TESLA - COLLABORATION

TESLA Damping Ring Impedances: Preliminary Design Consideration

V. D. Shiltsev

DESY

February 1996, TESLA 96-02

TESLA Damping Ring Impedances: Preliminary Design Consideration

V.D. Shiltsev *

DESY, Notkestrasse 85, 22603, Hamburg, GERMANY

February 7, 1996

Моим Ленам

Abstract

In this article we make a preliminary design study of collective effects and related issues in the damping ring for the linear collider TESLA. Key parameters, essential components and impedance budget are considered in detail for two options of the damping ring with circumferences of 20 km and 2.3 km. Single- and multi-bunch effects are extensively investigated.

1 Introduction: Damping Ring for Linear Collider TESLA

The TESLA linear collider design [1] intends to use damping rings for great reduction of the beams phase space volumes before injection in the main linac at the energy of 3.3 GeV. The positron beam is produced on a target with expected large normalized transverse emittance of the order of 0.01 m [2] and the damping ring appears to be the only solution for the needed decrease of the emittance within the linac cycle time of T_c=200 ms [3], while a RF gun source to achieve the design emittance of the electron beam could be a possible alternative [4], which would allow to save one of two damping rings. Magnetic wigglers had been considered to be installed at the damping ring in order to enforce radiative cooling [5].

Basic parameters of the ring are given in the Table 1. The circumference of the ring is not specified in the Table 1, because at present time there are some approaches to solve injection/ejection issues that leads to spread of the required ring circumferences. The problem is that the TESLA design intends a bunch train duration of about 800 μs (or about 240 km length) in the main linac. The train of some 1000 bunches must be compressed in a storage ring and then expanded when extracted out of it. Thus, the injection and ejection of every bunch has to be done individually. If one assumes to use a kicker for the ejection with rise/fall time τ, then the circumference of the ring is about $C[km] \simeq \tau[ns]/3$. For example, the "dog-bone" proposal for the damping ring assumes

*on leave from Budker Institute of Nuclear Physics, 630090 Novosibirsk, RUSSIA

ultimate conventional kickers with $\tau \sim 60$ ns, and therefore, $C \simeq 20$ km [6]. The "dog-bone" ring consists of two long (about 10 km) straight sections which share the tunnel of the main linac and a pair of end-rings. The damping wigglers are to be installed in the straight sections. Under assumption of using somewhat modified injection/ejection scheme with several deflecting elements [7, 8] the ring with circumference of about 6.3 km in the existing HERA tunnel was discussed in Ref.[9].

Table 1: Basic Parameters of TESLA Damping Ring

Energy	E	3.3	GeV
Cycle time	T_c	200	ms
No. of bunches	N_b	~ 1100	
Particles/bunch	N_e	$3.7 \cdot 10^{10}$	
Bunch length	σ_s	~ 1	cm
Damping times	$\tau_s/\tau_{x,y}$	18.5/37	ms
Norm. emittances			
at injection	$\epsilon_x^0/\epsilon_y^0$	$10^4/10^4$	μm
at ejection	ϵ_x/ϵ_y	10/0.2	μm

Several novel schemes of the fast kicker with $\tau \simeq 7$ ns and even less are under thorough consideration now (e.g. superfast *beam-beam kicker* [10] and travelling wave kicker with high voltage semiconductor pulse generator [11]). Consequently, an option for a low-cost damping ring in the existing tunnel of the PETRA ring with $C \approx 2.3$ km has to be taken into consideration.

In this article, we present a general study of the TESLA DR impedances versus its' circumference and make a quantitative analysis for the two extreme cases, namely, for the "dog bone" 20-km circumference ring and for 2.3 km long ring in the PETRA tunnel. Basic relevant parameters which are necessary for the impedance study, such as parameters of the RF system, wigglers section, vacuum chamber, etc., are discussed in Section 2. Sections 3 and 4 are devoted to the broadband and narrow band impedance budgets respectively. Brief overview of the derived results and discussion on further studies are presented in the Conclusion.

2 Major Components and Parameters

Purpose and design of an accelerator determine specific surroundings of a beam which are of utmost importance for current induced electromagnetic fields and, therefore, for particle dynamics. Specific feature of the damping ring for a linear collider is magnetic wigglers to enforce radiation damping. Based on the wiggler design [5], we specify basic parameters of the RF system and derive essential parameters of the damping ring.

2.1 Damping Wigglers

With the repetition frequency of the TESLA linac $1/T_c$ =5 Hz, the vertical damping time τ_y has to be about 5.5 times less than T_c because the vertical emittance must damped from its initial value of ϵ_y^0 to the design value ϵ_y:

$$\epsilon_y \simeq \epsilon_y^0 e^{-2T_c/\tau_y} \approx \frac{\epsilon_y^0}{50000}, \tag{1}$$

Here for simplicity we assume that there is no significant impact of residual vertical dispersion and coupling coefficient $\theta_c^2 = \epsilon_y/\epsilon_x$ is smaller then the design emittance ratio of 0.02. As the vertical damping time $\tau_y \approx 37$ ms is about two times of the longitudinal one $\tau_s = E/(U_0 f_0)$ ($E = 3.3$ GeV, f_0 is a revolution frequency) then one needs to provide the energy loss per turn U_0 of

$$U_0 = \frac{2E}{f_0 \tau_y} \simeq 0.6[MeV] \cdot C[km], \tag{2}$$

that is about 12 MeV for the "dog-bone" option and about 1.4 MeV for the 2.3 km ring. Thus, the magnetic structure of the DR should provide damping integral:

$$I_D = \int B^2 dL\,[T^2 m] = 43.4 \cdot C[km], \tag{3}$$

or about 870 T^2m for the "dog-bone" and about 100 T^2m for the 2.3 km machine in the PETRA tunnel. In Ref.[5] several damping wiggler designs for the TESLA damping ring were studied. For the sake of definiteness, we shall assume one of the considered options of the 1.5 T piecewise constant field wiggler based on permanent magnets with period less than 10 cm. It gives the total length of the wiggler section of about 387 m for the "dog-bone" ring and of about 44 m for the small ring with $C = 2.3$ km. The wiggler gap is 25 mm which allows for a vacuum chamber vertical aperture in the wiggler section of about 20 mm.

2.2 RF System

The choice of the RF frequency is determined by the TESLA linac RF frequency which is equal to 1.3 GHz. To synchronize ejection/injection processes, the RF frequency in the DR must be equal to (sub)harmonic of the linac frequency. As powerful DC klystrons in 400-600 MHz band are available from industry, we choose $f_{RF} = 433.33$ MHz that is one third of 1.3 GHz.

In this paper we assume normal conductive RF cavities. As some portion of RF power dissipates in cavity walls, the heat loading gives some limitation on number of cavities N_{cav}.

Following [12], we introduce *an overvoltage factor* q as ratio of the RF peak voltage V_0 and energy loss per revolution U_0:

$$q = \frac{1}{\sin\phi_s} = \frac{eV_0}{U_0}, \tag{4}$$

Let us denote peak accelerating voltage per single cavity as V_1, and N_{cav} is total number of cavities, then

$$q(U_0/e) = V_1 N_{cav} = V_0. \tag{5}$$

The power of losses in the cavity walls is equal to

$$P_{loss} = N_{cav}\frac{V_1^2}{2R_s^L} = \frac{q^2(U_0/e)^2}{2R_s N_{cav}}, \tag{6}$$

where R_s is the shunt impedance.

Total length of the 433 MHz RF structure is about

$$L_{RF} = L_{cav} \cdot N_{cav} \simeq 0.4[m] \cdot N_{cav}. \tag{7}$$

The heating due to the RF power dissipation is acceptable if the losses are below the limit of 150 kW/m , i.e. in our case

$$W = \frac{P_{loss}}{L_{RF}} \leq W_{max} = 0.15\ [MW/m]. \tag{8}$$

Let us assume that the cavity shunt impedance of $R_s \simeq 4\ M\Omega$ can be obtained, then we get following limitations:

on the peak voltage per cavity

$$V_1 \leq \sqrt{2R_s W_{max} L_{cav}} = 0.7\ [MV], \tag{9}$$

on the number of cavities

$$N_{cav} \geq \frac{q(U_0/e)}{\sqrt{2R_s W_{max} L_{cav}}} = 0.87 \cdot q \cdot C[km], \tag{10}$$

and on the power of losses

$$P_{loss} \leq W_{max} \cdot L_{cav} \frac{q(U_0/e)}{\sqrt{2R_s W_{max} L_{cav}}} = 0.052[MW] \cdot q \cdot C[km]. \tag{11}$$

2.3 Beam Parameters

The rms energy spread induced by synchrotron radiation is in a definite relation to the ring magnetic structure (see, e.g. [12]):

$$\sigma_E^2 = \frac{< \Delta E^2 >}{E^2} = \frac{C_E}{J_s}\gamma^2 \frac{\left\langle \frac{1}{\rho^3} \right\rangle}{\left\langle \frac{1}{\rho^2} \right\rangle}, \quad C_E \approx 3.84 \cdot 10^{-13}\, m. \tag{12}$$

where the damping partition number $J_s \approx 2$, ρ is the particle curvature radius in the magnetic field along the orbit.

As the main contribution to the radiation cooling comes from the wiggler with piecewise constant magnetic field $B_0 = 1.5$ T, the energy spread in the TESLA DR becomes circumference independent:

$$\sigma_E = \sqrt{\frac{C_E \gamma e B_0}{J_s m c^2}} \approx 0.85 \cdot 10^{-3}\sqrt{B_0[T]} = 1.04 \cdot 10^{-3}. \tag{13}$$

The next "fixed" value is the rms bunch length σ_s, which has to be of the order of 1 cm to satisfy bunch compressor requirements [13]. The rms bunch length relates to the rms energy spread as

$$\sigma_s = \sigma_E \frac{\alpha C}{2\pi \nu_s}, \tag{14}$$

where α denotes the momentum compaction factor of the ring, and ν_s is the synchrotron oscillation tune. The latter is determined by RF frequency $f_{RF} = h \cdot f_0$, the peak accelerating voltage V_0, and equilibrium RF phase ϕ_s:

$$\nu_s^2 = \frac{\alpha h e V_0 \cos\phi_s}{2\pi E}. \tag{15}$$

Combining Eqs.(14, 4) and Eq.(2) we obtain the synchrotron tune

$$\nu_s = \frac{\sigma_s f_{RF} U_0 \sqrt{q^2-1}}{\sigma_E E c} = 1.8 \cdot 10^{-4} \frac{\sigma_s f_{RF}}{\sigma_E c}\sqrt{q^2-1} \cdot C[km]. \tag{16}$$

Taking $f_{RF} = 433$ MHz, $\sigma_E = 1.04 \cdot 10^{-3}$ and required bunch length $\sigma_s \simeq 1$ cm, we get the required momentum compaction factor (see Eq.(14):

$$\alpha = 1.57 \cdot 10^{-4}\sqrt{q^2-1}, \tag{17}$$

and, finally, the synchrotron tune

$$\nu_s = 2.5 \cdot 10^{-3}\sqrt{q^2-1} \cdot C[km]. \tag{18}$$

The overvoltage factor q plays also some role for the momentum acceptance of the ring which is equal to [12]

$$\left(\frac{\Delta p}{p}\right)_{max} = \sqrt{\frac{2eV_0 f_0}{\pi f_{RF} \alpha E}\left(\sqrt{1-1/q^2} - \frac{arccos(1/q)}{q}\right)}. \tag{19}$$

Let us now define the last "free" parameter of q. Remind to a reader, that $q > 1$. Following arguments should be taken into account:

- smaller overvoltage factor q is favorable with respect to the required RF power,
- larger overvoltage factor is desired for larger momentum acceptance of the ring,
- smaller q is preferable for smaller number of RF cavities - see Eq.(10), and it leads to lower increments of coupled bunch instabilities due to high order modes (HOMs) in the cavities(see below),

- larger q leads to higher single bunch instabilities thresholds(see below),
- the overvoltage factor should be smaller in order to keep the synchrotron tune below some reasonable limits (say, $\nu_s \leq 0.10$), otherwise some difficulties in finding adequate operating points in the betatron tune space could be expected.

For the sake of definiteness in further consideration, we choose $q = 2$ for the "dog-bone" option (the main reason - to reduce the number of cavities to 38) and $q = 4$ for the proposed damping ring in the PETRA tunnel (allows to have sufficiently large ν_s with 8-cavities RF system). Below we make numerical estimations under following assumptions: 1) β-function is about 10 m in the wiggler sections, the arcs and the RF cavities; 2) $\beta \simeq 60$ m in the straight sections of the "dog-bone"; 3) total length of the arcs in the "dog-bone" ring is about 1.2 km; 4) the length of the wiggler is ≈2% of the circumference; 5) the mean radius of the beam pipe is about 3 cm in the arcs, about 5 cm in the straight sections of the 20 km long ring, and about 1 cm (half aperture) in the wiggler. Beam dynamics relevant parameters of the damping ring options are summarized in the Table 2.

3 Broadband Impedances

3.1 Longitudinal Impedance

Constraints on the Impedance

The total number of particles $N_{tot} = 4.1 \cdot 10^{13}$ gives us the mean bunch current to be inversely proportional to the ring circumference:

$$I_s = \frac{eN_{tot}f_0}{N_b} \approx \frac{1.78[mA]}{C[km]}. \tag{20}$$

The microwave longitudinal instability sets the first constraint on the effective broad-band impedance (see, e.g. [14]):

$$\left(\frac{Z}{n}\right)^{thr}_{eff} = \frac{\sqrt{2\pi}\alpha(E/e)\sigma_E^2\sigma_s}{I_s R}, \tag{21}$$

where $n = (\omega/\omega_0) = \omega R/c = \omega C/(2\pi c)$ is harmonic number.

For parameters of the TESLA DR we get

$$\left(\frac{Z}{n}\right)^{thr}_{eff} = 0.05[Ohm] \cdot \sqrt{q^2 - 1}, \tag{22}$$

i.e. 86 mΩ for the "dog-bone" (q=2.0) and about 0.2 Ω for the 2.3-km-long ring with $q = 4.0$.

By the definition the effective impedance is a machine impedance $(Z/n)_0$ averaged with the bunch spectrum:

$$\left(\frac{Z}{n}\right) = \frac{\int (\frac{Z}{n})_0 e^{-(n\omega_0\sigma/c)^2} dn}{\int e^{-(n\omega_0\sigma/c)^2} dn}. \tag{23}$$

Table 2: Two Options of TESLA Damping Ring

Parameter		"Dog-bone"	PETRA-DR	units
Circumf.	C	~20	2.3	km
Rev.frequency	f_0	~15	130	kHz
Bunch length	σ_s	~1	~1	cm
Bunch spacing	τ_b	60	7	ns
Loss per turn	U_0	12	1.38	MeV
DC current	I_B	0.1	0.86	A
DC/bunch	I_b	0.09	0.77	mA
SR power	P_{SR}	1.2	1.2	MW
Energy spread	σ_E	$1.04\cdot10^{-3}$	$1.04\cdot10^{-3}$	
Overvoltage	$q = eV_0/U_0$	2.0	4.0	
Synchr. tune	ν_s	.087	0.022	
Mom. compaction	α	$2.7\cdot10^{-4}$	$6.1\cdot10^{-4}$	
Mom. acceptance	$(\Delta p/p)_{max}$	$1.4\cdot10^{-2}$	$1.8\cdot10^{-2}$	
RF voltage	V_0	24.0	5.5	MV
No. cavities	N_{cav}	38	8	
Cavity voltage	V_1	0.63	0.69	MV
Mean β	$<\beta>$	arcs ~10	~10	m
		str.sect. ~60		m
Pipe radius	b	arcs ~3	~ 3	cm
		RF duct ~5.2	~5.2	cm
		str.sect. ~5		cm

Sometimes the SPEAR scaling $\left(\frac{Z}{n}\right) = \left(\frac{Z}{n}\right)_0\left(\frac{\sigma_s}{b}\right)^{1.68}$ is used to relate effective and machine impedances where b is the beam pipe radius [15]. For the average $< b > \simeq 3$ cm the machine impedance is 6.3 times larger than the effective impedance giving (Z/n)=0.54 Ohms for the "dog-bone" and 1.22 Ohms for the PETRA tunnel option. However, as the SPEAR scaling describes data on the cavities dominated impedance ring, and it may not necessarily be valid for the TESLA damping ring.

The broad-band impedance Z is often parametrized by expansion over $\sqrt{\omega}$ [16]:

$$Z(\omega) = -iZ_0\frac{L\omega}{4\pi c} + (1-i)R_W\sqrt{\omega} + R_\Omega + (1+i)R_c\sqrt{\frac{\omega_c}{\omega}}\theta(\omega-\omega_c). \tag{24}$$

here $Z_0 = 120\pi = \frac{4\pi}{c} = 377$ Ohm and $\omega_c/2\pi$ is the cut-off frequency of the beam pipe at the RF cavities, $\theta(\omega)$ is the step function. The first part of Eq.(24) reflects inductive impedance, the second – impedance due to resistive walls, the third describes some constant resistivity due to RF cavities modes, and the last term corresponds to impedance of the RF system beyond the cut-off frequency.

Purely inductive impedance $Z = -i\omega L$ does not lead to the microwave instability, but cause bunch lengthening due to distortion of the RF potential well which was observed in high current storage rings [17]. In the case of small distortion, the expected lengthening can be estimated as (see e.g [19]):

$$\frac{\Delta\sigma_s}{\sigma_s} \approx \frac{L}{2\sqrt{2\pi}\sigma_s}\frac{N_e r_0}{C\alpha\gamma\sigma_E^2}. \tag{25}$$

10% bunch lengthening at the TESLA DR corresponds to

$$\frac{Z}{n} = \omega_0 L \approx .01 \cdot \sqrt{q^2 - 1}\ \Omega. \tag{26}$$

(Note, that the last constraint is circumference independent).

As all the terms other than the first in the expansion (24) contain real part of impedance, they lead to energy losses of the beam. The longitudinal loss factor k_l gives the energy loss and defines the power deposited in the beam pipe by a train of N_b bunches

$$P = N_b k_l (eN_e)^2 f_0. \tag{27}$$

For nominal parameters of our machine (bunch charge of eN_e=5.9 nC and N_b=1100), the loss factor of $k_l = 1$ V/pC corresponds to $P[kW] = 11.7/C[km]$ of the microwave power, or 0.6 kW for the "dog-bone" and 5.1 kW for the ring with $C = 2.3$ km.

RF Cavities

As exact geometry of the RF cavities for TESLA DR is not known yet, below we estimate the cavity broad band impedance using the *diffraction model* (see, e.g. [14])

$$\left(\frac{Z}{n}\right)_0 = (1+i)\sqrt{\frac{cg}{\omega_0}}\frac{Z_0}{2\pi^{3/2}n^{3/2}b}, \tag{28}$$

where g is the resonator gap, b is the beam duct radius in the RF section. For the 433 MHz RF cavity, the parameters of $b \approx 5.2$ cm, and $g \approx 27$ cm are supposed. Corresponding effective impedance can be determined as

$$\left(\frac{Z}{n}\right)_{cav} = \frac{\int_{n_c}^{\infty}\left(\frac{Z}{n}\right)_c e^{-(n\omega_0\sigma/c)^2}dn}{\int e^{-(n\omega_0\sigma/c)^2}dn}, \tag{29}$$

here the cut-off harmonic number is $n_c = f_c/f_0$, the cut-off frequency is about $f_c \approx 2.3$ GHz for the 10.4-cm-diameter beam duct.

Further simplification gives

$$\left(\frac{Z}{n}\right)_{cav} = (1+i)\frac{2Z_0}{\pi C}\sqrt{\frac{g\sigma_s^3}{b^2}}G(\zeta), \tag{30}$$

where

$$G(\zeta) = \int_{\zeta}^{\infty} e^{-u^2}\frac{du}{u^{3/2}}, \tag{31}$$

and parameter $\zeta = n_e\sigma_s/R = 0.48\sigma_s[\mathrm{cm}]$.

The asymptotic of the integral (31) with $\zeta \ll 1$ is $G(\zeta) \to 2/\sqrt{\zeta}$, but for $0.1 < \zeta < 1.0$ the function of $G(\zeta)$ varies slightly. In our particular case of $\sigma_s = 1$ cm and, therefore, $\zeta = 0.48$, the integral is about $G(0.48) \approx 0.66$.

Finally, we get the effective single cavity impedance

$$\left(\frac{Z}{n}\right)_{cav} \approx (1+i)\frac{1.6[mOhms]}{C[km]}, \tag{32}$$

Taking into account number of cavities N_{cav} one estimates total impedance due to the RF system $(1+i)\cdot 0.7\times 8 = (1+i)\cdot 5.6$ mOhms for the PETRA-tunnel ring and $(1+i)\cdot 0.08\times 38 = (1+i)\cdot 3$ mOhms for the "dog-bone". [1]

The RF cavity longitudinal loss factor due to modes below duct cut-off frequency is connected by definition with R/Q parameter as:

$$k_l = \frac{\omega}{2}\left(\frac{R}{Q}\right), \tag{33}$$

For $(R/Q) \approx 200$ Ohms (which is close to several examples of ~ 500 MHz RF cavities see, e.g. [20, 19]) and $\omega = 2\pi\cdot 433$ MHz it gives about 0.3 V/pC. The diffration model give estimation of losses at frequencies above the cut-off:

$$k_l = \frac{1}{\pi b}\sqrt{\frac{g}{\pi\sigma_s}}\Gamma\left(\frac{1}{4}\right), \quad \Gamma\left(\frac{1}{4}\right) \approx 3.62, \tag{34}$$

and for the TESLA DR parameters (see above) one gets additional 0.6 V/pC, therefore, the single cavity loss factor is about 0.9 V/pC. Total loss factor of 8 cavities of the PETRA-tunnel DR is $k_l \approx 7.2$ V/pC and about $k_l \approx 34.2$ V/pC for the "dog-bone" ring with 38 cavities.

Resistive Wall

The longitudinal resistive wall (RW) impedance per meter of a circular pipe with radius b is given by [14]

$$\left(\frac{Z}{n}\right)_W/L = (1-i)\frac{Z_0\delta}{2bC}, \tag{35}$$

[1] The same consideration for the parameters of SLAC B-factory RF cavity gives $(Z/n)_{cav} \simeq 0.8$mOhms in a good agreement with TBCI code [18] calculations [21].

where $\delta = c/\sqrt{2\pi\sigma\omega}$ is the skin depth. For the PETRA-tunnel version we assume copper (or copper coated) vacuum chamber with conductivity of $\sigma = 5.2 \cdot 10^{17} c^{-1}$ that gives us

$$\delta[cm] = 1.21 \cdot 10^{-2} \sqrt{\frac{C[km]}{n}}, \tag{36}$$

and for $b_{eff} = 3$ cm we obtain

$$\left(\frac{Z}{n}\right)_W [Ohm] = (1 - i) \cdot 0.76 \cdot \sqrt{\frac{C[km]}{n}}. \tag{37}$$

At bunch frequencies $k\sigma_s = \omega\sigma_s/c \simeq 1$, i.e. $n_{eff} \approx R/\sigma_s = 1.6 \cdot 10^4 C[km]$, the broadband contribution can be estimated as

$$\left(\frac{Z}{n}\right)_W = (1 - i) \cdot 6\ mOhm. \tag{38}$$

For the "dog-bone" version we intend to have Al beam pipe in straight sections ($\sigma = 3.2 \cdot 10^{17} c^{-1}$) with radius of $b = 5$ cm and together with contribution from the wiggler section (2% of the circumference with $b = 1$ cm) and the arcs (~6% of circumference with $b = 3$ cm), the broad band impedance is estimated about $(Z/n)_W \approx (1 - i) \cdot 5.2$ mOhm.

The loss factor per unit length due to resistive wall impedance in a circular beam pipe is

$$\frac{dk_l}{ds} = \frac{1}{2\pi b}\sqrt{\frac{c}{2\pi\sigma}} \frac{\Gamma(\frac{3}{4})}{\sigma_s^{3/2}}, \quad \Gamma(\frac{3}{4}) \approx 1.23. \tag{39}$$

Using this formula one gets the RW loss factor $k_l \simeq 1.3$ V/pC for the $C = 2.3$ km ring and $k_l \simeq 7.6$ V/pC for the "dog-bone".

Sources of Inductive Impedance

The impedance of the ring may be estimated as the sum of the impedances of its components, such as bellows, BPMs, kickers, vacuum ports, tapers, etc. Most of these elements are discontinuities having resonant frequencies much higher the frequencies within the bunch spectrum, and, therefore, they give rise to a mostly inductive impedance. As the requirement on the bunch lengthening (26) is rather tight, we assume that some special efforts will be made in designing elements of the vacuum system to have minimum impedance like it has been made for B-factories at SLAC [19] and at KEK [22] (shielding of bellows, tapering of all shape transitions, screening of vacuum pumps and ports, etc.).

Bellows To be conservative, we assume bellows in the ring about every 8 meters, thus their total number is about $N_{bl} \simeq 120C$[km].

The TBCI calculation of the shielded bellows impedance for the B-factory in SLAC gives inductance of $L = 5.6 \cdot 10^{-2}$ cm and $k_l = 2.2 \cdot 10^{-3}$ V/pC per bellows with radius $b \simeq 3$ cm [21]. Therefore we may expect the total inductance of a bellows

$$\left(\frac{Z}{n}\right)_{bl} = -i30\frac{L}{R} \times N_{bl} \simeq \ i12.6\, mOhm \tag{40}$$

for the PETRA-tunnel ring and (with taking into account $L \propto 1/b$ scaling) about $-i7.5$ mOhms for the "dog bone" with $< b > \simeq 5$ cm.

The loss factors are about $k_l \simeq 0.6$ V/pC for PETRA-ring option and $k_l \simeq 5.3$ V/pC for the "dog-bone".

Beam Position Monitors (BPMs) Assuming 4 BPMs per betatron wavelength, the total number N_{BPM} can be roughly estimated to be about 350 for dog-bone (80 in arcs and 30 in wiggler section where $< \beta > \simeq 10$m, and 240 in straight sections with $\beta \simeq 60$ m) and about 140 for PETRA ring ($< \beta > \simeq 10$m).

Four-button BPMs with the button radius a=1.5cm and slot of w = 2 mm give inductive impedance [23]

$$\left(\frac{Z}{n}\right)_{BPM} = -i\frac{Z_0 w a^2}{\pi C b^2} \times N_{BPM} \times 4 \tag{41}$$

that is about $-i4.5$ mOhm for dog-bone and $-i16.2$ mOhm for PETRA DR.

The longitudinal loss factor can be estimated by formula [23]

$$k_l = \frac{\alpha_e^2 + \alpha_m^2}{(2\pi b)^2 \sqrt{\pi}\sigma_s^{3/2}}, \tag{42}$$

where the electric and magnetic polarizability of the single button are about $\alpha_e \sim \alpha_m \sim wa^2$. Thus, we estimate $k_l = 0.3$ V/pC for the PETRA-DR and $k_l = 0.8$ V/pC for the "dog-bone".

Kickers The design of the TESLA damping ring intends to install several kickers: at least two feedback kickers (longitudinal and transverse) and kicker(s) for injection ejection. As the designs of the kickers are not considered in details at present time, we present some estimations based on a simple broad band resonator model of the kicker as a pair of discontinuities with characteristic size of about the beam pipe radius, i.e. about $b \simeq 3$ cm. Following [14], we can estimate the kickers' impedance as

$$\left(\frac{Z}{n}\right)_K \simeq -i\frac{Z_0}{2\pi n_{eff}} = -i\frac{7.5[m\Omega]}{C[km]}, \tag{43}$$

that is about 10 mOhms for the three kickers at the PETRA version ring and about 1 mOhm for the 20-km long machine (here $n_{eff} = R/\sigma_s \simeq 1.6 \cdot 10^4 C[km]$ is the effective bunch harmonics as in Eq.(38) above).

The loss factor of the three kicker is about

$$k_l \simeq \frac{4ln(2)}{b} \times 3 \approx 2.5\, V/pC. \tag{44}$$

Other elements At this early stage of the machine design we can not study impedance budget in details, and below we will assume that all other sources (pumping slots, synchrotron radiation absorbers,, etc.) contribute about 20% of the inductive impedance that is close to the SLAC B-factory impedance budget [19]. All above mentioned estimations of the longitudinal impedance are collected in Table 3.

Table 3: Impedance Budget of TESLA Damping Ring

Parameter	PETRA -DR Z/n, mΩ	PETRA -DR k_l, V/pC	"Dog -bone" Z/n, mΩ	"Dog -bone" k_l, V/pC
Threshold micro-wave instability	200		86	
RF Cavities	5.6	7.2	3	34.2
Resistive wall	6	1.3	5.2	7.6
Total non-inductive	$\simeq$ 12		$\simeq$8	
10% lengthening	40		17	
Bellows	13	0.6	7.5	5.3
BPMs	16.2	0.3	4.5	0.8
Kickers	10	2.5	1	2.5
Other induct.	10	~1	~ 4	~2
Total inductive	50		17	
Loss factor		~13		~52
Loss power, kW		66		31

3.2 Transverse Impedance

The transverse mode-coupling (or "strong head-tail") instability leads to fast beam break-up, and, therefore, sets strong constraint on transverse impedance (see,e.g. [12]):

$$\mathrm{Im} < Z_\perp \beta_\perp >^{th} = \frac{16\sqrt{\pi}(E/e)\sigma_s \nu_s}{3 I_s R} \tag{45}$$

where brackets < .... > mean averaging with the beta-function at the locations of the impedance sources. With the parameters of the two options of the TESLA DR, the threshold $< Z/\beta >$ is 24.6 MOhms for the 2.3-km ring and about 96 MOhms for the "dog-bone".

The transverse impedance can be estimated with use of certain relation to the longitudinal impedance [14]

$$Z_\perp \simeq \frac{2R}{b^2}\left(\frac{Z}{n}\right). \tag{46}$$

This is exact equation for the resistive wall impedances. Using values of Z/n from the preceding section, we have calculated the transverse impedance budget which is presented in Table 4. The value of beta-function at kickers equal to $\beta \simeq 150$ m was used.

Table 4: Transverse Impedance Budget

Parameter	PETRA-DR	"Dog-bone"
Threshold Im $< \beta_\perp Z_\perp >_{thr}$, $k\Omega$	24,600	96,000
RF Cavities, $k\Omega$	15	70
Resistive wall, $k\Omega$	80	880
Bellows, $k\Omega$	110	1140
BPMs, $k\Omega$	130	280
Kickers, $k\Omega$	1,230	1,060
Other, $k\Omega$	80	610
Total, $k\Omega$	$\simeq$ 1,600	$\simeq$4,000

4 Coupled-Bunch Instabilities

Long range wakefields in a storage ring cause different bunches to interact. For certain values of relative phase between bunches, the coupled-bunch motion can be unstable, that leads to the beam loss. The instabilities take place in longitudinal (synchrotron) motion as well as in transverse planes. The beam remains stable if radiative damping times (about 18 ms and 37 ms for longitudinal and transverse oscillations respectively) are smaller than the instability growth times. Otherwise, a feedback system is necessary. The major contributors to the bunch coupling in a positron storage ring are resistive wall wake-fields, main accelerating mode and high order modes (HOMs) in the RF cavities, and slow photoelectrons produced by synchrotron radiation photons.

4.1 Longitudinal Instability

The resonant frequencies for the longitudinal case are $\omega_{pk} = \omega_0(pN_b + k + \nu_s)$ where N_b is the total number of bunches. If the ring impedance $Z(\omega)$ is not zero at these frequencies, then the growth rate (increment) of longitudinal instability at the mode number $k = 0, 1, 2 \ldots N_b - 1$ is equal to [14]

$$\frac{1}{\tau_{||}^k} = \frac{\alpha I_B f_0}{2(E/e)\nu_s} \sum_{p=-\infty}^{+\infty} (pN_b + k + \nu_s) \cdot \mathrm{Re}Z(\omega_0(pN_b + k + \nu_s)), \tag{47}$$

here I_B is the total beam dc current. To emphasize the bunch spectrum, one has to use the effective impedance $Z(\omega) = Z_{||} \cdot e^{-(\omega\sigma_s/c)^2}$.

Resistive Wall The longitudinal impedance due to resistive wall scales as

$$\mathrm{Re}Z_W = \frac{Z_0\delta(\omega)\omega}{2b\omega_0} \propto \frac{\sqrt{C \cdot n}}{b}. \tag{48}$$

It is equal to $\mathrm{Re}Z_W[Ohm] = 1.15 \cdot \sqrt{n}$ for the 2.3-km ring and about $\mathrm{Re}Z_W[Ohm] = 2.9 \cdot \sqrt{n}$ for the 20-km "dog-bone".

The summation in Eq.(47) has been performed numerically [24]:

$$\sum_{p=-\infty}^{+\infty} (pN_b + k + \nu_s) \cdot \sqrt{pN_b + k + \nu_s} = N_b^{3/2} F(k/N_b), \tag{49}$$

where the maximum value of the function $F(k/N_b)_{max} \approx 0.038$ at $k \approx 0.19 N_b$ see Fig.1.

Thus, the maximum increment is equal to $\frac{1}{\tau_{||}}\,[s^{-1}] = 7.5 \cdot 10^{-4}$ for the PETRA-tunnel DR and about $3 \cdot 10^{-6} s^{-1}$ for the "dog-bone", i.e. the instability is much weaker than the synchrotron radiation damping.

Instability due to Accelerating Mode In storage rings, the cavity tuning is usually set to compensate the reactive part of the beam loading and to minimize the required RF power. Without this detuning, large amount of the RF power is reflected from the cavity. The optimum detuning is given by (see e.g. [12, 27]):

$$\Delta f = -\frac{I_B\sqrt{1 - 1/q^2}}{2V_1} \cdot \left(\frac{R}{Q}\right) \cdot f_{RF} \tag{50}$$

that is 2.9 kHz for the "dog-bone" and 28 kHz for the PETRA-tunnel DR (or about 1/5 of the revolution frequency).

This shift leads to unequal impedances of the upper and the lower synchrotron side-bands:

$$\Delta\mathrm{Re}Z = \mathrm{Re}Z_+ - \mathrm{Re}Z_-, \quad \mathrm{Re}Z_\pm = \frac{R_s}{1 + 4Q_L^2 \frac{(\Delta f \pm \nu_s f_0)^2}{f_{RF}^2}}, \tag{51}$$

and accordingly to (47), the instability growth rate is equal to

$$\frac{1}{\tau_{\|}} = \frac{\alpha I_B f_0}{2(E/e)\nu_s} \cdot \Delta \mathrm{Re} Z. \tag{52}$$

Fig.2 presents the increment versus normalized beam current I/I_B for the "dog-bone" (I_B=0.1 A, lower curve) and 2.3-km ring (I_B=0.87 A, upper curve). The parameters used in calculations are $(R/Q) = 100$ Ohms, $R_s = 4$ MOhms, $Q_L = 5000$, $V_1 = 0.65$ MV, other parameters accordingly to Table 2. For the design current, the increment is about 4000 s^{-1} for the PETRA-tunnel ring, and 22 s^{-1} for the 20-km long ring.

HOMs in RF Cavities Instability time due to higher order modes in the RF system can calculated using Eq.(47)

$$\frac{1}{\tau_{\|}}[s^{-1}] = \frac{18.6 \cdot 10^{-2} N_{cav}}{C^2[km]} \sum_{res} f_r[GHz] \cdot (\frac{R}{Q})_{HOM} \cdot Q_L = \frac{1.35 N_{cav}}{C^2[km]} Q_L, \tag{53}$$

where we used the parameter $(R/Q)_{HOM} \simeq 60$ Ohms and HOM effective frequency of the order of $f_r \sim 1.2$ GHz. Recent R&D on HOMs damping [19, 22] let us hope that the damped value of HOM's quality factors can be as small as $Q_L \simeq 100$. Therefore, we get the growth rate of 190 s^{-1} for the PETRA-tunnel ring and about 13 s^{-1} for the "dog-bone".

4.2 Transverse Instability

The resonant frequencies in transverse case are $\omega_{pk} = \omega_0(pN_b + k + \nu_\perp)$ where $\nu_\perp$ is the betatron tune (vertical or horizontal). Increment of the transverse coupled-bunch instability can be calculated accordingly to [14]

$$\frac{1}{\tau_\perp^k} = \frac{I_B f_0}{(E/e)} \sum_{p=-\infty}^{+\infty} \beta_\perp \mathrm{Re} Z_\perp(\omega_0(pN_b + k + \nu_\perp)) \approx \frac{180 \sum \beta[m] \mathrm{Re} Z_\perp[MOhm/m]}{C^2[km]}[s^{-1}], \tag{54}$$

(the second equation corresponds to the design parameters of the TESLA DR).

Resistive Wall Instability Using Eq.(46, 35), the transverse resistive wall impedance of the copper vacuum chamber of the 2.3-km TESLA DR is estimated to be equal to

$$\beta \mathrm{Re} Z_\perp[MOhm] = 4.1 \cdot \sqrt{\frac{C^3[km]}{n}} = 14.3 \frac{1}{\sqrt{n}}, \tag{55}$$

where significant contribution (about 1/3) of the wiggler section with smaller aperture was taken into account.

For the Al chamber of the "dog-bone" ring $\beta \mathrm{Re} Z_\perp[MOhm] \approx \frac{500}{\sqrt{n}}$ (with about 30 % contribution of the wiggler and the arcs).

The maximum increment occurs at the mode number which is equal to the integer part of the tune $k = [\nu_\perp]$, thus, to derive the impedance one should replace $\sqrt{n}$ to the square root of the fractal part of the transverse tune $\sqrt{\Delta \nu_\perp}$. Taking for definiteness $\Delta \nu_\perp = 0.25$, we get $\frac{1}{\tau_\perp} = 970$ s^{-1} for the DR in the PETRA tunnel and $\frac{1}{\tau_\perp} = 450$ s^{-1} for the "dog bone" ring.

HOMs One can expect to obtain the damped transverse impedance of the HOMs to be about $Z_\perp \simeq 4 \cdot Q_L$ kOhm/m (that corresponds to the SLAC and KEK B-Factories estimations [21, 22]), and, therefore, the increment of transverse oscillations is equal to

$$\frac{1}{\tau_\perp}\,[s^{-1}] = \frac{7.2 N_{cav} Q_L}{C^2[km]}, \tag{56}$$

here we took $\beta_\perp \sim 10$ m at the RF cavities.

Therefore, with $Q_L = 100$, Eq.(56) gives us the growth rate of $1100\,s^{-1}$ for the PETRA-DR and about $70\,s^{-1}$ for the "dog-bone".

Instability due to Slow Photoelectrons This instability was observed experimentally at the KEK Photon Factory positron ring and commonly considered as caused by secondary slow photoelectrons which are born on the surface of the vacuum chamber under the synchrotron radiation. These electrons with energies of 1–10 eV come to the beam orbit, live there for some time longer than several bunch spacing times, and, therefore, cause long range wakefield.

Table 5: Slow Photo-electrons Instability

Parameter	KEK-PF	PETRA-DR	"Dog-bone" arcs
Energy, GeV	2.5	3.3	3.3
e^+/bunch, N_e	$1.2 \cdot 10^9$	$3.7 \cdot 10^{10}$	$3.7 \cdot 10^{10}$
Mean radius R, m	8.5	366	64
$u_c = (3\hbar c\gamma^3)/(2R)$, keV	4.16	0.22	1.3
Loss per turn, U_0, keV	415	29	495
$N_\gamma = (15\sqrt{3}U_0)/(8u_c)$	320	425	1240
Tune $\nu_\perp$	3.3	~40	~60
Increment $\tau_\perp^{-1}$, s^{-1}	100	360	480

The developed theory of the instability [25] contains several not well known parameters (such as spectrum of secondary photoelectrons, quantum efficiency of their production, etc.). Therefore, here below we use only the scaling of the theory and experimentally observed increment of vertical coupled bunch motion at the KEK PF equal to $1/\tau_{PF} \simeq 100\,s^{-1}$, and try to apply it to the parameters of the TESLA DR.

Accordingly to [25], the increment of the instability is proportional to

$$\frac{1}{\tau_\perp} \propto \frac{\eta N_\gamma N_e}{\gamma \nu_\perp}, \tag{57}$$

where η is quantum efficiency of the electron production, N_γ is number of emitted γ-quanta per turn per one positron. Table 5 presents calculated increments for the two options of the TESLA DR where the effect can take place in arcs. One can see from this Table, that the effect of slow photoelectrons is not negligible, and may lead to 2–3 ms instability growth time. Nevertheless, it was quickly realized that external magnetic field of the order of 10 G can sufficiently deflect the trajectories of these electrons and avoid the instability [25]. Another questionable point is that the instability does not take place in high-current multibunch positron rings at DESY (PETRA and HERA-e) [26]. The issue of the photoelectron-induced instability in the TESLA DR needs further investigation.

4.3 Summary on Coupled-Bunch Instabilities

The results on the coupled-bunch instability increments are summarized in Table 6.

Table 6: Multibunch Instabilities at TESLA DR

	PETRA-DR	**"Dog-bone"**
Longitudinal $\frac{1}{\tau}$		
Main mode	**4000 s^{-1}**	**22 s^{-1}**
HOMs, Q=100	**190 s^{-1}**	**13 s^{-1}**
SR decrement	**55 s^{-1}**	**55 s^{-1}**
Transverse $\frac{1}{\tau_\perp}$		
HOMs, Q=100	**1100 s^{-1}**	**70 s^{-1}**
Resist. wall	**970 s^{-1}**	**450 s^{-1}**
Slow photo e^-	**360 s^{-1}**	**480 s^{-1}**
SR decrement	**27 s^{-1}**	**27 s^{-1}**

One can see that for the both options of the TESLA DR a transverse feedback system is necessary. Without taking into account the photoelectron-instability, the feedback damping time has to be about 0.5 ms for the PETRA-DR and less than 2 ms for the "dog-bone" ring. Note that 0.1 – .5 ms damping times were achieved with the use of the feedback at PETRA and HERA electron storage rings [28]. Main contributors to the multibunch impedance of the TESLA DR are higher order modes of the RF cavities and resistive walls of the vacuum chamber.

Our estimations show that with the damping of the HOMs down to $Q \sim 100$, the longitudinal instability increment in the "dog-bone" option is smaller than the damping decrement due to synchrotron radiation: hence, we could avoid a longitudinal feedback there. For the ring in the PETRA tunnel, the feedback will be necessary, otherwise the value of Q_{HOM} should be down to ~20 that seems to be a hard task. Nevertheless, an example of the KEK B-factory HOM-damped cavity shows that many transverse modes can be damped down to $\sim 10 - -40$ [29]. Another possible factor to reduce the required strength of the feedback system could be stronger focusing at the RF cavities. Altogether, these measures can increase the transverse HOM instability growth time to some dozens of milliseconds, and these possibilities should be carefully checked in computer simulations and/or verified experimentally.

The longitudinal instability due to the main accelerating mode is found to be unacceptably fast at the PETRA DR option Nevertheless, several ways to avoid this effect are currently used in high current storage rings. First, one can use superconducting RF cavities which typically have much smaller (R/Q) [27]. Accelerating cavity coupled with large storage cavity [30] also allows drastic reduction in the instability increment [22]. Finally, a local RF cavity feedback may counteract the effect of the detuning [31].

5 Conclusions

Major collective effects are considered for the two proposed options of damping ring for the TESLA linear collider. The first option is the so called "dog-bone" ring with ~20 km circumference; another one fits into the existing 2.3-km tunnel of PETRA at DESY. To enforce the radiation cooling, some 2% of the circumference of the ring should allow for the inclusion of 1.5 T wiggler. To compensate the synchrotron radiation losses an advanced RF system has to be installed. On the basis of the linac requirements, we conclude that the RF system should contain from 8 to 38, 433 MHz cavities and simultaneously allow to damp effectively higher order modes of these cavities.

Preliminary evaluation of the broadband impedance budget has shown that single bunch collective effects don't lead to severe constraints for both options for the TESLA DR (although not all possible sources were considered in detail).

The designed single bunch current is found to be more than 10 times less than the microwave longitudinal turbulent instability and transverse mode-coupling instability thresholds. The estimation of the inductive impedance allows to predict some 10% bunch lengthening due to potential RF well distortion which is still acceptable.

More severe constraints are due to multibunch effects. A transverse feedback system with about 0.5 ms damping time is necessary for the 2.3-km ring and with some 2 milliseconds for the "dog-bone" DR. The major source of the transverse narrow band impedance of the "dog-bone" ring is the resistive vacuum chamber walls, while for the 2.3-km ring more than a half of the impedance comes from high order modes in the RF cavities. We expect that damping of HOMs of the cavities down to $Q \sim 100$ will allow us to avoid a longitudinal feedback system for the 20-km-long ring. A feedback system seems to be necessary for the 2.3-km ring, because the requirement to have quality factors of $Q \leq 20$ for all HOMs looks to be about the limit of present RF technology. Stronger beam focusing at the RF cavities also can reduce the instability increment. Certainly, some measures (for example, a local feedback in the RF system) have to be taken in order to prevent fast longitudinal instability in the PETRA-DR driven by the main accelerating mode.

We would like to mention that despite the serious efforts to maintain multibunch stability in the PETRA-tunnel damping ring option, the problems are significantly easier than in the low-energy rings of asymmetric B-factories presently under construction at SLAC and KEK, which intend to keep stable a factor of 3 times more current.

Further work on the issues of the beam stability should include:

more detailed calculations (based on certain engineering design) of numerous contributions to the broadband impedance;

numerical simulation and (desirable) experimental verification of the HOMs damping in the RF structure for the damping ring;

transient beam loading effects in the RF cavities during bunch-by-bunch ejection/injection;

simulations and design of the feedback systems to maintain longitudinal and transverse beam stability;

study of beam-ion collective instability in the positron ring [32];

thorough investigation of the photoelectron-induced instability.

Acknowledgment

Author would like to thank R.Brinkmann, R.Wanzenberg and N.Walker for careful reading the manuscript, numerous discussions and useful corrections.

References

[1] R. Brinkmann, *Proc. of IEEE PAC'95*, Dallas (1995), see also DESY Internal Report M-95-08 (1995), pp.57-59.

[2] K. Flöttmann, DESY 93-161 (1993).

[3] J. Rossbach, *Nucl. Instr. Meth.*, A309, p.25 (1991).

[4] E. Colby, et.al, *Proc. of IEEE PAC'95*, Dallas (1995).

[5] R. Brinkmann, J. Pflüger, V. Shiltsev, N. Vinokurov, and P. Vobly, DESY TESLA Print 95-24 (1995).

[6] K. Flöttmann, J. Rossbach, *Proc. of EPAC'94*, London, vol.1, p.503 (1994), see also DESY Internal Report 94-03-K (1994).

[7] J.P. Delahaye, J.P. Potier, *Proc. of EPAC'94*, London, 1994, and CERN/PS 94-15 (LP).

[8] D.G. Koshkarev, DESY Print, TESLA 95-16, 1995.

[9] P. Zenkevich, DESY Print TESLA 95-15 (1995).

[10] V. Shiltsev, DESY Print TESLA 95-22 (1995).

[11] V. Shiltsev, talk at the TESLA Collaboration meeting, DESY October 1995; high voltage pulse generators of *Kentech Technology, England.*

[12] H. Wiedemann, Particle Accelerator Physics I, Springer-Verlag (1993).

[13] P. Emma, DESY TESLA Print 95-17 (1995).

[14] A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, *John Wiley & Sons, Inc.* (1993).

[15] A.W. Chao, J. Gareyte, SLAC Int. Note SPEAR-197 (1976).

[16] S. Heifets, SLAC/AP-93 (1992).

[17] R.D. Kohaupt, *IEEE Transact. Nucl. Sci.*, Vol.NS-26, No.3, p.3480 (1979).

[18] T. Weiland, *Particle Accelerators*, Vol.15, p.245 (1984).

[19] S. Heifets, et. al, SLAC/AP-99 (1995).

[20] R. Klatt, T. Weiland, DESY M-84-06 (1984).

[21] *PEP-II : An Asymmetric B-Factory. Conceptual Design Report*, LBL-PUB-5379 and SLAC-418, SLAC (June 1993).

[22] *KEK B Factory Design Report*, KEK Report 95-7 (1995).

[23] S. Kurennoy, IHEP 92-84, UNK, Protvino (1992).

[24] M.M. Karliner, N.V. Mityanina, V.P. Yakovlev, *Proc. of HEACC'92*, Hamburg, p.1142 (1992).

[25] K. Ohmi, *Phys. Rev. Lett.*, v. 75, No. 8, p.1526 (1995).

[26] R. Brinkmann, private communication.

[27] F. Pedersen, in *Lecture Notes in Physics 425: Factories with e^+-e^- Rings*, Springer-Verlag (1992), p.269.

[28] D. Heins, et. al, DESY 89-157 (1989).
R.D. Kohaupt, DESY 91-071 (1991).

[29] K. Hanaoka, KEK-Report 95-10, (1995).

[30] T. Shintake, *Part. Accelerators*, v.34, (1994), p.131.

[31] F. Pedersen, *IEEE Trans. Nucl. Sci* NS-32 (1985), p.2138.

[32] F. Zimmermann, et.al, *Proc. 1995 IEEE Part. Accel. Conf.*, Dallas (1995).

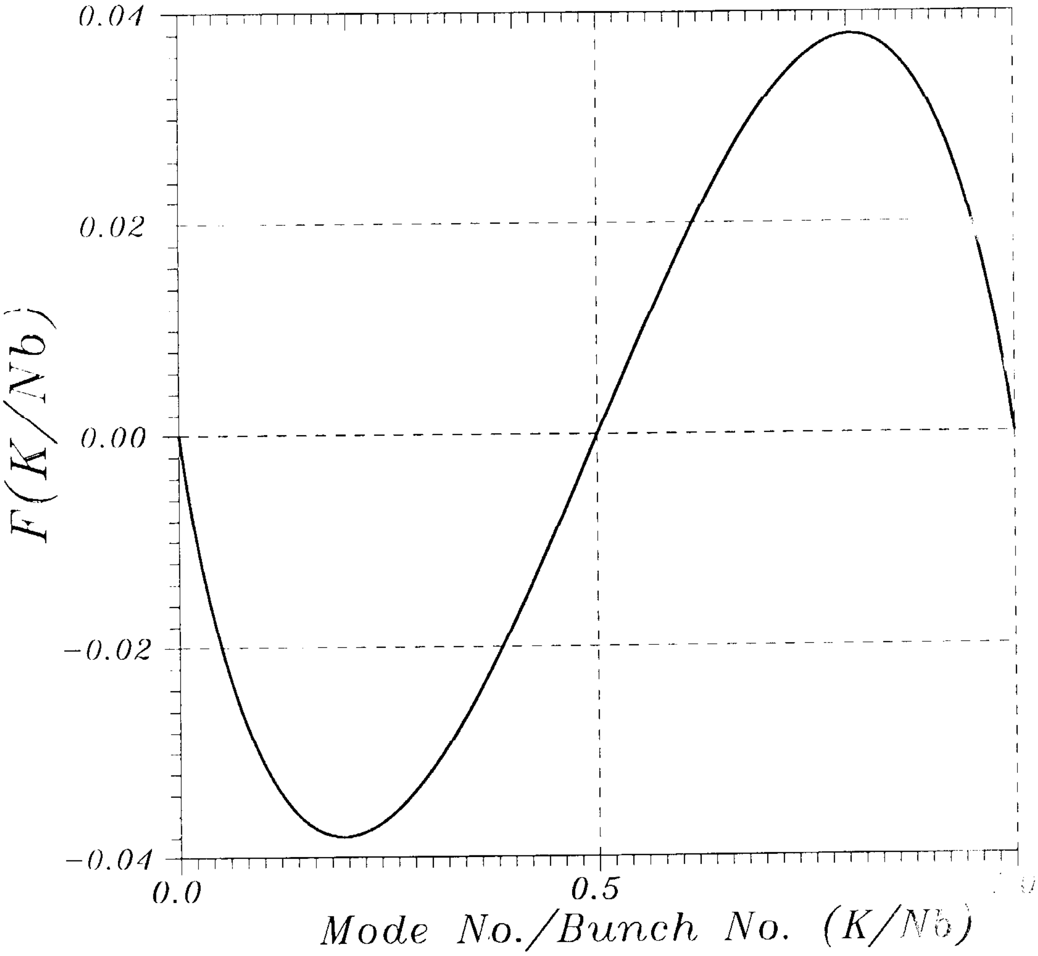

Fig.1: Function $F(k/N_b)$ for calculation of the longi-
udinal instability due to resistive wall (see in text).

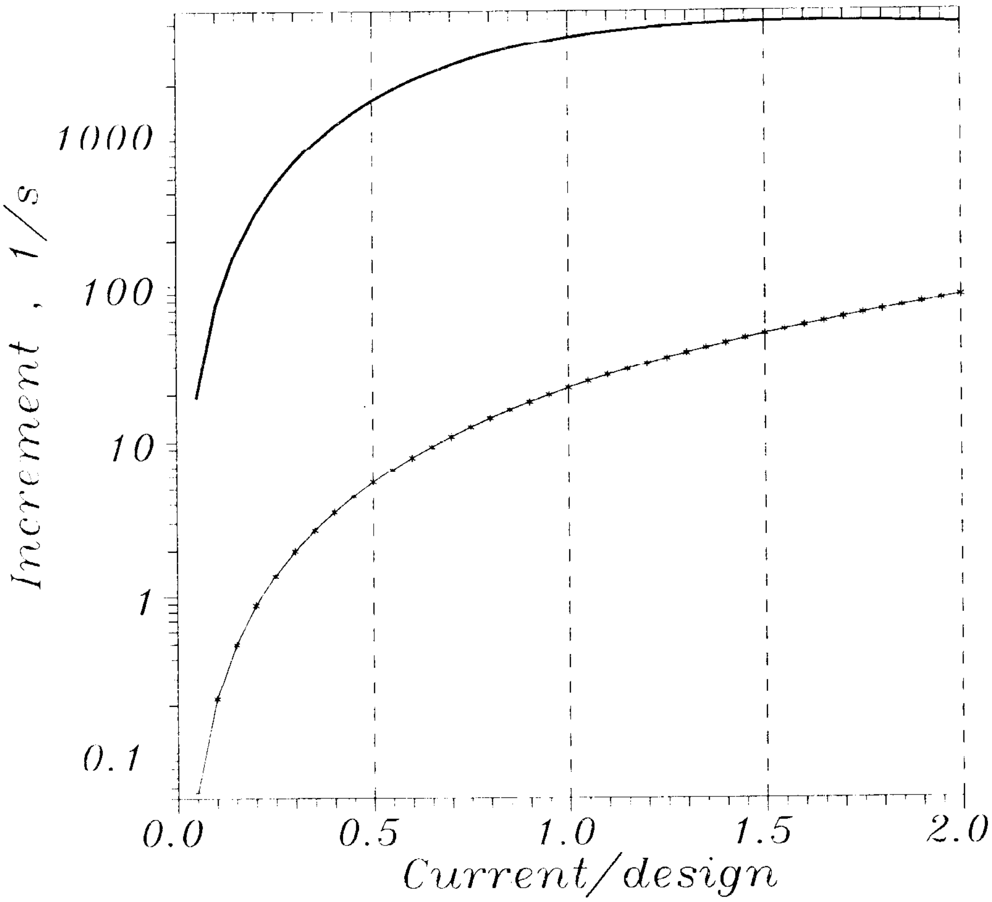

Fig.2: Growth rates of longitudinal instability due to main accelerating mode: upper curve – for 2.3-km circumference TESLA damping ring, lower curve – for the "dog-bone" option with $C = 20$ km