Damping Rings for TESLA
(Design Considerations)

P.R.Zenkevich *
MPY, DESY,
Hamburg, Germany

Abstract. *Two options of damping rings for supercondacting linear collider TESLA have been considered: 1) "classical" electron ring with $L = 6335m$ which is equal to a circumference of HERA electron ring; 2) "dogbone" ring with long straight sections placed within a tunnel of the main linac and two small rings, connecting this straight sections from both ends. An attempt of a systematical design analysis has been made for both options. Calculated main ring parameters (including longitudinal and transversal impedances) are listed in Tables. For HERA-like ring it is represented an example of linear lattice calculated by COMFORT code.*

1 Introduction

A damping ring for TESLA is to produce a very long train of bunches ($\sim 240\ km$) for injection in the main superconducting linac. To obtain such long train we should to stretch the beam by use of a special ejection system. According to a present status of kicker technology a time interval between bunches has to be within a range from 25 ns up to 75 ns that corresponds to a ring circumference L from 6.0 km up to 20 km.

We consider two extreme cases:

1) "classical" electron synchrotron with $L = 6335m$ (this length coincides with a circumference of HERA electron ring in order to make possible a use of the existing tunnel and magnets of the existing ring);

2) so named "dogbone" ring ([1], [2]) with long straight sections placed within a tunnel of the main linac and two small rings, connecting these straight sections from both ends (in the further text our two options will be nominated as "HR" ring and "DB" ring, correspondingly).

*Permanent address: Institute of Theoretical and Experimental Physics, Moscow,Russia

A choice of these rings properties represents a complicated problem of many parameters, and its solution depends, as it is well-known, on a designer art. Nevertheless let us make an attempt of systematical design analysis which is useful from tutorial point of view and permits to clear up the most important features of such rings.

In the first part we discuss a geometry of the rings and list its initial parameters. Then we investigate a dependance of the main rings parameters on the "free" parameter (beam energy E) and, at the last chapter, we give the examples of the magnetic lattices for chosen beam energy.

2 Initial Parameters and Rings Geometry

It is convenient to divide all the initial parameters on three groups:

1) parameters defined by the main linac requirements;

2) parameters connected with chosen technology of the ring systems (RF system, wiggler and so on);

3) parameters defined by the chosen ring geometry (length of the arcs and the straight sections, radii of curvature etc).

Let us discuss a choice of one of the initial parameters: longitudinal beam emittance ϵ_z, which value is determined by requirements of the subsequent bunch compression up to small value $\sigma_z^l = 0.1\ cm$. An energy spread during the compression σ_E^c is defined by $\sigma_E^c \triangleq \epsilon_z/\sigma_z^l$. If we guess that maximum permissible value $\sigma_E^c = 10^{-2}$, than we obtain $\epsilon_z = 10^{-5}m$.

The list of the initial parameters is given in Table 1, the list of the rings geometrical parameters is given in Table 2.

Let us discuss as well a chosen geometry of the rings. For HR we tried to preserve the same geometry as in the existing HERA electron ring. The ring consists of four arc quadrants connected by four long straight sections.

DB includes two small rings with circumference each equal to $1 km$. The ring consists of one arc with bending angle 300° and two sections which bend in the opposite direction (bending angle of such section is equal to 60°). All sections are made from the similar magnets with the same radius of the curvature.

Table 1: List of initial parameters

denom	*name*	*quantity*	*dimension*
ν_{rep}	repetition frequency of main linac	10	Hz
N	number of particles per bunch	$5 \cdot 10^{10}$	
N_b	number of bunches in train	800	
ϵ_y^N	normalized vertical emittance in main linac	10^{-6}	m
ϵ_i^N	normalized emittance of injected beam	10^{-2}	m
λ_w	length of wiggler period	0.4	m
B_w	magnetic induction in wiggler	1.5	T
λ	wavelength of RF field	0.6	m
ϵ_z	beam longitudinal emittance	10^{-5}	m
σ_z^l	bunch length in main linac	0.1	cm

Table 2: List of rings geometrical parameters

denom	*name*	HR	DB	*dimension*
L_a	length of arcs	4700	2700	m
L	ring circumference	6.335	20	km
ρ_a	arc radius of curvature	600	110	m
α_a	arc bending angle	360	840	*degree*

3 Ring Characteristics

As it is well-known [3], main properties of the stored beam in the electron storage ring are defined by the following formulae:

$$U_s = \frac{2}{3} r_e m_0 c^2 \gamma^4 I_2 \tag{1}$$

$$\sigma_E^2 = c_q \gamma^2 I_3 / I_2 G_s \tag{2}$$

$$\epsilon_0^N = c_q \gamma^3 I_5 / I_2 G_x \tag{3}$$

Here

U_s - energy losses per turn,
σ_E - energy spread of the beam
($\sigma_E = \sqrt{< \Delta E/E >^2}$),
ϵ_0^N - normalized equilibrium emittance,
$c_q = 55h/3\sqrt{3}mc = 3.84 \cdot 10^{-13} m$,
G_s, G_x - damping partition numbers, which are defined by:

$$\begin{aligned} G_s &= 2 + I_4/I_2 \\ G_x &= 1 - I_4/I_2 \end{aligned} \tag{4}$$

The integrals are given by:

$$I_2 = \int ds/\rho^2 \tag{5}$$

$$I_3 = \int ds/|\rho|^3 \tag{6}$$

$$I_4 = \int \frac{\psi(s)}{\rho^2}\left(\frac{1}{\rho} + \frac{2}{B}\frac{\partial B}{\partial x}\right) ds \tag{7}$$

$$I_5 = \int H ds/|\rho|^3 \tag{8}$$

$$H = \frac{1}{\beta}[\psi^2 + (\beta\psi' - \frac{1}{2}\beta'\psi)^2] \tag{9}$$

Here ρ is the radius of curvature in magnets, B is the magnetic induction, β is β-function, ψ and ψ' are dispersion function and its derivative, correspondingly.

Firstly, let us find U_s. Comparing a normalized beam emittance of the injected beam ($0.01m$) and the vertical normalized equilibrium emittance of the ring ($10^{-6}m$) we see that we need about five vertical damping times τ_d to reach the required equilibrium emittance. Thus we obtain:

$$U_s = 10\nu_{rep}\frac{L}{c}E \tag{10}$$

Radiation losses are the sum of radiation losses in the arcs and in the wiggler. Let us introduce parameters ρ_{eff} and y, where

$$\rho_{eff} = 2\pi / \int_0^L \frac{ds}{\rho_a^2} = \frac{2\pi}{\alpha_a}\rho_a \tag{11}$$

$$y = \frac{2\pi\rho_w^2}{\rho_{eff}L_w} \tag{12}$$

Here ρ_{eff} (effective radius of the arcs) is determined by a chosen ring geomrtry: for HR $\rho_{eff} = \rho_a = 600m$, for DB $\rho_{eff} = \rho_a/2 \cdot \frac{7}{6} = 47m$.

Parameter y represents a relation of radiation losses in the arcs to the radiation losses in the wiggler. To find this parameter, we may write Eq.(1) in the following form:

$$U_s = \frac{2}{3}r_e m_0 c^2\gamma^4(\frac{2\pi}{\rho_{eff}} + \frac{L_w}{\rho_w^2}) = \frac{2}{3}r_e m_0 c^2\gamma^4\frac{2\pi}{\rho_{eff}}\frac{1+y}{y} \tag{13}$$

Comparing Eq.(10) with Eq.(13), we obtain:

$$y = \frac{1}{b(\gamma)} - 1 \tag{14}$$

$$b(\gamma) = \frac{15L\nu_{rep}\rho_{eff}}{2\pi c r_e\gamma^3} \tag{15}$$

Now we may find σ_E^2. Taking into account that for separate function system with good accuracy $G_s = 2, G_x = 1$ and neglecting a small contribution of the arcs in integral I_3, we may write:

$$\sigma_E^2 \approx \frac{c_q\gamma^2}{2\rho_w}\frac{1}{1+y} \tag{16}$$

Taking into account that

$$\rho_w = \frac{m_0c^2\gamma^2}{eB_w}$$

we obtain the final result:

$$\sigma_E^2 = \frac{c_q eB_w\gamma}{2m_0c^2(1+y)} = \frac{A\gamma}{1+y} \tag{17}$$

Using theory of the linear synchrotron oscillations we may write:

$$\sigma_s = R\sqrt{\frac{2\pi\alpha E}{qeV_0 cos\varphi_s}}\sigma_E \tag{18}$$

Here

α - momentum compaction factor
q - harmonic number of RF field
φ_s - equilibrium phase
V_0 - amplitude of RF field

This amplitude is connected with U_s by the expression

$$V_0 sin\varphi_s = U_s \tag{19}$$

Multiplying σ_s on σ_E and using Eq.(10) and Eq.(19) we obtain:

$$\alpha = \frac{20\pi\nu_{rep} tan\varphi_s}{c\lambda}\left(\frac{\epsilon_z}{\sigma_E}\right)^2 \tag{20}$$

Parameter α permits us to find Q_{eff} - effective betatron frequency of the arcs, which, in principle, for rings with complicated geometry does not coincide with real betatron number in the arcs. ($Q^a_{eff} = \mu_c/\varphi_c$, where μ_c is a shift of betatron phase per bending cell, φ_c is a bending angle of cell). For standart $FODO$ structure

$$Q^a_{eff} = \sqrt{\frac{1}{\alpha}\frac{L_a}{L}} \tag{21}$$

The last interesting parameter of the longitudinal motion (dimensionless synchrotron frequency ν_s) is defined by the following formula

$$\nu_s = \sqrt{\frac{\alpha_q U_0 cos\varphi_s}{E}} = L\sqrt{\frac{10\nu_{rep}}{c\lambda tan\varphi_s}\alpha} \tag{22}$$

Now let us find the wiggler parameters and equilibrium emittances. Using Eq.(10) and Eq.(13) and substituting ρ_w, we obtain

$$L_w = \left(\frac{m_0c^2}{eB_w}\right)^2\frac{15L\nu_{rep}}{cr_e y}\frac{1}{1+y} \tag{23}$$

This formula differs from formula derived in paper [2] by correction factor $\frac{1}{1+y}$ which takes into account radiation losses in the arcs.

The horisontal equilibrium emittance may be also expressed through initial parameters and y. Using Eq.(3) and separating contributions from the arc and the wiggler, we may write:

$$\epsilon_x^N = \frac{\epsilon_w^N + \epsilon_a^N y}{1+y} = \epsilon_0 \tag{24}$$

Here ϵ_a^N and ϵ_w^N are the equilibrium emittances for the arc and the wiggler, correspondingly, ϵ_0 is a required value of the normalized emittance.

For the wiggler consisting of rectangular magnets ϵ_w^N is determined by

$$\epsilon_w^N = \frac{1}{48} c_q \left(\frac{eB_w}{m_0 c^2}\right)^3 \beta_w \lambda_w^2 \tag{25}$$

For standard $FODO$ structure

$$\epsilon_a^N = c_q \gamma^3 \frac{1}{Q_{eff}^3} F_1 \frac{R_a}{\rho_a} \tag{26}$$

Here F_1 is a correction factor taking into account a strong focusing (for $\mu_s = \pi/2 \quad F_1 \approx 1.2$)

We may rewrite Eq.(24) in the following form:

$$\epsilon_w^N = \epsilon_0(1+y) - \epsilon_a^N y \tag{27}$$

which permits to find the maximal value of ϵ_w^N and then, using Eq.(25) calculate the necessary valye of β_w (average β-function in the wiggler).

Let us underline that a solution of Eq.(27) has physical sense only if $\epsilon_w^N > 0$ that corresponds to the following condition:

$$\frac{\epsilon_a^N}{\epsilon_0} < b(\gamma) \tag{28}$$

Using this condition, we may find that a maximal value of γ ($\gamma = \gamma_{lim}$) is determined by

$$\gamma_{lim} = \sqrt{Q_{eff}} \left(\frac{15 L \nu_{rep} \epsilon_0}{c r_e c_q F_1} \frac{\rho_a}{R_a}\right)^{1/6} \tag{29}$$

Values of the rings parameters calculated by use of the derived formulae are given in Table 3.

4 One-bunch Instabilities

We consider microwave instability and transverse mode-coupling instability.

Table 3: Rings parameters

$Ring$	HR	DB	HR	DB	HR	DB
$Energy(GeV)$	4	4	6	6	8	8
$\gamma(10^4)$	0.784	0.784	1.17	1.17	1.57	1.57
$U_0(MeV)$	17	53.6	25.5	80	34	107
b	200	50	62.4	15.6	25.8	6.46
$y(10^{-2})$	0.502	2.04	1.63	6.85	4.03	18.3
$\sigma_E(10^{-3})$	1.15	1.14	1.394	1.346	1.60	1.50
$\sigma_z(cm)$	0.87	0.88	0.72	0.74	0.62	0.67
$\alpha(10^{-4})$	3.37	3.47	1.55	1.71	0.90	1.16
Q_{eff}	47	17	69	24	90	29
$L_w(m)$	173	535	114	360	83	230
$\rho_w(m)$	8.89	8.89	13.3	13.3	17.8	17.8
ϵ_a/ϵ_0	3.37	3.47	1.55	1.71	0.90	1.16
ϵ_w^N/ϵ_0	1.004	0.962	1.014	0.839	1.033	0.341
$\lvert\frac{Z_\parallel}{n}\rvert_{eff}^{thr}(\Omega)$	0.102	0.104	0.0845	0.0902	0.0760	0.0915
$\lvert\frac{Z_\parallel}{n}\rvert_{eff}^{exp}(\Omega)$	0.088	0.059	0.102	0.0720	0.120	0.080
$< Im(\frac{Z_\perp}{n}\beta_\perp) >_{eff}^{thr} (M\Omega)$	53	173	46	154	39	153
$< Im(\frac{Z_\perp}{n}\beta_\perp) >_{eff}^{exp} (M\Omega)$	0.097	0.20	0.11	0.25	0.13	0.28
$\nu_s(10^{-2})$	4.49	14.5	3.13	10.2	2.32	8.39

4.1 Microwave Instability

Microwave instability sometimes referred to as turbulent bunch lengthening causes an increase in both the momentum spread and the bunch length of a bunched beam. The threshold (peak) current I_p is given by

$$I_p = \frac{2\pi|\alpha|E(\beta\sigma_E)^2}{e|Z_n/n|_{eff}} \tag{30}$$

Here $|Z_n/n|_{eff}$ is effective longitudinal impedance divided by the azimuthal mode number n. Substituting formula

$$I_p = \frac{ecN}{\sqrt{2\pi}\sigma_z} \tag{31}$$

we find:

$$|\frac{Z_n}{n}|_{eff}^{thr} = \frac{(2\pi)^{3/2}|\alpha|E\sigma_E\epsilon_z}{ce^2N} \tag{32}$$

Excluding σ_E and α, we obtain that

$$|\frac{Z_n}{n}|_{eff}^{thr} \sim \epsilon_z^3/\gamma^{1/2} \tag{33}$$

We see that for given longitudinal phase volume the threshold longitudinal impedance slowly decreases with growth of γ. For numerical calculations we used Eq.(32). For $N = 5 \cdot 10^{10}$ and $\epsilon_z = 10^{-5}m$ we obtain

$$|Z_n/n|_{eff}^{thr} = 33.5\gamma\alpha\sigma_E$$

Calculated values of $|Z_n/n|_{eff}^{thr}$ are given in Table 3.

However, the impedance of the rings is also dependent on γ since a number of cavities, in correspondance with our assumptions, is proportional γ. Expected numbers of this parameter, calculated in Appendix, are also listed in Table 3. Comparing these two groups of parameters, we may conclude:

1) for HR ring expected impedances are less than the threshold ones only for $E = 4GeV$,

2) for DB ring expected impedances are less than the threshold ones for all investigated range of energies.

4.2 Transverse Mode-Coupling Instability

Because the ring is large, we must also consider the transverse mode-coupling instability, which is known to limit the single-bunch current in

PEP. This instability arises when due to imaginary part of the transverse impedance $Z_\perp$ the frequency of the synchrotron sidebands of the $m = 0$ and $m = -1$ coincides.

The threshold of this instability is defined by:

$$I_b = \frac{4(E/e)\nu_s}{< Im(Z_\perp)\beta_\perp > R} \frac{4\sqrt{\pi}}{3}\sigma_s \tag{34}$$

where β_{perp} is the β-function at the location of the impedance, sign $<>$ means an averaging over the ring, I_b - average beam current, which is connected with N_e by the following expression:

$$I_b = \frac{cN_e}{2\pi R} \tag{35}$$

Using this formulae, we obtain:

$$< Im(Z_\perp)\beta_\perp >^{thr} = \frac{32(\pi)^{3/2} E\nu_s\sigma_s}{3e^2cN_e} \tag{36}$$

For $N_e = 5 \cdot 10^{10}$

$$< Im(Z_\perp)\beta_\perp >^{thr} (M\Omega) = 2.5 \cdot 10^4 E(Gev)\sigma_s(m)\nu_s$$

Corresponding values are given in Table 3.

For rough estimation of expected values of $Z_\perp$ we may use a well-known scaling law:

$$Z_\perp = \frac{Z_n}{n}\frac{R}{b^2} \tag{37}$$

For $b = 3 \cdot 10^{-2}, \beta_\perp = 20m$ we obtain:

$$\begin{aligned} < Im(Z_\perp)\beta_\perp >^{exp} (M\Omega) &= 1.1\tfrac{Z_n}{n}(\Omega) \quad for HR \\ < Im(Z_\perp)\beta_\perp >^{exp} (M\Omega) &= 3.5\tfrac{Z_n}{n}(\Omega) \quad for DB \end{aligned} \tag{38}$$

Comparing expected and threshold numbers, we see that expected numbers for $< Im(Z_\perp)\beta_\perp >$ are on an order of magnitude less than the threshold ones. Using these estimations we restrict ourselves the investigation of the longitudinal impedances Z_n/n only.

5 Longitudinal Impedances

This chapter is written using the scheme given in [4] and Balevsky calculations of wakes for PETRA cavities. In this scheme all longitudinal impedances are represented as a series expansion over $\sqrt{\omega}$:

$$Z(\omega) = -i\omega L + R(\omega)\sqrt{\frac{\omega}{\omega_0}} + R(\Omega) + (1+i)Z'_{cav}\sqrt{\frac{\omega_0}{\omega}} \tag{39}$$

Here

ω_0 - rotation frequency in the ring

ω - circular frequency of a wave

The first term describes an inductive impedance, the second one describes a resistive impedance, the third term corresponds to a constant resistivity, the last term represents so named "high-frequency tail" of the RF cavities.

Let us underline that the term $R(\Omega)$ appears mostly due to low-frequency modes with $\omega < \omega_{max}$ (ω_{max} is parameter, defined by cavity characteristics; for PEP cavities $\omega_{max} = 2\pi \cdot 1.260 HHz$). These low frequency modes do not contribute in high frequency beam oscillations of short bunches with $\sigma \approx 1cm$ and therefore may be neglected.

Thus, we may take into account the last three terms.

a) Inductive impedance

$$\frac{Z_n}{n} = -i\omega_0 L_0 \tag{40}$$

To calculate L_0 it is necessary to find inductivity of each inductive element and then summarize them. However, in our case such procedure is difficult since we have no detailed information about a vacuum chamber and its elements.

Therefore, for preliminary estimations we used scaling based on the calculations of the impedance in ring PEP-2.

All the inductive elements may be divided on two groups: 1) elements number of which is approximately the same in each ring (injection and ejection system, and, perhaps, feedback system and tapers); 2) all the last elements whose number (and, therefore, inductivity) is proportional, roughly speaking, to a number of quadrupoles in the ring m (vacuum ports, bellows etc).

For elements of the first group:

$$(\frac{Z_n}{n})_1 \sim (\frac{Z_n}{n})_2 \cdot \frac{R_1}{R_2} \tag{41}$$

for the second group

$$(\frac{Z_n}{n})_1 \sim (\frac{Z_n}{n})_2 \cdot \frac{R_1}{R_2} \cdot \frac{m_2}{m_1} \tag{42}$$

Inductive impedance of the first group $\sim 2 \cdot 10^{-2}\Omega$, for the second group:

$(\frac{Z_n}{n})_{ind} \approx 0.042\Omega$ for HR-ring

$(\frac{Z_n}{n})_{ind} \approx 0.013\Omega$ for DB-ring

(we have used in scaling the following numbers: in PEP-2 ring whole number of quadrupoles $m_2 \approx 300$ for $R_2 = 350m$; for HR-ring and DB-ring $m_1 \approx 500$).

b) Resistive wall impedance

It is defined by:

$$\frac{Z_n}{n} = Z_0(\frac{1-i}{2})\frac{\delta}{b_{eff}} \tag{43}$$

Here

Z_0 - free space impedance ($Z_0 = 370\Omega$),

δ - depth of the skin-layer,

b_{eff} - effective radius of the chamber.

For rough estimations

$$\frac{1}{b_{eff}} = \frac{1}{2} < \frac{1}{b_x} + \frac{1}{b_y} > \tag{44}$$

For copper [5]

$$\delta = \frac{6.6cm}{\sqrt{f}} = \frac{6.6cm}{\sqrt{nc/L}} \tag{45}$$

Then

$$\frac{|Z_n|}{n} = \frac{\sqrt{2}}{2} Z_0 \frac{6.6cm}{\sqrt{nc/L}} \frac{1}{b_{eff}} \tag{46}$$

In this term a dependence on n is not very strong, and we may consider $n = R/\sigma$.

For $b_{eff} = 3cm$

$\frac{|Z_n|}{n} \sim 8 \cdot 10^{-3}\Omega$

c) High frequency impedance of the cavities

Table 4:

$\sigma_z(cm)$	$w_m(\times 10^{11}\frac{V}{A\cdot s})$	$k_\parallel(\times 10^{11}\frac{V}{A\cdot s})$
0.5	8.00	−5.75
1.5	4.74	−3.33
2.0	4.14	−2.90
2.5	3.69	−2.58

Let us try to find this impedance from Balevsky calculations of wakes which are given in Table 4.

Impedance of high frequency tail may be found from $k_\parallel$. For $k_\parallel$ we have the following formula:

$$k_\parallel = \frac{1}{\sqrt{2\pi}} \int_\infty^\infty Z_n |\rho(\omega)|^2 d\omega \tag{47}$$

Here $\rho(\omega)$ - Fourier harmonics of beam longitudinal density.

For Gaussian beam

$$|\rho(\omega)|^2 = exp(-\frac{\omega^2\sigma_z^2}{c^2}) \tag{48}$$

With account that $Z_n(\omega) = Z_n^*(-\omega)$, we obtain

$$k_\parallel = \frac{1}{\pi} \int_0^\infty Re Z_n |\rho(\omega)|^2 d\omega \tag{49}$$

Let us represent $k_\parallel$ in the following form:

$$k_\parallel = k_\parallel^0 + \frac{1}{\pi} Z' \sqrt{\omega_0} \int_{\omega_m}^\infty \frac{exp(-\omega^2\sigma^2/c^2)}{\sqrt{\omega}} d\omega \tag{50}$$

Here the first term ($k_\parallel^0$) describes resistivity of the cavities due to low frequency mode (this resistivity does not influence on high-frequency oscillation of short bunch), the second term is connected with mentioned above "high-frequency tail" of the cavities.

Changing variables in the integral

$\frac{\omega\sigma}{c} = u$

and introducig new variable

$x = \frac{\omega_m\sigma}{c}$

we obtain

$$k_\parallel = k_\parallel^0 + AF(x) \tag{51}$$

Here

$$F(x) = \frac{1}{\sqrt{x}} \int_x^\infty \frac{exp(-u^2)}{u} du \tag{52}$$

$$A = \frac{1}{\pi} \sqrt{\omega_0 \omega_m} Z' \tag{53}$$

Constants A and $k_\parallel^0$ may be found from Balevsky calculations by use of regression method. We may introduce functional form

$$\Phi = \sum_{i=1}^{4} [k_i - k_\parallel^0 - AF(x_i)]^2 \tag{54}$$

Minimising this functional, we obtain:

$k_\parallel^0 = 2.94 \cdot 10^{11}$; $\quad A = 1.025 \cdot 10^{11} (\frac{V}{A \cdot s})$

$\omega_0 = \frac{c}{350m} = 0.857 \cdot 10^6 s^{-1}$

$\omega_m = 2\pi \cdot 1.260 HHz = 0.790 \cdot 10^{10} s^{-1}$

Thus, for PEP parameters

$$Z' = \frac{\pi A}{\sqrt{\omega_0 \omega_m}} = 4k\Omega \tag{55}$$

Really Balevsky calculations are performed for one cell of the PETRA cavity consisting from 5 cells (whole length of the cavity is equal to $1.2m$). Thus, we see that whole impedance of cavity is equal to $20k\Omega$, and impedance per meter $= 16.7k\Omega/m$.

Effective impedance is equal to

$$(\frac{Z_n}{n})_{eff} = \frac{\int_{\omega_m}^\infty (\frac{Z_n}{n}) exp(\frac{-\omega^2\sigma^2}{c^2}) dn}{\int_0^\infty exp(\frac{-\omega^2\sigma^2}{c^2}) dn} \tag{56}$$

Taking into account, that

$$n = \frac{\omega}{\omega_0} \tag{57}$$

and

$$\frac{|Z_n|}{n} = \frac{\sqrt{2}\pi A}{\sqrt{\omega_0 \omega_m}} \frac{1}{n^{3/2}} \tag{58}$$

we obtain

$$(\frac{Z_n}{n})_{eff} = \frac{\sqrt{2}\pi A}{\sqrt{\omega_0 \omega_m}} \frac{\int_{n_m}^\infty exp(-\alpha^2 n^2) n^{-3/2} dn}{\int_0^\infty exp(-\alpha^2 n^2) dn} \tag{59}$$

$$\int_0^\infty exp(-\alpha^2 n^2) dn = \frac{\sqrt{\pi}\alpha}{2} \qquad (60)$$

$$\alpha = \frac{\omega_0 \sigma}{c} \qquad (61)$$

Assuming that one cavity gives $1MV$, after simple algebra we obtain final result

$$(\frac{Z_n}{n})_{eff} = \sqrt{2} F(x) Z' \sqrt{\frac{R}{R_p}} (\frac{\sigma}{R})^{3/2} U_0(MeV) \qquad (62)$$

$$F(x) = \int_x^\infty \frac{exp(-u^2)}{u^{3/2}} du \qquad (63)$$

Here $x = \omega_m \sigma / c$. Substituting numbers, we have

$$(\frac{Z_n}{n})_{eff} = 1.5 \cdot 10^2 F(x) \frac{U_0(Mev)}{R(cm)} \sigma^{3/2}(cm) \qquad (64)$$

Let us underline that due to Eq.(10) U_0/R does not depend on R, and for small σ $\sigma^{3/2} \cdot F(x)$ is also independent on σ. Calculated value of $(Z_n/n)_{eff}$ is given in Table 3.

6 Calculated Parameters of HR-ring

To illustrate common considerations the linear lattices for HR ring and DB ring have been calculated by COMFORT code. However, we describe HR ring optics only, since for DB ring there is similar lattice which was calculated earlier by R.Brinkmann.

Energy of HR ring was chosen to be equal to $4GeV$ in order to satisfy requirements connected with microwave instability and with the subsequent compression of the bunch (see Table 3). Number of cells in the arcs of the existing HERA ring is equal to 200; thus, we choice phase shift per cell $\mu_s = \pi/2$, in order to obtain $Q^a_{eff} = 50$ (this value is about $Q^a_{eff} = 47$ given in Table 3).

The ring includes four long straight sections. All sections are made dispersion free by use of the dispersion suppressors; each of them includes two cells and two magnets. Length of this magnet is approximately equal to a half length of the standard bending magnet.

In one of the straight sections it is placed the wiggler ($L_w = 155.3m$). Optics of the wiggler cells is satisfied two requirements: 1) it is produced

Table 5: Calculated parameters of HR ring

denomination	*quantity*	*dimension*
$Energy$	4	GeV
γ	$7.84 \cdot 10^3$	
$Circumference$	6335	m
ν_x	64.037	
ν_y	62.578	
α	0.00033	
L_w	155.3	m
ρ_w	8.81	m
ν_s	0.04248	
U_0	15	MV
σ_E	0.00115	
σ_s	0.923	cm
ϵ_x	$1.061 \cdot 10^{-5}$	m
N_m^a	400	
L_m	9.185	m
ρ_a	608.9	m
N_m^{DS}	16	
L_m^{DS}	45925	m
ρ_{DS}	603.6	
λ	0.6	m

β_w-function necessary to obtain the given equilibrium phase volume; 2) optics is "self-matched" to the standard cells (such optics permits to exclude special matching sections and the tapers which give a contribution in longitudinal impedance).

The last three sections have the another optics than the wiggler one. In the middle of the each section there are 6 similar cells with phase shift equal to $\pi/3$ and increased (comparing to the arc cells) β-function. These cells matched with arcs by use of the special matching sections. Such choice of the optics diminishes the chromaticity of the straight sections and, consequently, must increase the dynamical aperture. Moreover, increased β-function improves conditions for ejection and injection systems.

A plot of the optical functions (β_x, β_y and ψ_x) for straight sections is given in Fig.1 and Fig.2.

A list of the ring parameters is given in Table 5.

7 Acknowlegments

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8 Discussion

Systematic investigation parameters of the damping rings for TESLA results in a conclusion, that the optimal energy for both options is in a range $3 - 5GeV$. For the short ring ($L = 6.3km$) microwave instability is dangerous and therefore it is necessary to be careful in design of vacuum chamber, cavities and another parts of the equipment, which gives a remarkable contribution in the longitudinal impedance. Let us underline, that the threshold of microwave instability is sharply increased with a growth of the longitudinal phase volume ϵ_z, and therefore it enhances, if the compression scheme will operate with larger value of ϵ_z.

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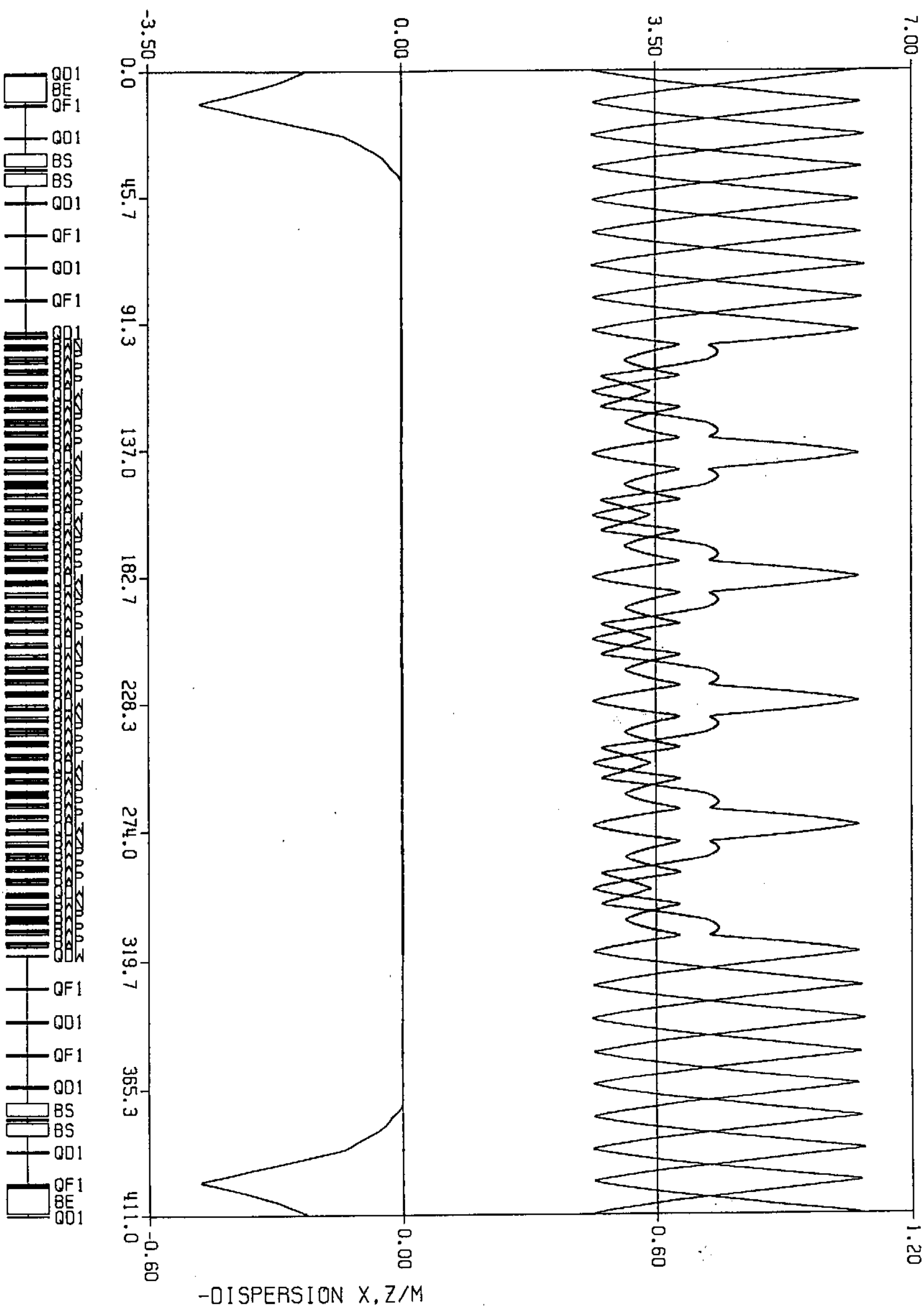

Fig. 7: HERA-E RING AS TESLA D.R., SECTION WITH WIGGLERS

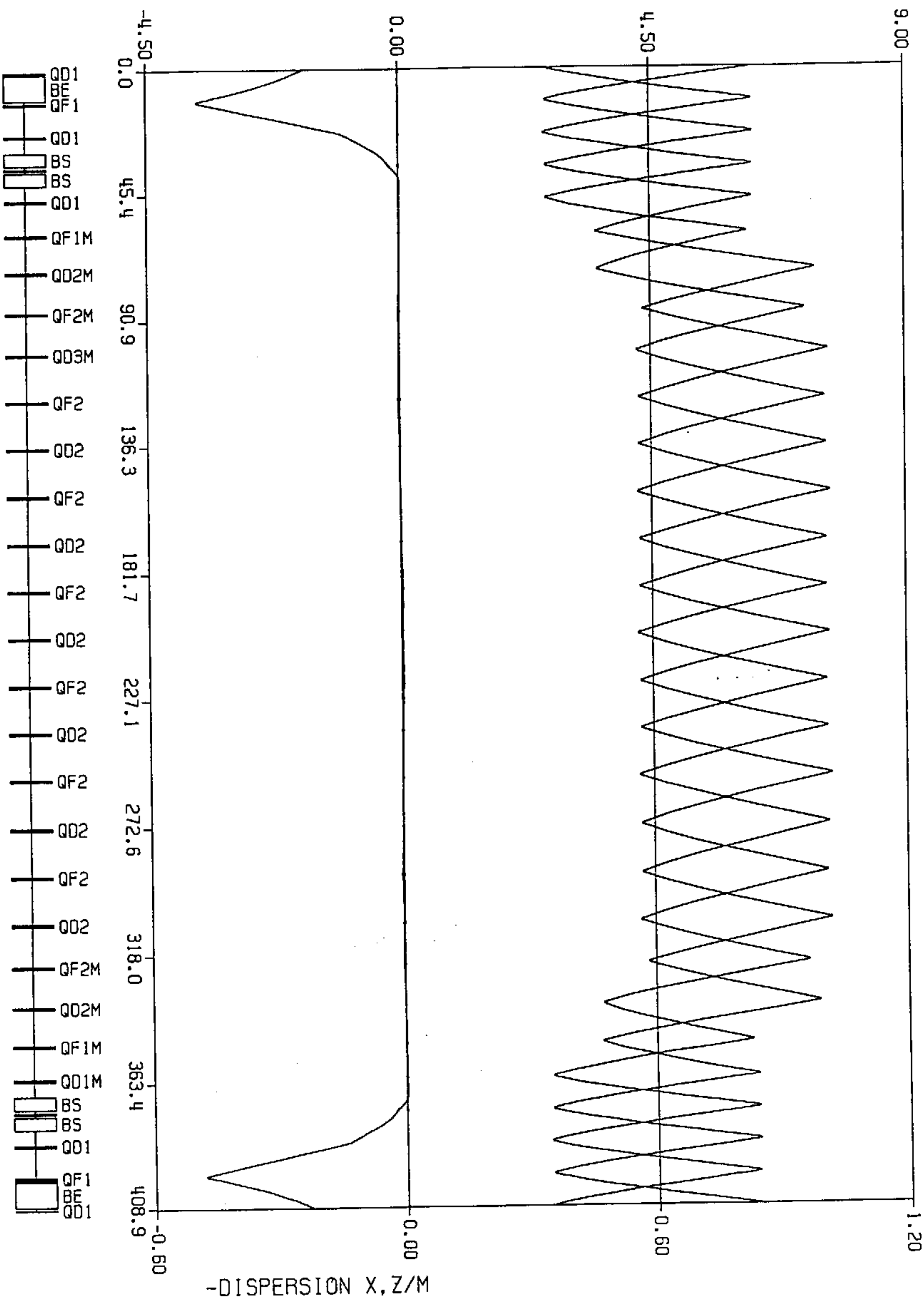

Fig. 2: HERA-E RING AS TESLA D.R., SECTION WITHOUT WIGGLERS