# Diffraction Radiation as a non-intercepting diagnostics for TTF: the bunch length measurement

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### **Abstract**

Diffraction Radiation, i.e. Transition Radiation diffracted by an aperture in a metallic foil, can be a candidate as a non-intercepting diagnostics for the TTF beam. It can be used to measure bunch length because its coherent emission spectrum do not differ significantly from that of standard transition radiation in most of the nominal TTF beam conditions.

### 1 - Introduction

The high power of the TTF beam poses severe limitations in the use of intercepting diagnostics. As pointed out in the TTF Conceptual Design Report, ceramic fluorescent screens can only be used with the minimum pulse duration, while even the thin (25 µm or less) aluminized Kapton foils for OTR production can sustain the full beam power only for large beam transverse dimensions. This problem will become even grater with beams produced by the Injector 2 or by the injector for the FEL experiment, whose lower emittance will allow larger charge density. On the other hand the same large charge density, being a possible source of strong wakefields, requires the characterization of these beams along their full length, so that new non-intercepting diagnostic devices must be developed.

Diffraction radiation, that is the radiation emitted by a charged relativistic particle going through an aperture in a metallic foil, is a possible candidate for this kind of diagnostics. If the aperture in the foil is larger than the beam transverse dimension, we have a real non-intercepting device, giving also only small perturbation to the beam.

In this paper I will show that diffraction radiation from a slit can be used to measure the bunch length by means of the same technique which is already planned for this measurement and that makes use of the coherent transition radiation by a metallic foil.

The measurement will be possible in the first stage of TTF, with only one accelerating module, up to rms bunch length as short as 500  $\mu$ m, while at full TTF energy (500 MeV) it will be possible to measure also the 50  $\mu$ m rms compressed bunch.

### 2 - Diffraction Radiation

The mathematical derivation of diffraction radiation properties in different environment conditions is rather complex, but has been extensively studied [1+4]. In the accelerator physics, diffraction radiation plays an extremely important role as a source of beam energy losses, and has been studied principally for circular machines, with the main focus on the effects on the beam and not on the radiation itself.

Although in these studies the diffraction radiation has been considered as an independent form of radiation, it is evident that it is nothing else that transition radiation emitted by a surface with a cut on it which give rise to a "diffraction" distribution.

Diffraction radiation derives by the properties of the electromagnetic field of a charged particle in relativistic uniform motion in vacuum: for  $\gamma > 1$ , the electric field produced by a charge q at a point p at distance p from the trajectory and at a longitudinal distance vt from the particle is (see fig. 1)

$$E_{\perp} = \frac{\gamma q \rho}{\left[\rho^2 + (\gamma vt)^2\right]^{\frac{3}{2}}}$$

$$E_{\parallel} << E_{\perp}$$

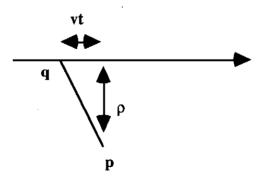


Fig. 1 - Notations for the field of a relativistic particle

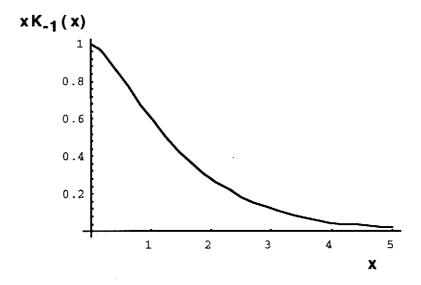
This is due to the fact that the field is significantly different from zero only at angles near 90°.

The armonic content of this field is

$$E_{\perp}(\omega) \sim \frac{\rho \omega}{\gamma v} K_{-i} \left( \frac{\rho \omega}{\gamma v} \right)$$

in which  $K_{-1}$  the modified Bessel function of second order.

From the plot of the function  $x K_{-1}(x)$ 



we see that the function has a significant value only for  $x \le 1$ .

For β≈1

$$x=\frac{2\pi\rho}{\gamma\lambda}$$

and a strong field is present at a distance  $\rho$  only for wavelengths that satisfy

$$\gamma \lambda \ge 2\pi \rho$$

This means that when an electron goes through a hole of radius  $\rho$  in a metallic foil, wavelengths for which

$$\gamma \lambda < 2\pi \rho$$

have a negligible intensity while wavelengths for which

$$\gamma \lambda >> 2\pi \rho$$

are emitted with almost the same intensity as the Transition Radiation from an uncut foil.

### 3 - Diffraction Radiation from the TTF beam

For TTF energies ( $\gamma$ =200+800) and hole diameter not less than 1 mm, the radiation satisfying the above condition is confined in the infrared and far infrared region, so that diffraction radiation cannot be used with the standard optical diagnostic system to obtain images of the beam.

But there is an important beam parameter which is measured by means of long radiation wavelengths: the bunch length through the analysis of the coherent transition radiation spectrum. The coherent emission is significative only for wavelength equal to or longer than the bunch length, so that for a given energy there will be a minimum bunch length for which the diffraction radiation spectrum will be very similar to that of transition radiation. This minimum value will also depend on the hole diameter that the beam transverse dimension can allow.

I will show that at the initial energy of 100 MeV it is possible to measure bunch length as short as 500  $\mu m$  rms, with a hole of 3 mm diameter.

It is thus possible to have a non intercepting diagnostics, giving small perturbation to the beam. This possibility is of particular importance for the FEL beam, whose length must be kept as short as possible and a bunch length measurement must be performed at the entrance of the undulator with the nominal beam transverse dimension, and no kind of screen can sustain the density power of such a small beam, even with a reduced macrobunch length.

In this situation, with an energy of 500 MeV, a bunch length of 50  $\mu$ m rms can easily been measured.

### 4 - Diffraction Radiation from a Slit

The chose of a slit and not of an iris is due only to practical reasons, the results being independent from the geometry of the screen. In fact two individually moving slits will allow the positioning of the screens as near as possible to the beam without intercepting it independently from the beam position.

The number of photons emitted by an electron passing trough the center of a slit cut along the x axis and of width a is [3]:

$$\frac{dN}{d\Omega} = 2\pi^2 \frac{k^2 c}{\hbar} \frac{d\omega}{\omega} \left\{ |E_x|^2 + |E_y|^2 \right\}$$

with  $k = 2\pi/\lambda$  and

$$\begin{split} E_x \Big( k_x, k_y \Big) &= \frac{iek_x}{4\pi^2 cf} \left( \frac{e^{-\frac{a \left( f - ik_y \right)}{2}}}{f - ik_y} + \frac{e^{-\frac{a \left( f + ik_y \right)}{2}}}{f + ik_y} \right) \\ E_y \Big( k_x, k_y \Big) &= \frac{e}{4\pi^2 c} \left( \frac{e^{-\frac{a \left( f - ik_y \right)}{2}}}{f - ik_y} - \frac{e^{-\frac{a \left( f + ik_y \right)}{2}}}{f + ik_y} \right) \\ f &= \sqrt{k_x^2 + \alpha^2} \qquad \alpha^2 = \frac{k^2}{\beta^2 \gamma^2} \\ k_x &= k \sin \theta \cos \phi \\ k_y &= k \sin \theta \sin \phi \end{split}$$

After some mathematical manipulation, the photons number can be written as

$$\begin{split} \frac{dN}{d\Omega} &= \frac{e^2}{8\pi^2 \hbar c} \frac{k^2}{f^2 \left(f^2 + k_y^2\right)} \\ &\left\{ 2e^{-fa} \left(f^2 + k_x^2\right) - \frac{2\alpha^2 e^{-af}}{f^2 + k_y^2} \left[ \left(f^2 - k_y^2\right) \cos ak_y - 2fk_y \sin ak_y \right] \right\} \frac{d\omega}{\omega} \end{split}$$

### 5 - Coherent Diffraction Radiation

It is well known that for wavelength much longer than the electron bunch, all electrons emit coherently and the total intensity become proportional to the square of the number of electrons in the bunch.

More generally the total intensity is given by

$$N_{fch} = N_e (1 + N_e f(\lambda)) N_f$$

where  $N_f$  is the number of photons emitted by a single electron, and given by the above formula, while  $f(\lambda)$  is a form factor related to the charge distribution in the microbunch  $\rho(z)$ :

$$f(\lambda) = \left| \int \rho(z) \exp[i2\pi z / \lambda] dz \right|^2$$

in which I have neglected the effect of the transverse beam dimension.

For an uniform distribution of length lb, we have

$$f(\lambda) = \frac{\sin^2\left(\frac{\pi l_b}{\lambda}\right)}{\left(\frac{\pi l_b}{\lambda}\right)^2}$$

while for a gaussian distribution of  $\sigma=l_b/2$ 

$$f(\lambda) = e^{-\frac{1_0^2 \pi^2}{\lambda^2}}$$

The spectra for coherent transition radiation from a 100 Mev beam from a foil and from slits of 5 mm and 3 mm width is compared in the following pictures

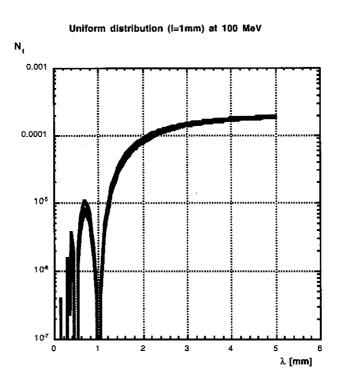


Fig 2 - Comparison between transition radiation from a metallic foil and diffraction radiation from a 5 mm and a 3 mm slit for a 100 MeV beam with uniform longitudinal distribution of 1 mm length

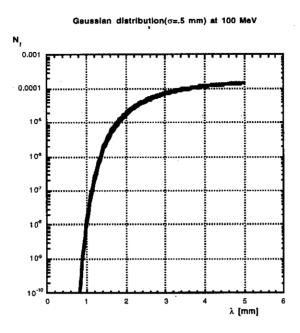


Fig 3 - The same as before, but for a beam with a gaussian longitudinal distribution with  $\sigma = .5$  mm

At 1 GeV energy, the decrease of transverse emittance allows a more narrow slit, giving the capability of measuring a much more short bunch, as it is shown in the next pictures.

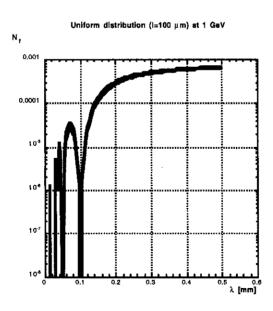


Fig 4 - Comparison between transition radiation and diffraction radiation from a 1 mm slit from a 1 GeV bunch with uniform longitudinal distribution of .1 mm length

# 10<sup>-4</sup>

Gaussian distribution (c=50 µm) at 1 GeV

## Fig 5 - as in the previous picture but from a gaussian longitudinal distribution with $\sigma$ =.05 mm

### 6 - Effects of beam transverse dimensions

As can be seen by the electric field equation, the more an electron crosses the slit offcenter, the more its emission spectrum is similar to that of transition radiation. This means that a beam with a finite transverse dimension in the plane orthogonal to the slit will emit a coherent diffraction radiation spectrum better suited for bunch length measurement.

On the other hand, the use of a slit in place of a foil was mainly required by the danger of a foil damage by the high power density of the beam, so that a slit aperture just sufficient to strongly reduce the power intercepted is fully acceptable, even if some small beam tail hit the surfaces.

### References

- [1] Y.N. Dnestrovskii, D.P. Kostomarov Sov. Phys. Dokl. <u>4</u>, 132 (1959)
- [2] B.M. Bolotovskii, G.V. Voskresenskii Sov. Phys. Uspekhi 2, 73 (1966)
- [3] M.J. Moran, B. Chang Nucl. Inst. Meth. Phys. Res. <u>B40/41</u>, 970 (1989)
- [4] M. L. Ter-Mikaelian: High-Energy Electromagnetic Processes in Condensed Media Interscience Tracts on Physics and Astronomy N. 29