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Dynamic Measurements of the Lorentz Forces on a MACSE cavity

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1. Introduction

In a pulsed machine like the TESLA proposal, the cavity detuning due to the Lorentz forces can be very harmful for the stability of the rf field. For example, the presently observed cavity detuning is about 4 Hz/(MV/m)^2 and calculations on the 1300 MHz TESLA cavity with a 2.5 mm thick wall show a detuning of about 2 Hz/(MV/m)^2. But as pointed out D. Proch [1], all these measurements and calculations are steady state results and there is no data on the dynamic effect. In fact, the response of the mechanical wall deformation to the Lorentz forces is not instantaneous and and has a time constant. The purpose of the tests on a cavity of MACSE was to determine whether this mechanical time constant is small or large, compared to the typical 2 mS duration of the rf TESLA pulse.

2. Results of tests

The useful parameters of the cavity were first carefully and accurately measured :

* The loaded Q of the cavity

Figure 1 shows the phase shift of the cavity when the frequency of the master oscillator is varied, from which a Q_L of 2.08 10^7 is deduced.

* The static Lorentz forces detuning

Figure 2 shows the detuning in Hz when the accelerating field is varied, from which a factor of 4.5 Hz/(MV/m)^2 is deduced.

The input power of the cavity was then pulsed from a continuous low power of 50 W to a peakpower of 1.8 kW. The pulse duration was set to 1mS and the repetition rate was around 10 Hz. The amplitude and phase of the cavity voltage are monitored with a fast linearized PIN-diode and a phase demodulator. These devises were also cautiously calibrated and the amplitude and phase are given by

Eacc (MV/m)= 0.502 + 0.0125 V (mV)
Phase shift (deg) = 0.286 ΔV (mV)

Figures 3, 4 and 5 show the amplitude(bottom) and phase (top) signals for 3 cases : the cavity is first accurately tuned for the continuous low level, the resonance frequency is then 23 Hz down and finally 58 Hz up on the right tuning of first case. We note first that although the cavity is correctly tuned in figure 1, the phase shift does not remain to zero because of the Lorentz forces detuning. The time constant can be deduced from comparison between these measurements and simulations.

3. The cavity equations

We consider a cavity feeded by a generator with frequency ω and coupling factor β and a beam current I_0. The equivalent electric circuit is showed on figure 6. In the time domain, the voltage and the current of the resonator are given by :

$$\frac{d^2V_c}{dt^2} + \frac{\omega_r}{Q}\frac{dV_c}{dt} + \omega_r^2 V_c = \frac{\omega_r R}{Q}\frac{dI_t}{dt} \qquad \text{, where}$$

I_t is the total driving current and is the sum of the generator and beam currents.
V_c is the resulting cavity voltage.

$Q = \dfrac{Q_0}{1+\beta}$ is the loaded Q of the cavity.

ω_r is the resonance frequency.

$R = \dfrac{R_0}{1+\beta}$ is the loaded shunt impedance of the cavity.

The driving currents having the angular frequency w, the cavity current and voltage can be written in complex notation

$$V_c = \tilde{V}_c(t)\, e^{j\omega t} \qquad \text{et} \qquad I_t = \tilde{I}_t(t)\, e^{j\omega t}$$

The resonator equation is then

$$\frac{d^2\tilde{V}_c}{dt^2} + \left(2j\omega + \frac{\omega_r}{Q}\right)\frac{d\tilde{V}_c}{dt} + \left(\omega_r^2 - \omega^2 + j\omega\frac{\omega_r}{Q}\right)\tilde{V}_c = \frac{\omega_r R}{Q}\left(\frac{d\tilde{I}_t}{dt} + j\omega\tilde{I}\right)$$

With high frequency and high Q resonators, the previous second order equation reduces to a simpler first order equation

$$\frac{d\tilde{V}_c}{dt} + \left(\frac{\omega_r}{2Q} + j\frac{\omega^2 - \omega_r^2}{2\omega}\right)\tilde{V}_c = \frac{\omega_r R}{2Q}\tilde{I}$$

With the usual definitions of the filling time τ and the detuning parameter y of the cavity,

$$\tau \frac{d\,\tilde{V}_c}{dt} + (1 + j\,y)\,\tilde{V}_c = R\,\tilde{I}_t \qquad \text{with } \tau = \frac{2\,Q}{\omega_r} \quad \text{and} \quad y = -\,tg\,\psi = \tau\,(\omega - \omega_r)$$

The driving term is the sum of the generator voltage and the beam voltage

$R\,\tilde{I}_t = R\,\tilde{I}_g + R\,\tilde{I}_b = \tilde{V}_g - \tilde{V}_b$

We define the amplitude and phase of the complex quantities and we want to know the response of the amplitude A and the phase Φ of the cavity to the driving generator and beam voltages

cavity voltage : $\tilde{V}_c = A\ e^{j\Phi}$

generator voltage : $\tilde{V}_g = A_g\,e^{j\Phi_g}$

beam voltage : $\tilde{V}_b = A_b\,e^{j\Phi_b}$

The voltages in the previous cavity equation are written in terms of amplitudes and phases, the real and imaginary parts are separated and the 2 equations system is solved. We obtain finally 2 differential equations, which describe the response of the cavity voltage to an excitation of the driving generator and beam, and where the coupling between amplitude and phase of the cavity appear explicitely

$$\begin{aligned} \tau\,\dot{A} &= A_g \quad \cos(\phi - \phi_g) - A_b \quad \cos(\phi - \phi_b) - A \\ \tau\,\dot{\phi} &= -\,A_g / A\ \sin(\phi - \phi_g) + A_b / A\ \sin(\phi - \phi_b) - y \end{aligned} \qquad (1)$$

The detuning parameter y has the initial value y_0 $\quad y = \tau\,(\omega - \omega_r) = y_0 - \tau\,\Delta\omega_r$

We choose a first order equation to describe the evolution of the resonance frequency, which follows the quadratic law of the Lorentz forces, with a mechanical time constant

$\tau_m\ \Delta\dot{\omega}_r + \Delta\omega_r = -\,2\pi\,K\,A^2 \qquad (2)$

The system (1) and (2) of coupled differential equations was easily numerically integrated using Mathematica [2].

4. Results of simulations

The experiment described in section 2, was done obviously without beam and the phase of the generator was kept constant. The phase reference is the generator phase, which can be set to zero. The source is raised up from Ag_0 to Ag_p during the time T (1mS) and the system (1) is then

$$\tau \dot{A} = \left(A_{gp} - \eta\,(t - T)\, A_{g0} \right) \cos\phi \quad - A$$

$$\tau \dot{\phi} = - \left(A_{gp} - \eta\,(t - T)\, A_{g0} \right) \sin\phi \,/\, A \; - y$$

where $\eta\,(t)$ is the Heaviside function.

Simulations were performed for the 3 experimental cases.and the comparison with the experimental measurements will determine the time constant.

<u>1st case : cavity initially tuned</u>

The cavity is initialy correctly tuned and the pulse duration is 1 mS. Figures 7 and 8 show the plots of the amplitude (in MV/m) and phase (in radians) vs time (in mS) for 3 values of the mechanical time constant τ_m (0, 1 and 3 mS). The accelerating field raises up from 2.875 MV/m to 5.75 MV/m. The measurements (see figure 3) gave a minimum of field of 2.5 MV/m and a minimum of phase occuring at 4.5 mS after the beginning of the impulse. Comparing with the simulations, we can deduce without ambiguity a time constant of about 0.5 mS.

<u>2d case : initial negative cavity detuning</u>

The resonance frequency is intentionally decreased with helps of the stepping motor. The measured frequency shift is - 23 Hz. Figures 9 and 10 show the plots of amplitude and phase for the 3 values of the time constant. From comparisons with the measurements (see figure 4), we can deduce a time constant lower than 1 mS.

<u>3d case : initial positive cavity detuning</u>

The resonance frequency is this time intentionally increased and the measured frequency shift is + 48 Hz. Figures 11 and 12 show the plots of amplitude and phase for the 3 values of the time constant. From comparisons with the measurements (see figure 5), we can deduce a time constant of about 0.5 mS.

5. Conclusion

Precise amplitude and phase measurements on a MACSE cavity and cavity response simulations showed that the Lorentz forces detuning is quasi instantaneous (time constant of about 0.5 mS). In the TESLA proposal, the beam is assumed to come at $t_0 = \tau$ Log2 after the beginning of the filling of the cavity, with a beam loading voltage of half the generator voltage, in order that the cavity voltage remains constant during the whole beam pulse. Figure 13 shows the transient amplitude of the cavity voltage without any Lorentz forces detuning (the cavity phase is zero all along the pulse). If now we introduce the Lorentz forces effect with a static detuning factor of 2 $Hz/(MV/m)^2$, and even with a time constant of 1 mS, and assuming no feedback system, the cavity behaviour completely changes. Figure 14 shows the evolution of the resonance frequency shift, reaching a maximum value 0.5 mS later the beam arrival. Figures 15 and 16 show the plots of the amplitude and phase. A feedback system could hardly improve the situation, if no stiffening scheme reduces the cavity Lorentz forces.

The author would like to thank B. Phung, J.P. Charrier and M. Lefèvre for their contributions to the rf measurements.

[1] D. Proch, private communication.

[2] S. Wolfram, Mathematica a System for doing mathematics by computer, second edition, Addison-Wesley, 1991

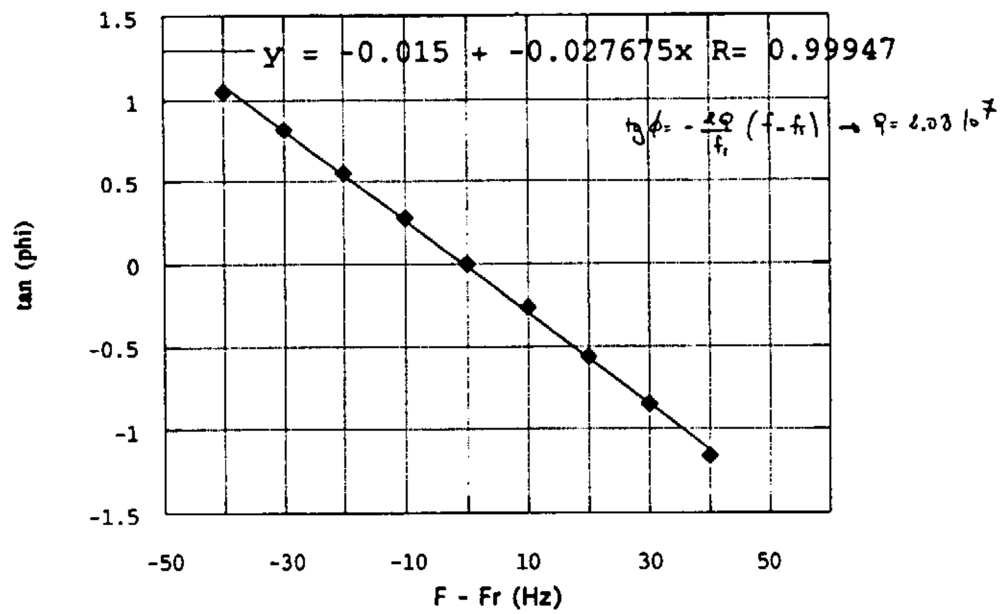

Figure 1 : Tangent of the cavity phase shift vs generator frequency

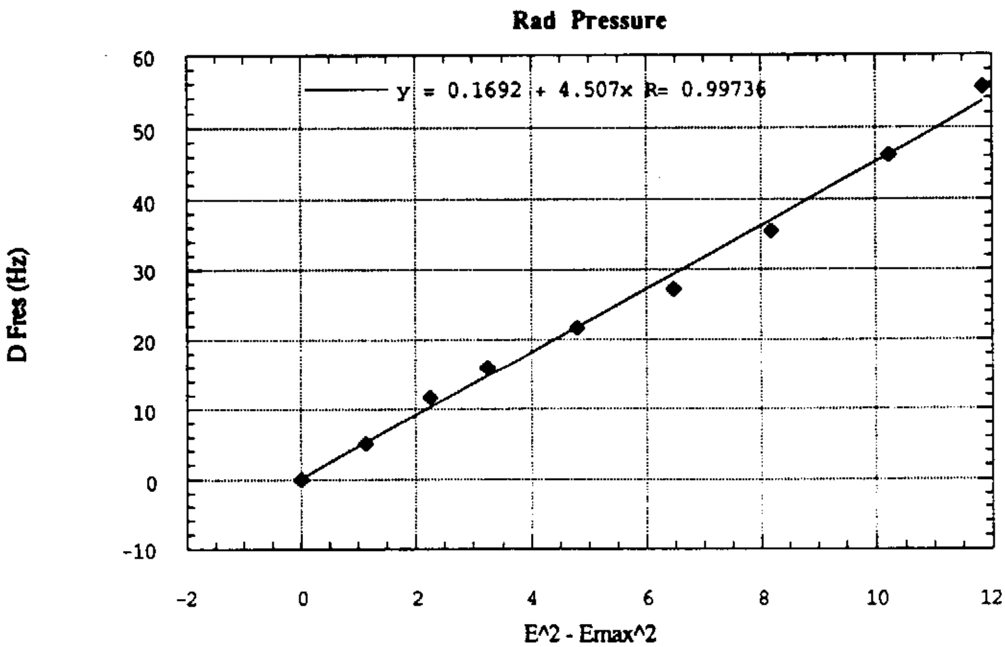

Figure 2 : Frequency shift (Hz) vs square accelerating field

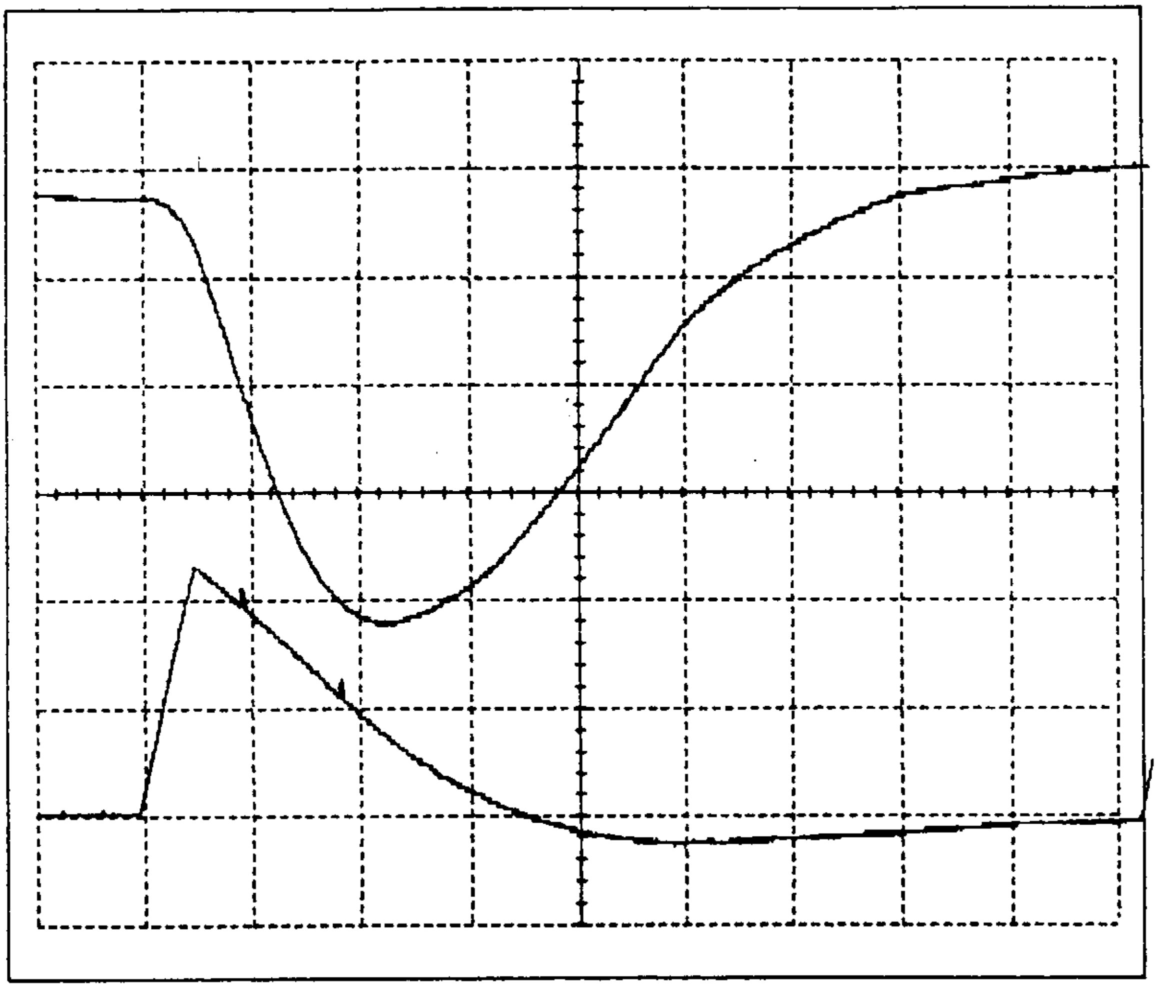

Figure 3 : Amplitude (bottom 1.25 MV/m / div) and phase (top 14.3 deg / div) without initial detuning

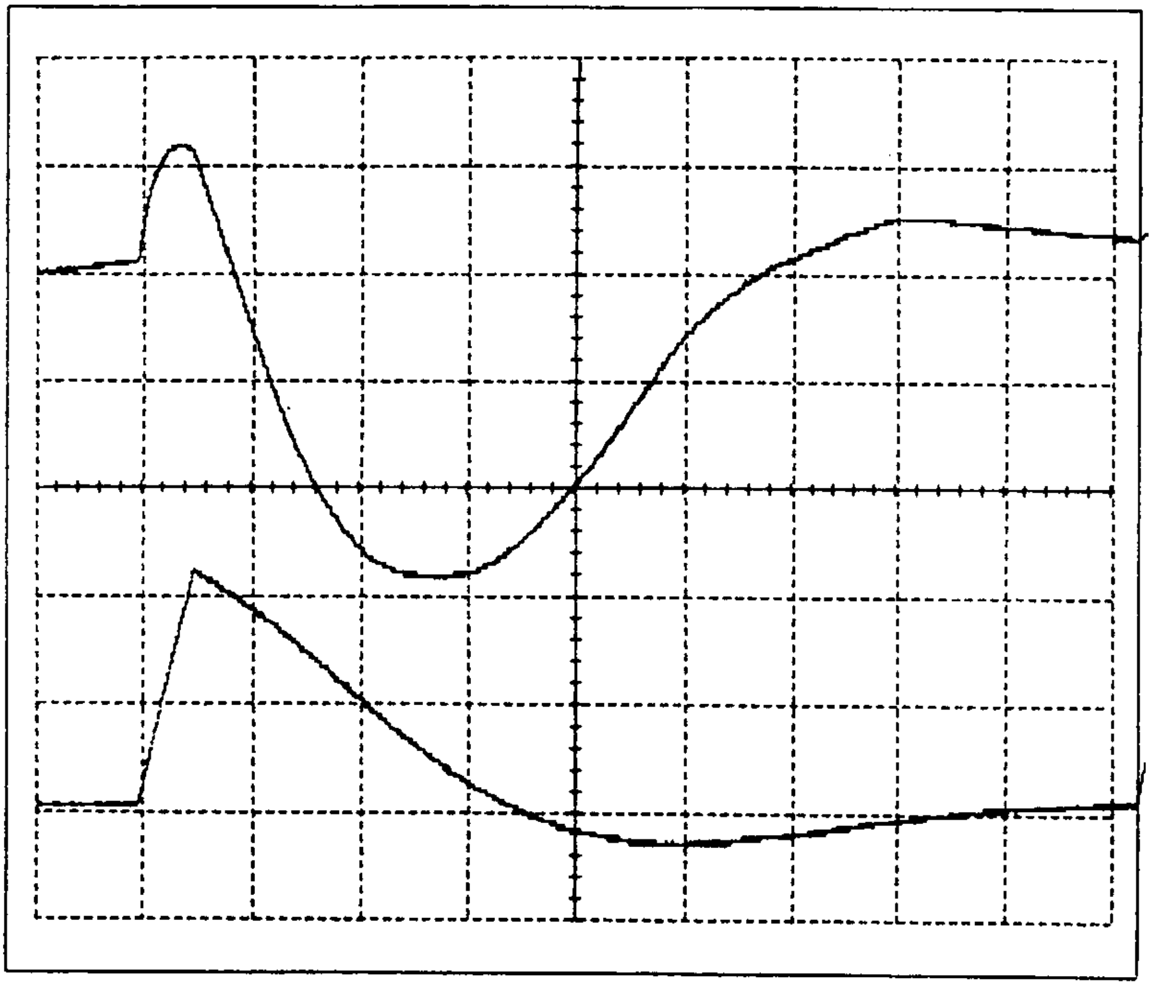

Figure 4 : Amplitude (bottom 1.25 MV/m / div) and phase (top 14.3 deg / div) with initial negative detuning (23 Hz)

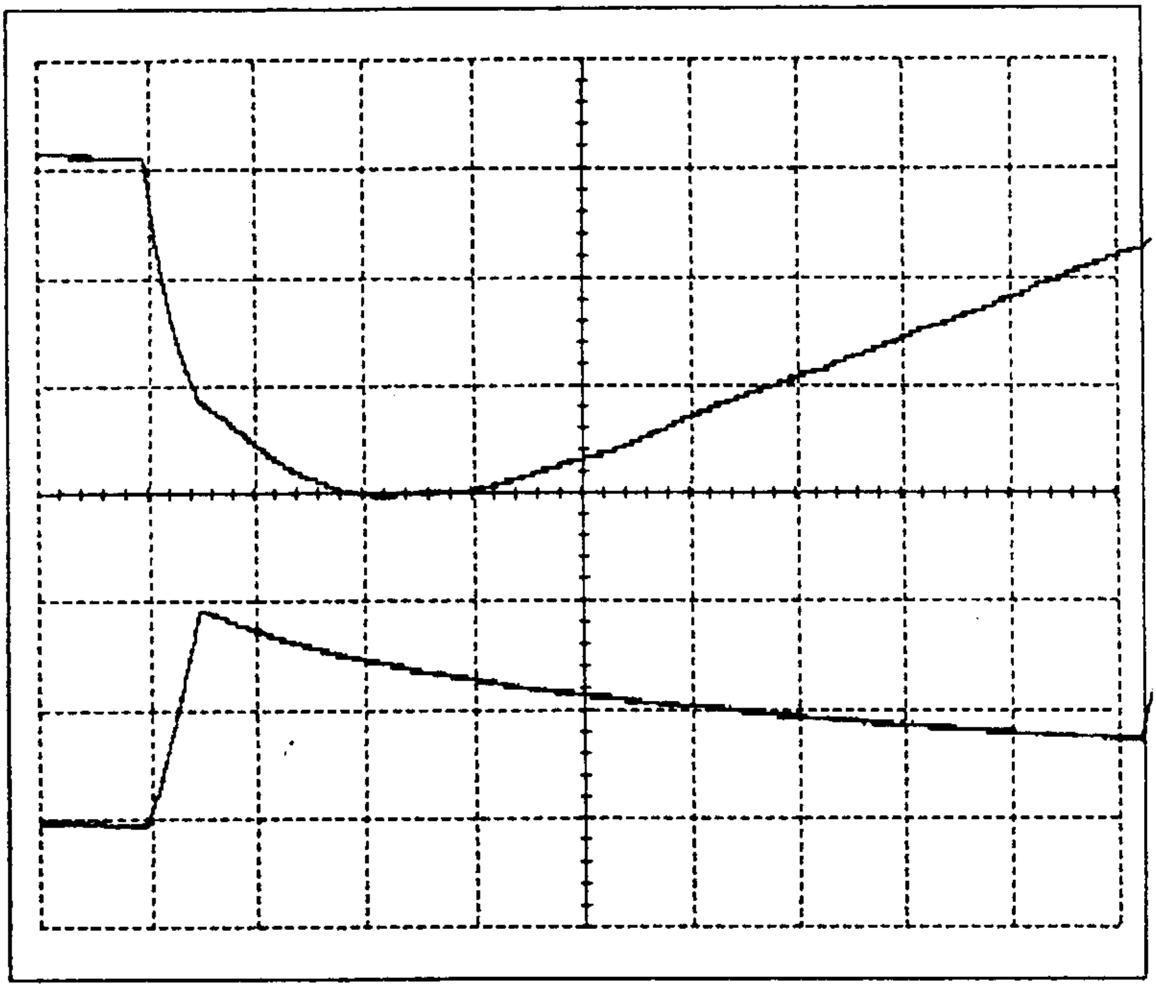

Figure 5 : Amplitude (bottom 1.25 MV/m / div) and phase (top 14.3 deg / div)
with initial positive detuning (48 Hz)

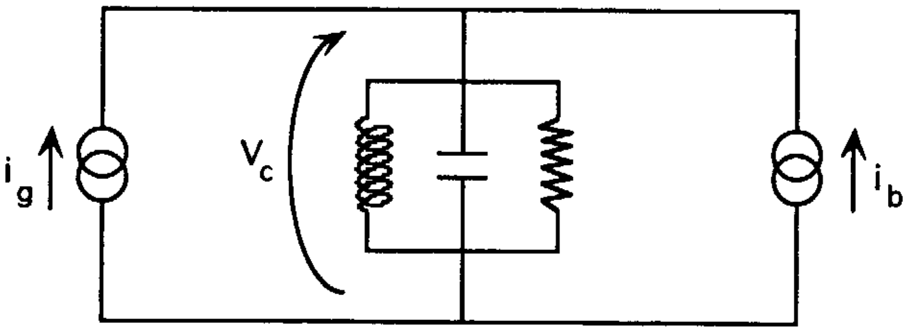

Figure 6 : equivalent electric circuit of the cavity

Loaded cavity admittance : $Y_L = G_0\left[\,(1+\beta) + j\,Q_0\,\Omega\right] = G\,(\,1 + j\,Q\,\,\Omega\,)$

with : $G = \dfrac{1}{R} = G_0\,(1+\beta) = \dfrac{1+\beta}{R_0}$ $\quad Q = \dfrac{Q_0}{1+\beta}$ $\quad \Omega = \dfrac{\omega}{\omega_r} - \dfrac{\omega_r}{\omega}$

beam current rf component : $i_b = 2\,I_0$

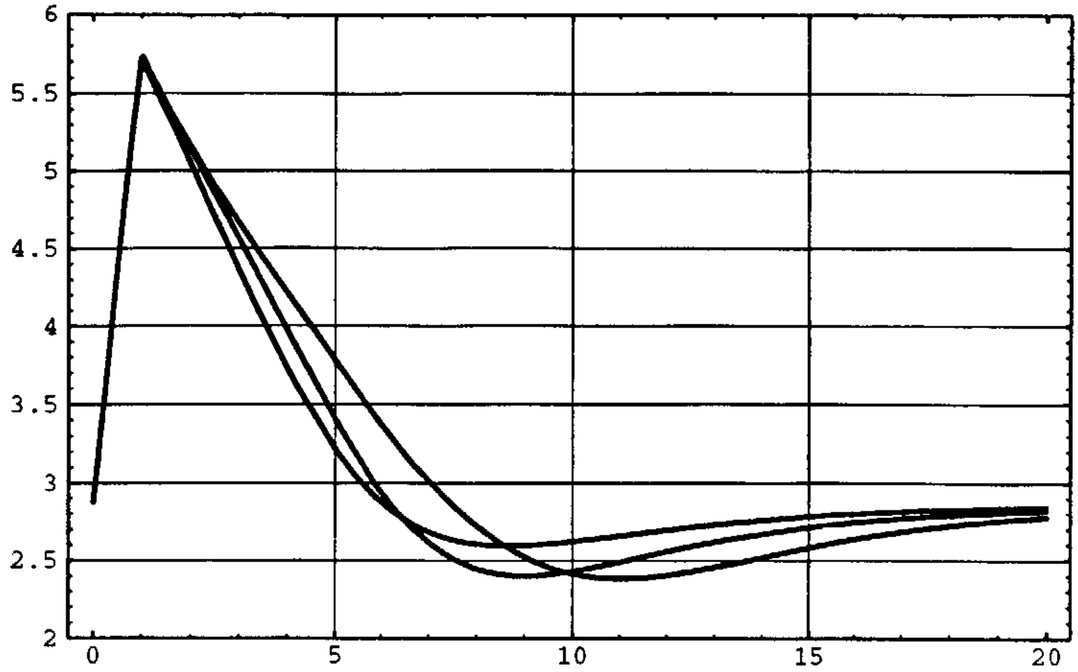

Figure 7 : amplitude (MV/m) vs time (mS) without initial detuning
results of simulations

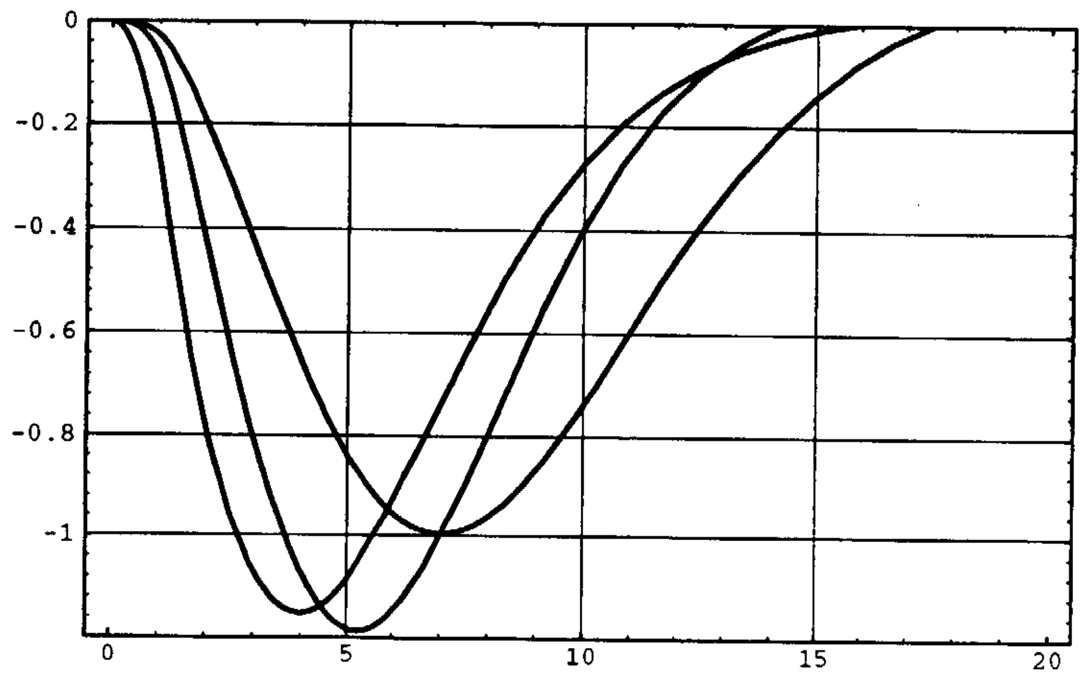

Figure 8 : phase (radians) vs time (mS) without initial detuning
results of simulations

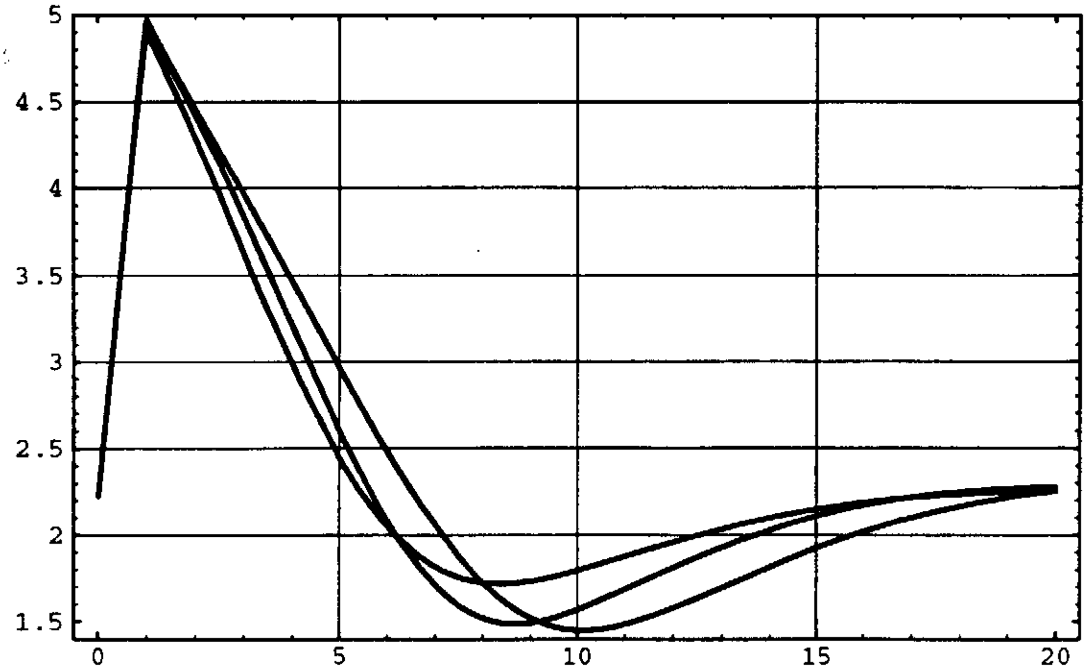

Figure 9 : amplitude (MV/m) vs time (mS) with initial negative detuning (-23 Hz) results of simulations

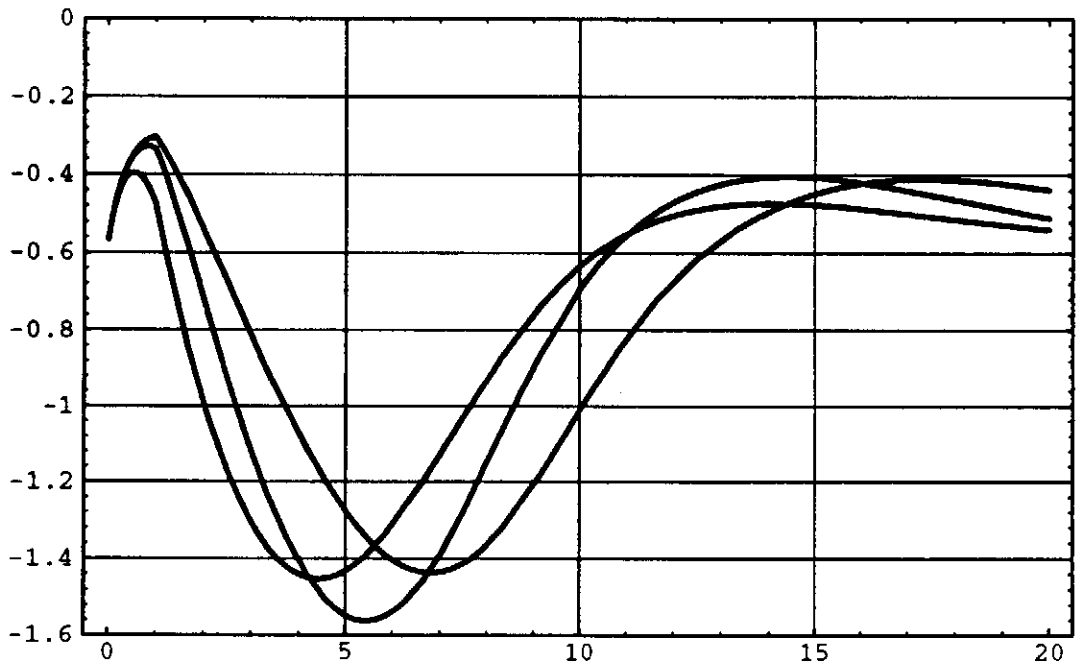

Figure 10 : phase (radians) vs time (mS) with initial negative detuning (-23 Hz) results of simulations

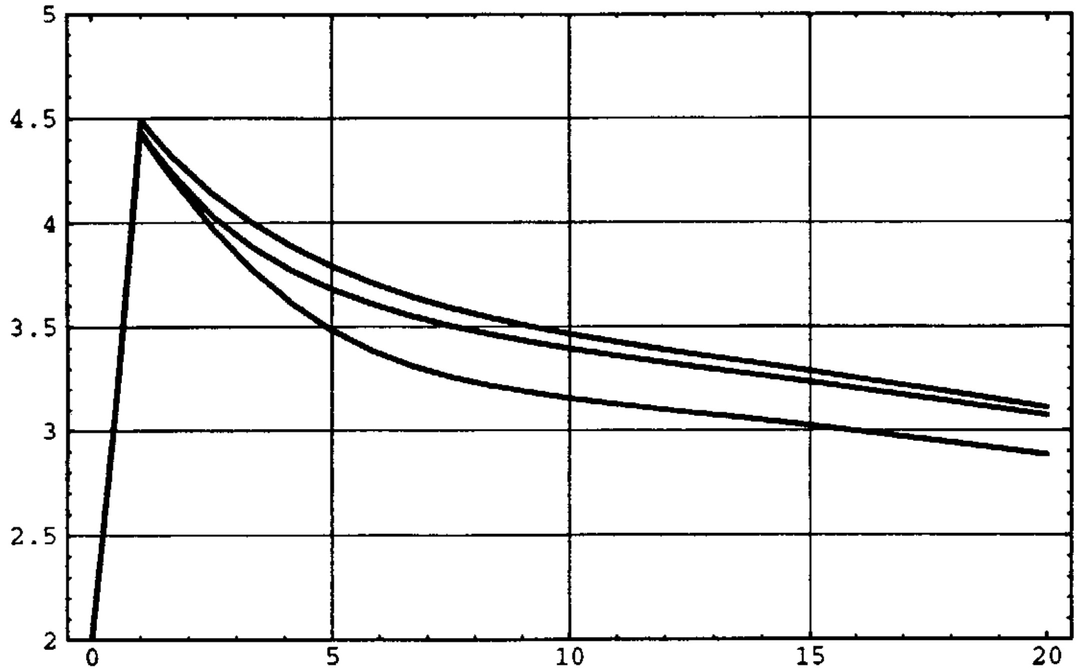

Figure 11 : amplitude (MV/m) vs time (mS) with initial positive detuning (48 Hz) results of simulations

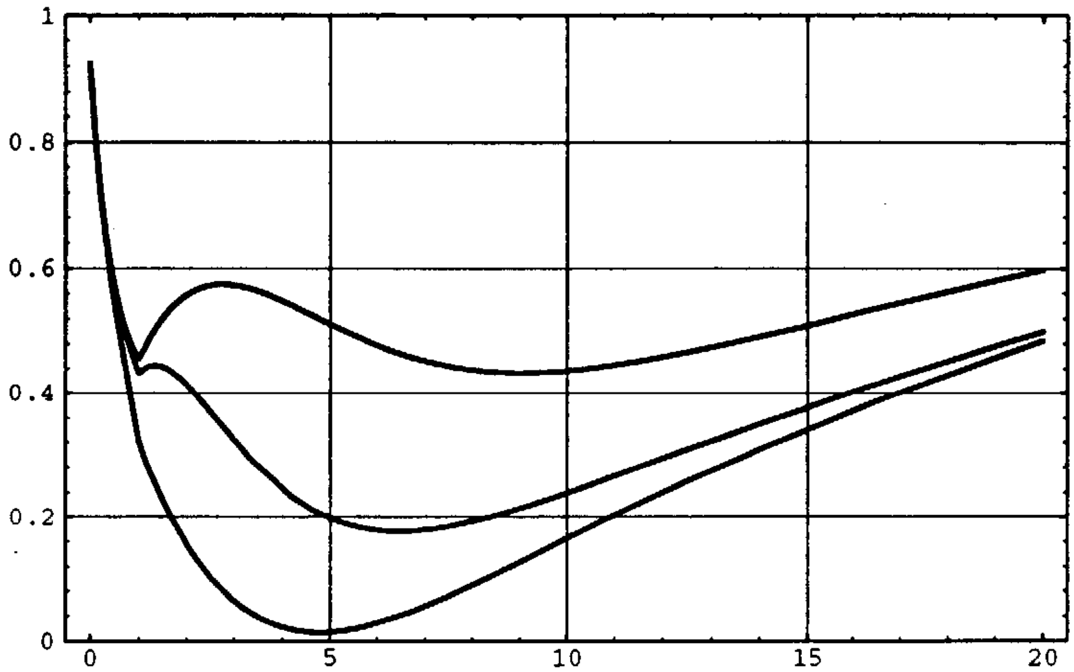

Figure 12 : phase (radians) vs time (mS) with initial positive detuning (48 Hz) results of simulations

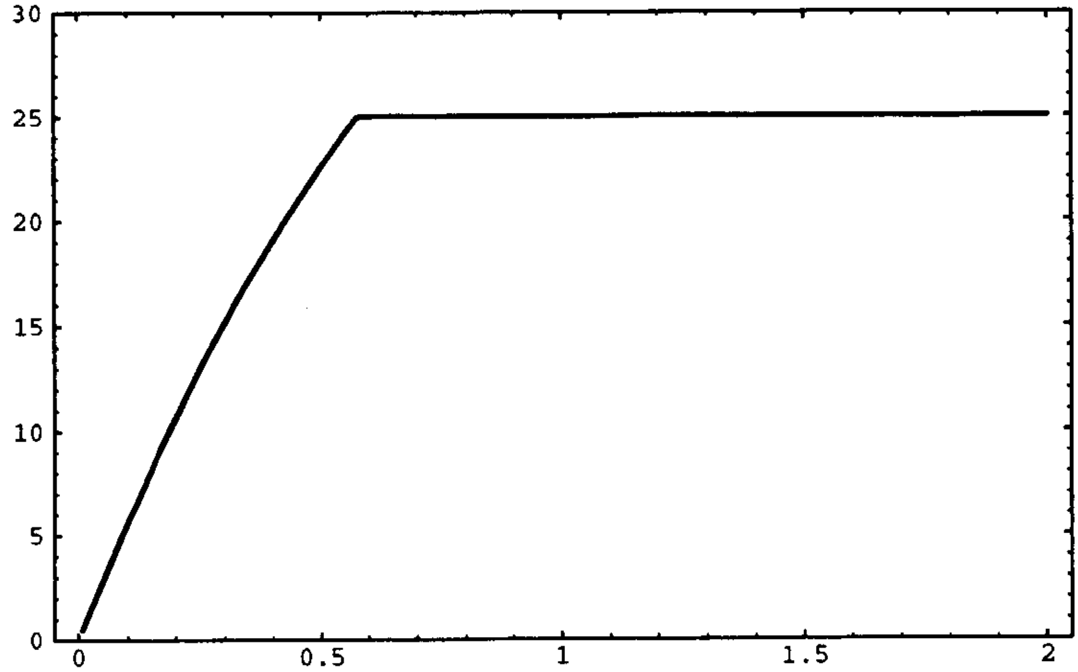

Figure 13 : amplitude (MV/m) vs time (mS) without Lorentz forces effect with a beam current step (results of simulations)

loaded $Q = 3.4\ 10^6$

filling time $= \tau = \dfrac{2\,Q}{\omega} = 0.8325$ mS

beam voltage /m $= A_b = A_g / 2 = 25$ MV/m

beam delay $= t_0 = \tau \operatorname{Log} 2 = 0.577045$ mS

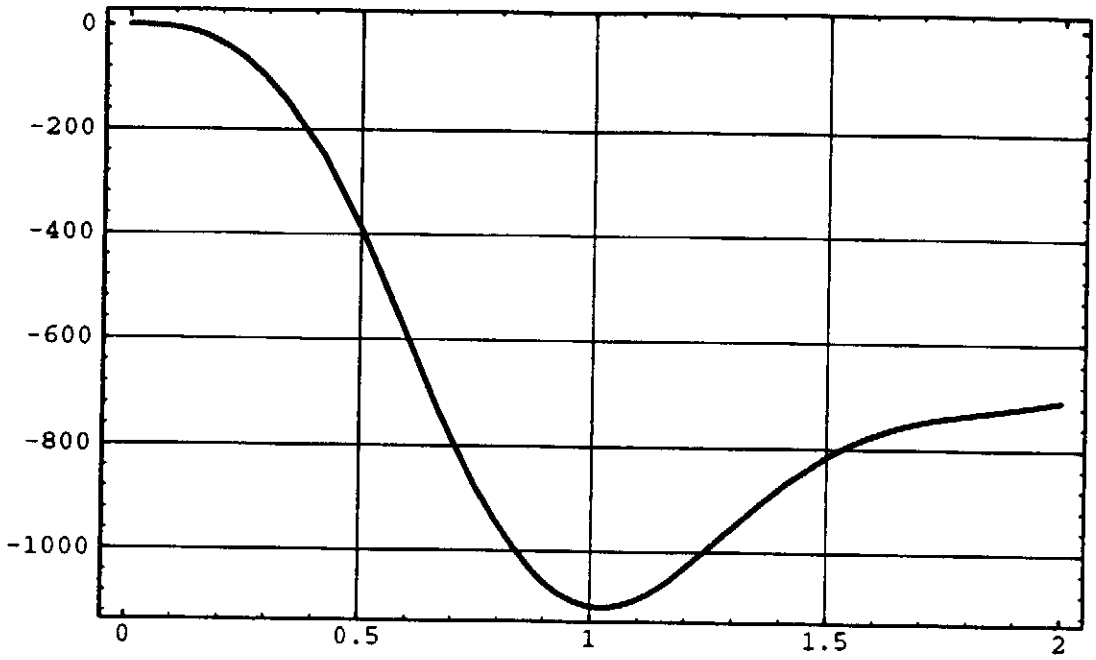

Figure 14 : Resonance frequency shift (Hz) vs time (mS) with Lorentz forces effect with a beam current step (results of simulations)

static detuning factor = 2 Hz / $(MV/m)^2$

mechanical time constant = τ_m = 1 mS

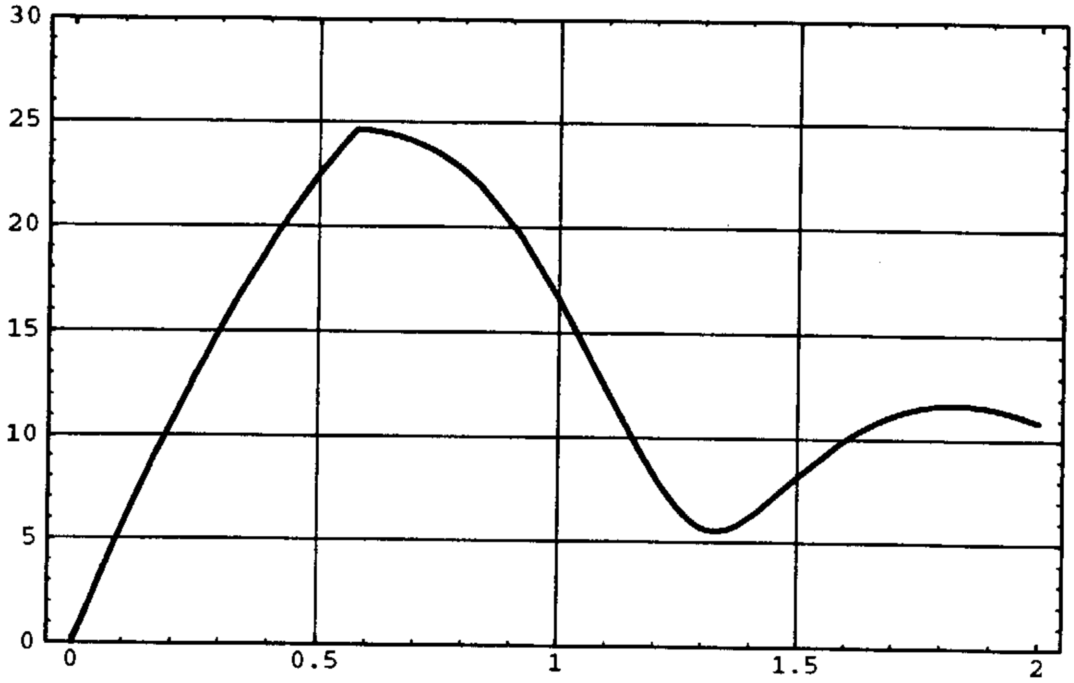

Figure 15 : amplitude (MV/m) vs time (mS) with Lorentz forces effect
with a beam current step (results of simulations)

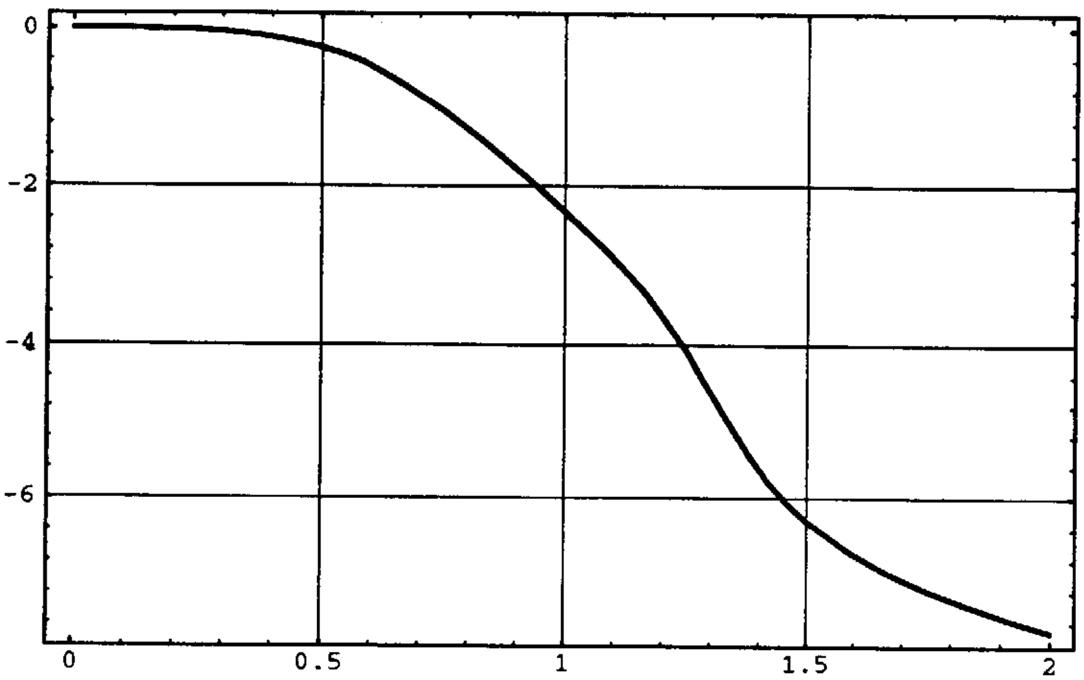

Figure 16 : phase (radians) vs time (mS) with Lorentz forces effect
with a beam current step (results of simulations)