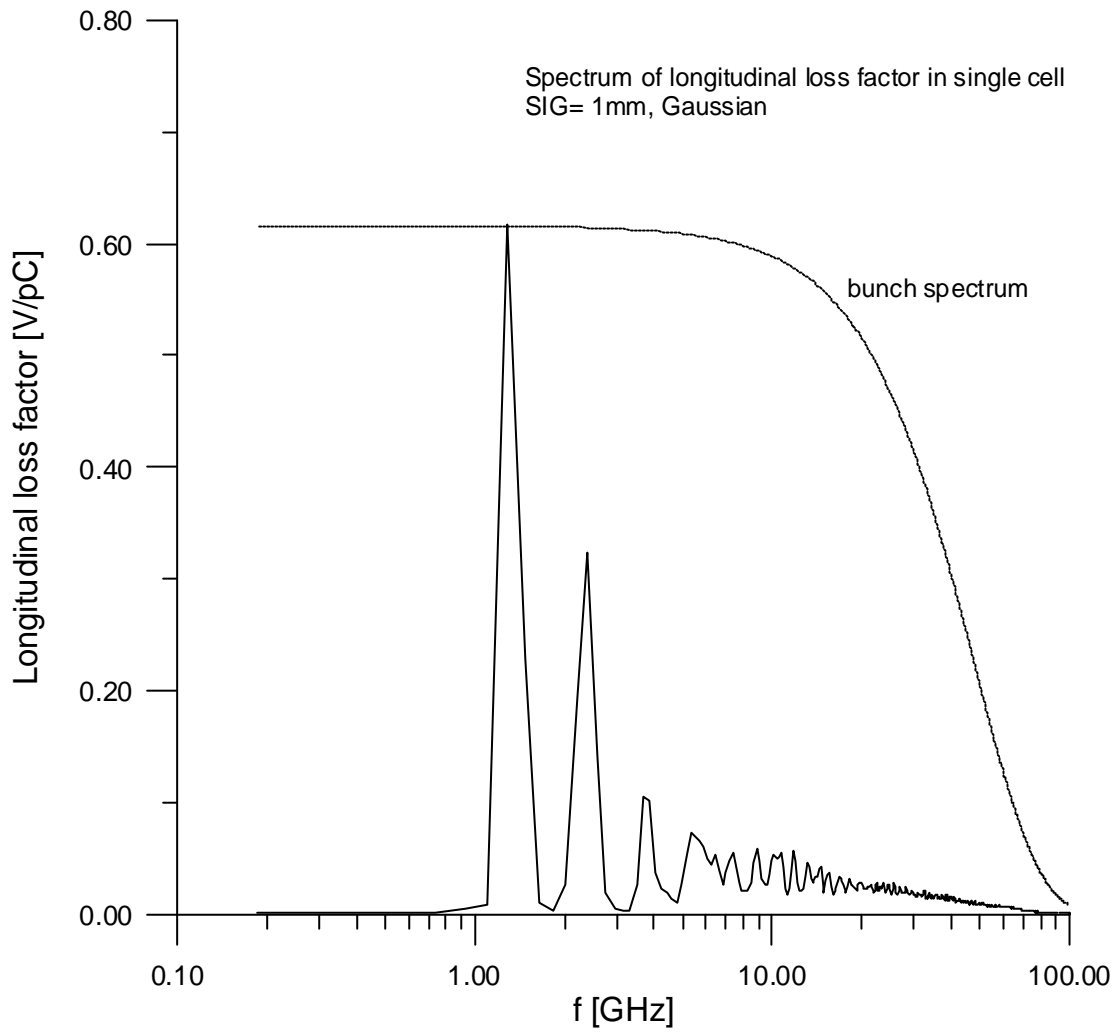


Fig.6. Frequency spectrum of K long. of single TESLA cell.  
Sig = 1 mm bunch.

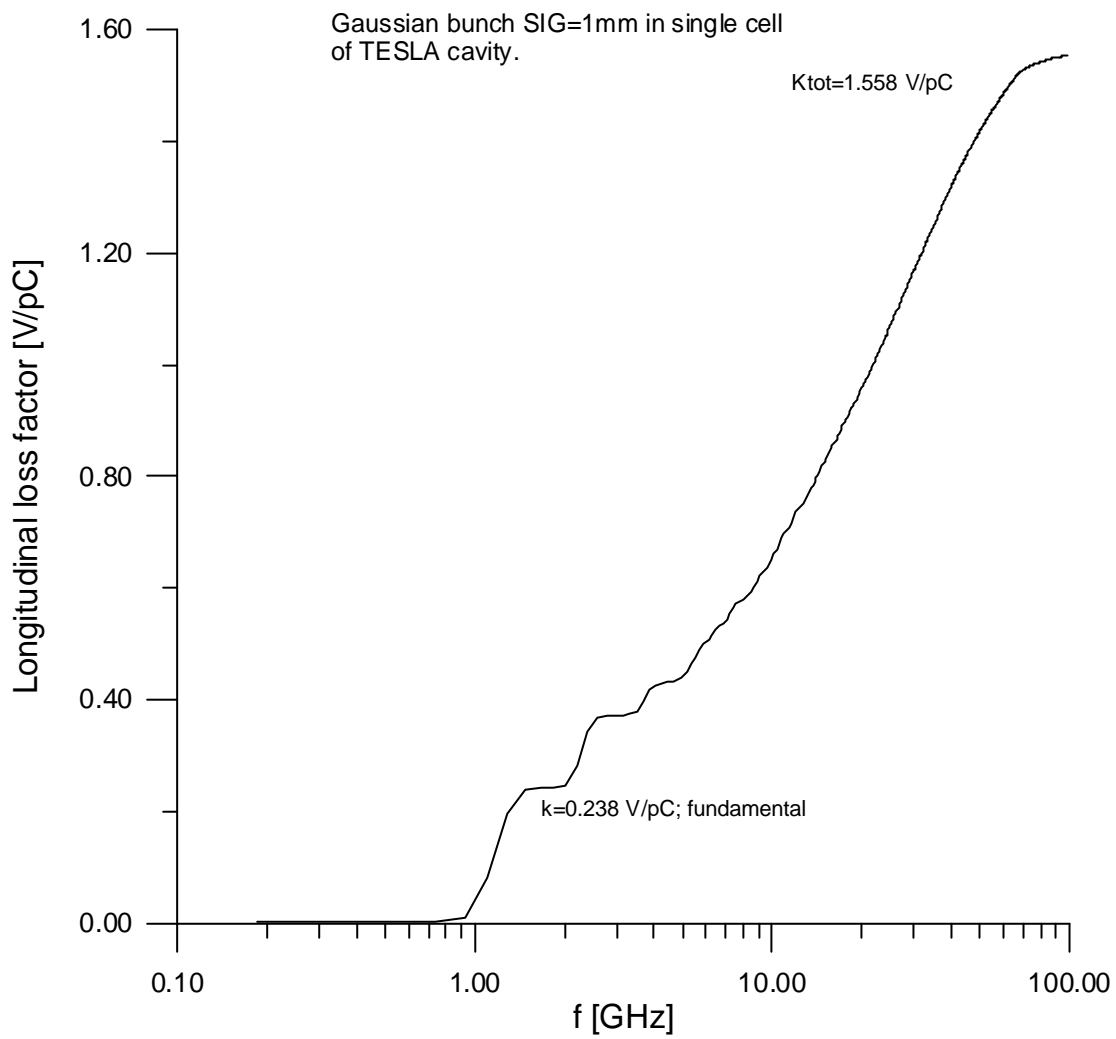


Fig.7. Integrated longitudinal loss factor for single TESLA RF cell.  
Sig = 1mm.



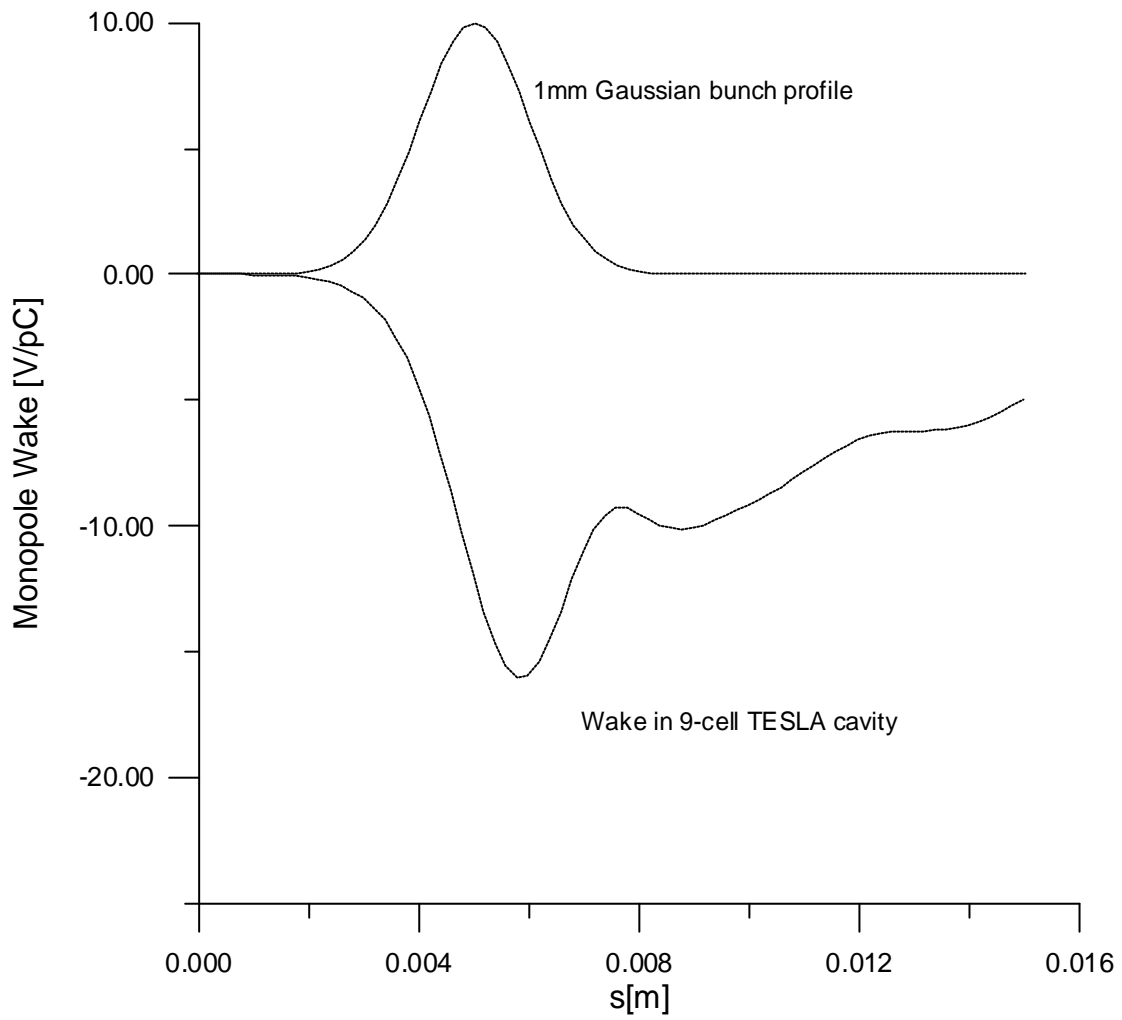


Fig.8. Longitudinal monopole wake in 9-cell TESLA cavity for Sig=1mm.

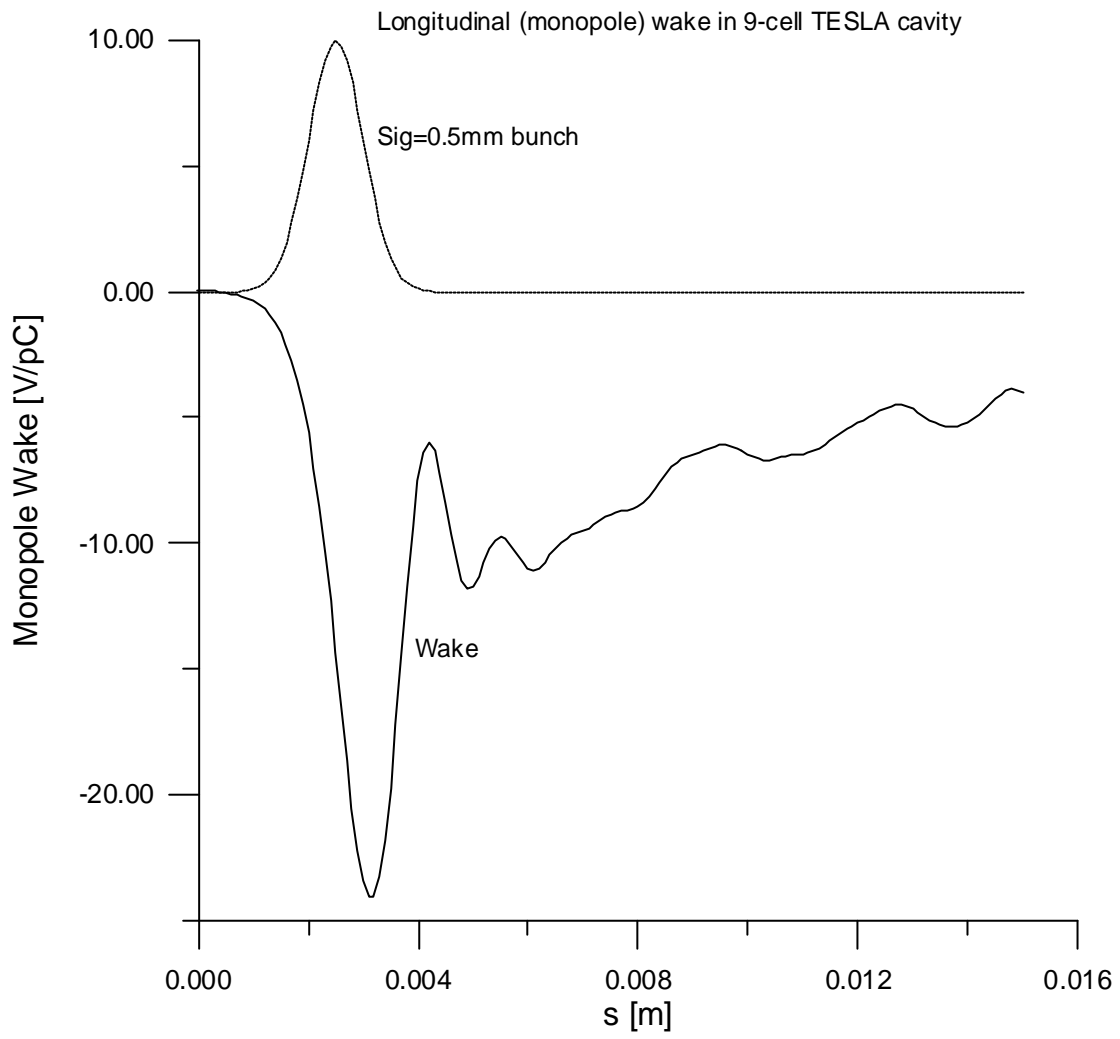


Fig.9. Longitudinal monopole wake in 9-cell TESLA cavity.  
Sig = 0.5 mm bunch.

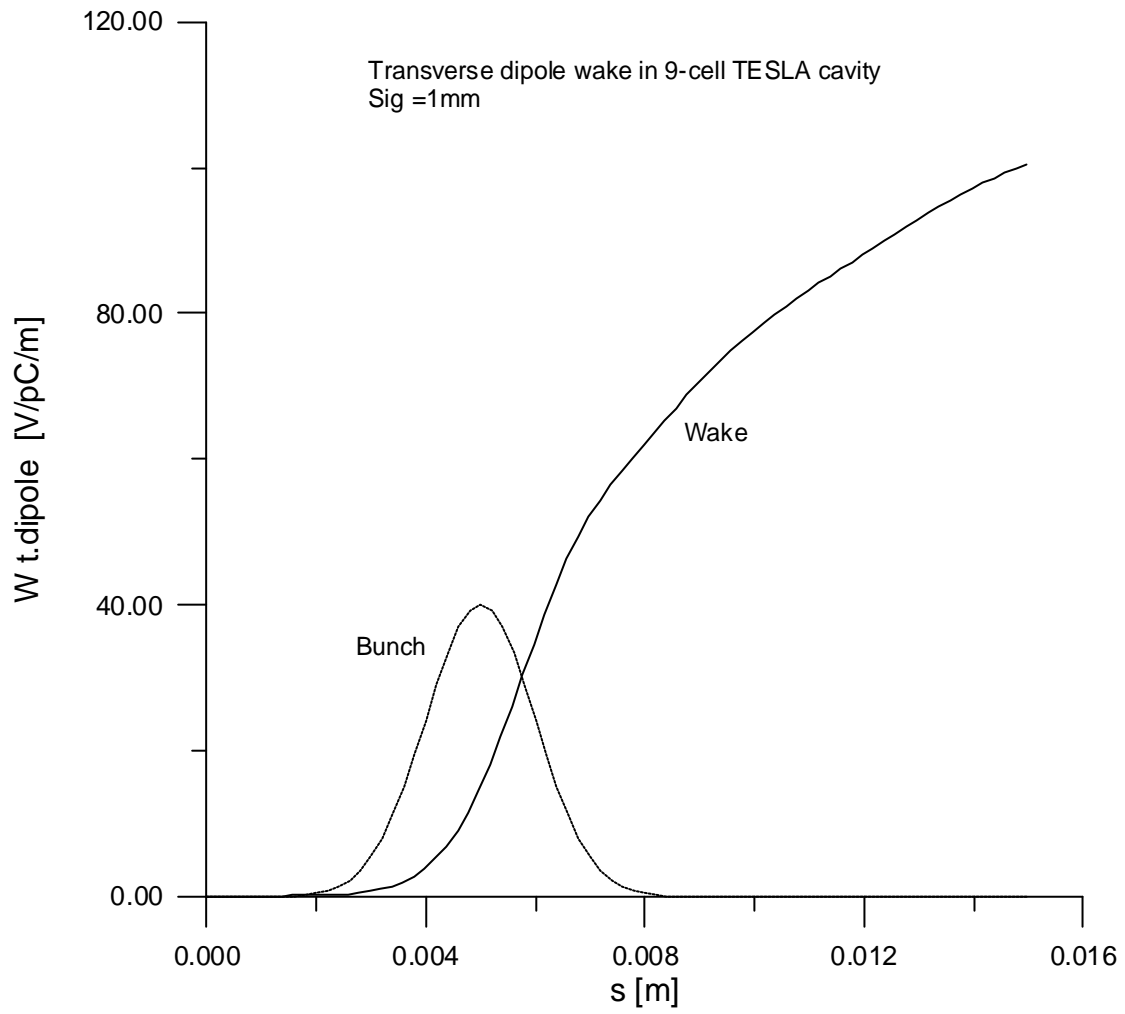


Fig.10. Transverse dipole wake in TESLA 9-cell cavity.  
Sig = 1 mm Gaussian bunch.

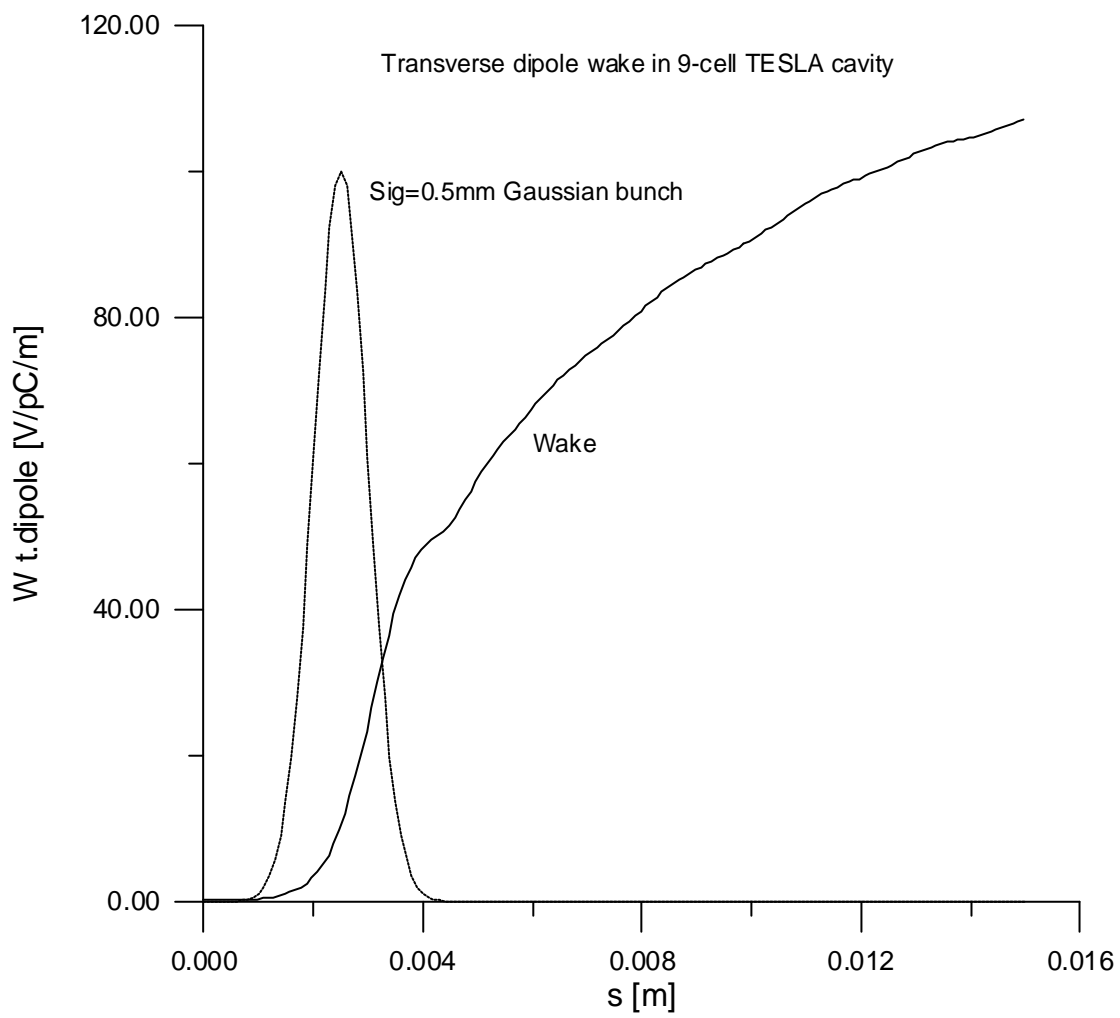


Fig.11. Transverse dipole wake in TESLA 9-cell cavity for Sig=0.5mm.

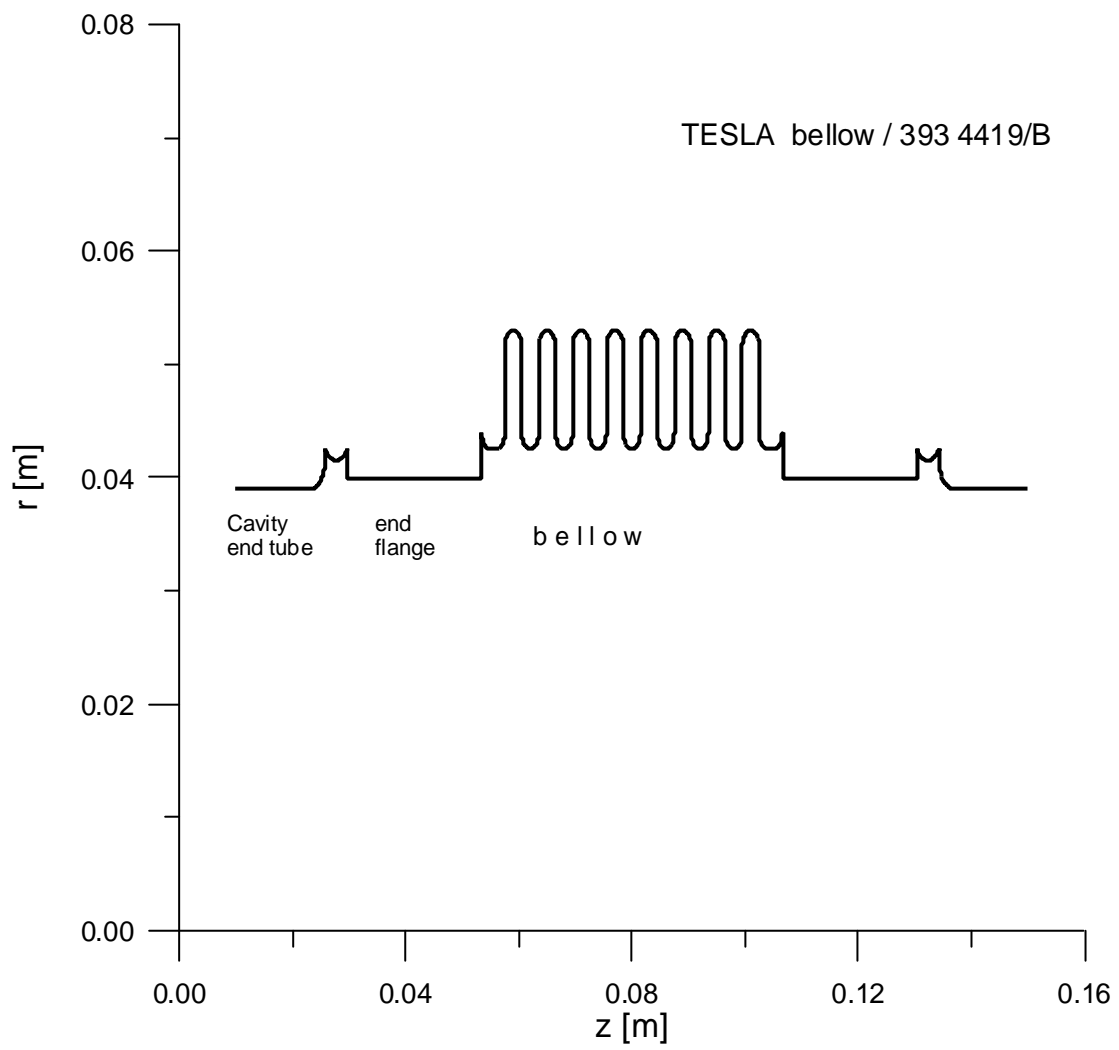


Fig.12. The TESLA intercavity flexible bellow geometry

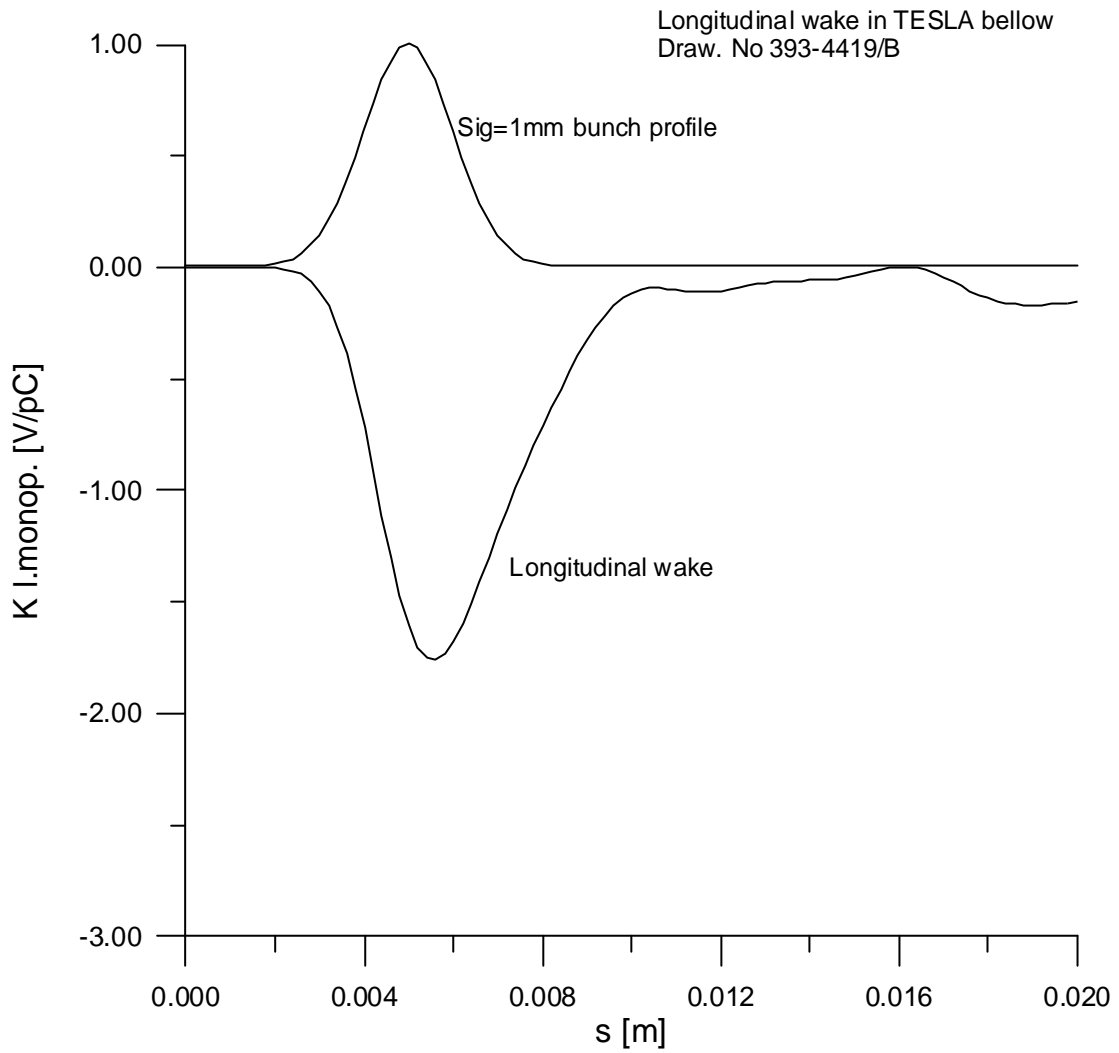


Fig.13. Longitudinal monopole wake in TESLA bellow for Sig=1mm.

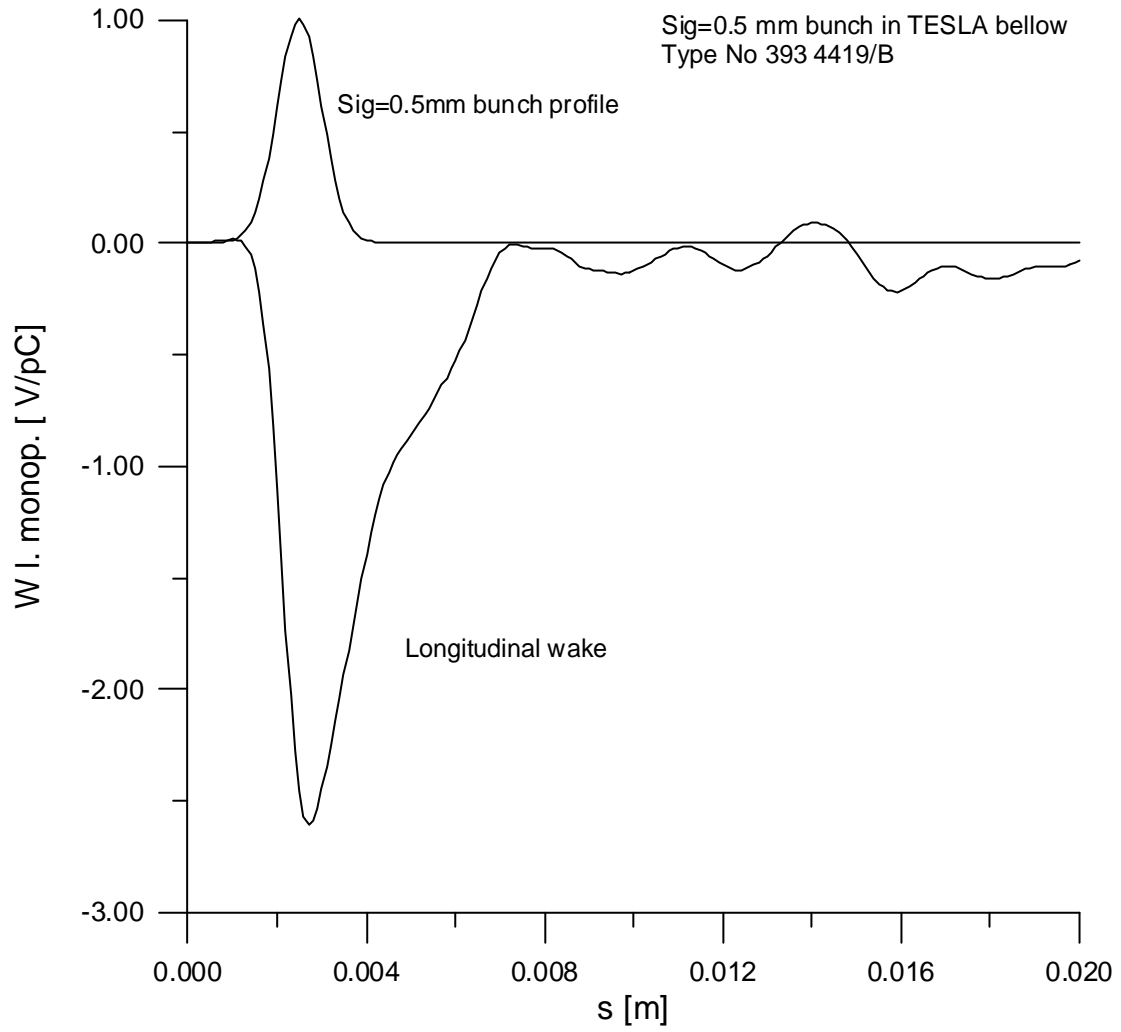


Fig.14. Longitudinal monopole wake in TESLA bellow for Sig=0.5mm.

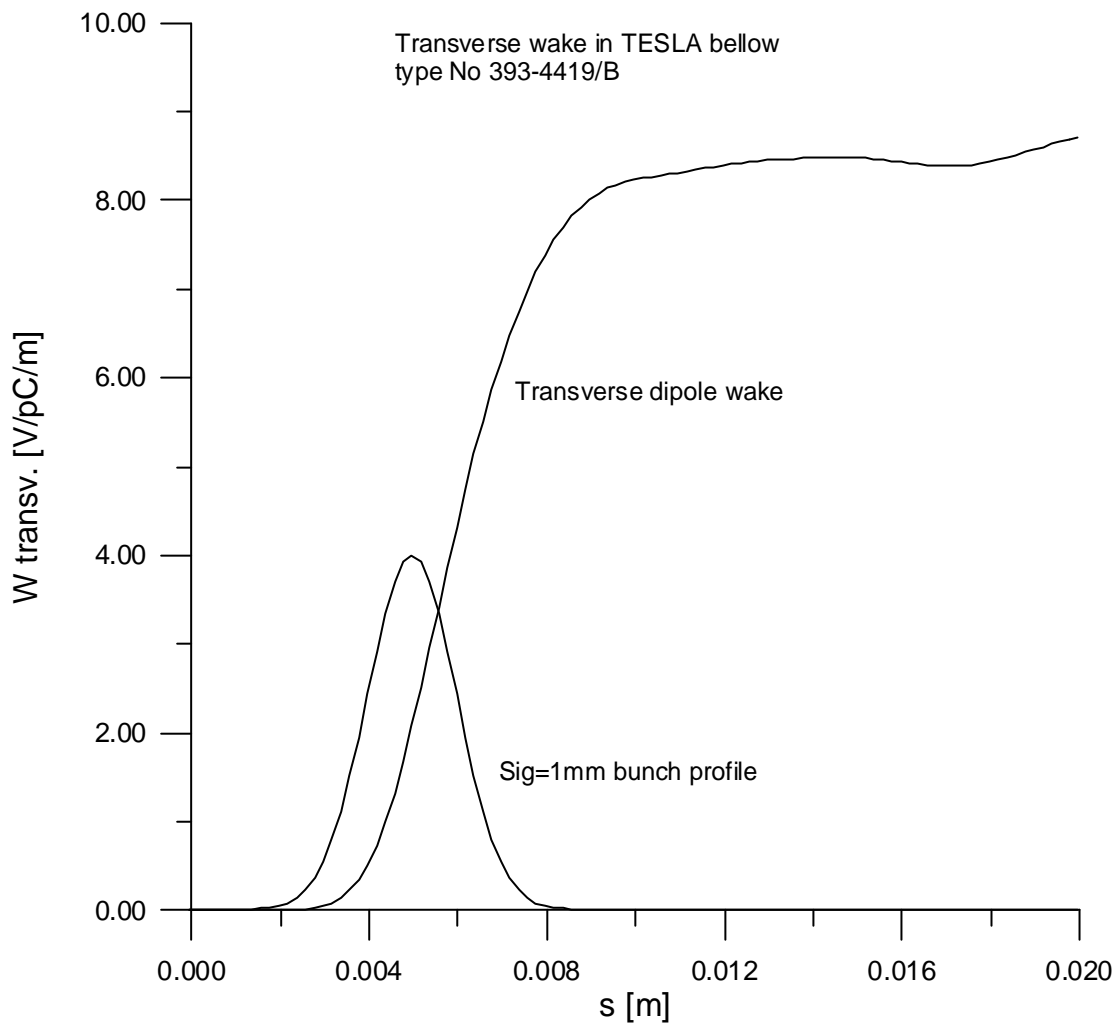


Fig.15. Transverse dipole wake in TESLA bellow for Sig=1mm.



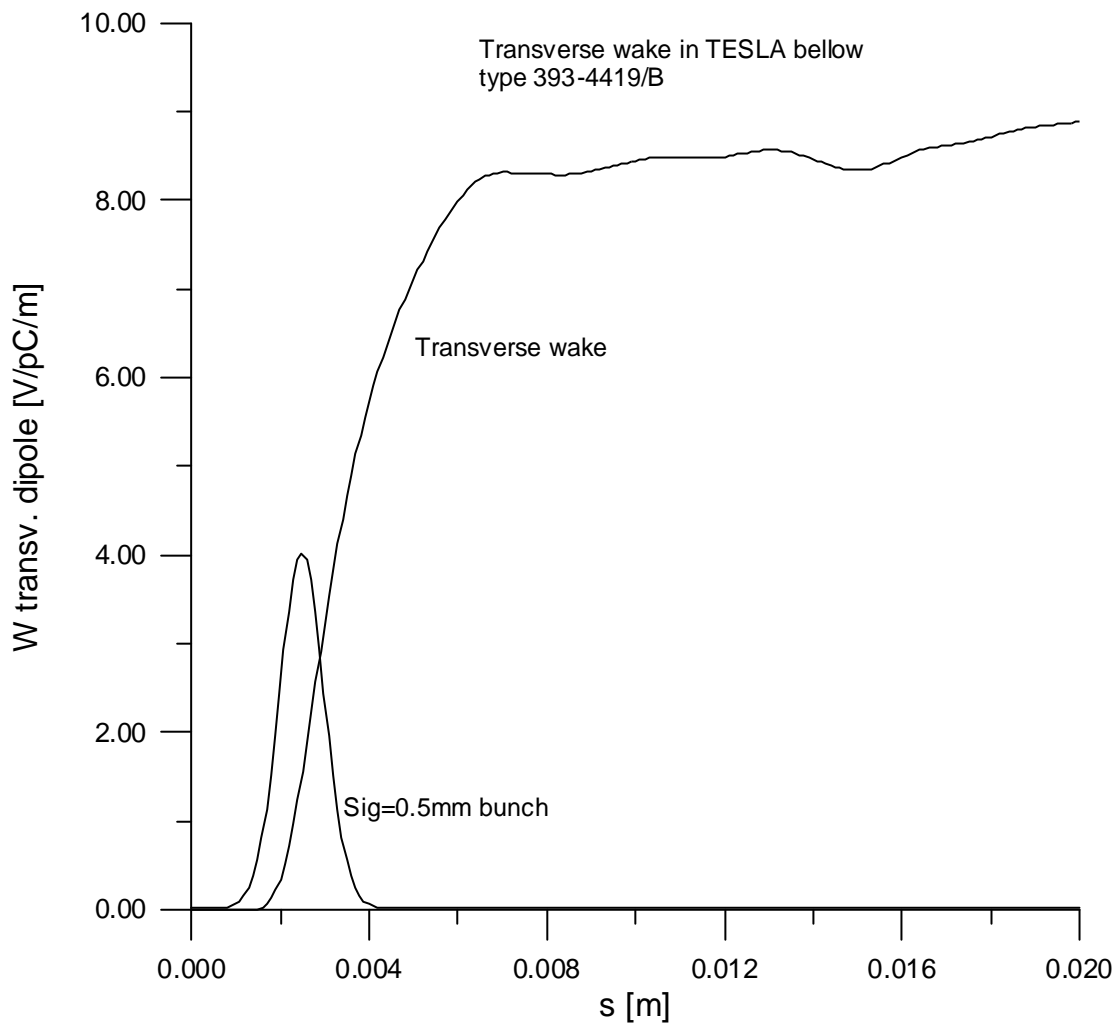


Fig.16. Transverse dipole wake in TESLA bellow for Sig=0.5mm.

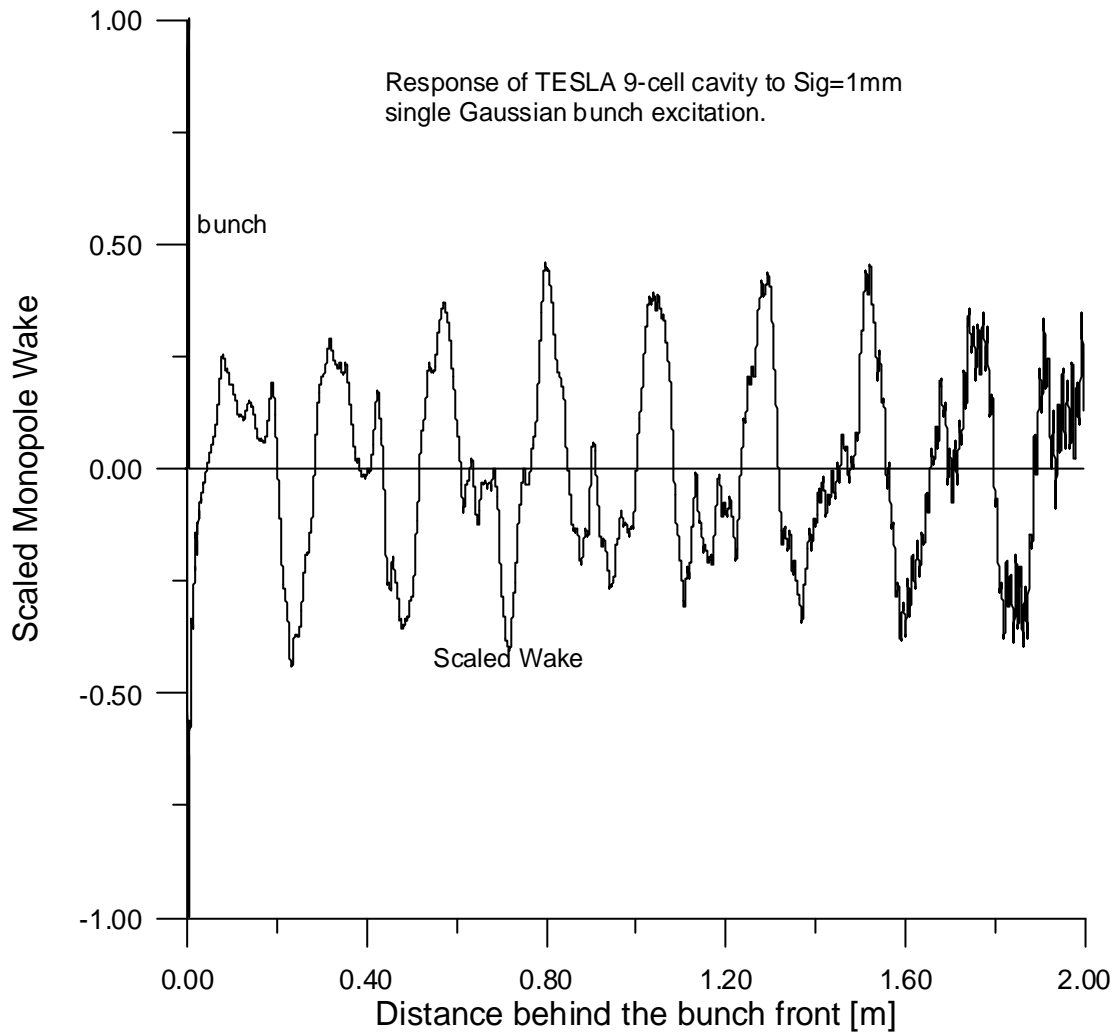


Fig.17. The behaviour of longitudinal monopole wake as a function of distance behind the bunch front.