TESLA-Report 1993-24

RF focusing in TESLA cavities

B. Aune
CE Saclay, France

Introduction

In a SW field pattern, an alternating focusing force acts on the particles traveling off-axis due to the backward wave. The result for an accelerated particle is a net focusing effect which can be strong at low energy and high accelerating fields. Ray-tracing codes can be used to get the trajectories and envelopes in the TESLA cavities. As an example, fig 1 shows two trajectories for a 5 MeV beam accelerated in a 9-cell cavity in which the accelerating field is 15 MeV/m (also shown is the envelope for a beam with a 1 mm*mr emittance). In this case, the RF focusing effect results in an equivalent focal length of about 1.5 m . This is a much stronger focusing force than the one produced by the quadrupoles. Clearly, for the low energy part of TTF, the beam optics is dominated by this effect.

In order to make calculations which do not involve time consuming ray-tracing codes, we rather use a first order matrix simulation of the cavity that we have included in a Transport-like code. Comparisons have been made with ray-tracing results for different conditions. The conclusion is that the matrix approximation can be used for the main linac, even if the injection energy is as low as 5 MeV.

For the capture cavity, where the input energy is 250 keV and where space charge effect is important, only a ray-tracing code like Parmela can be used. A first order matrix can then be calculated and included in the transport code. Resulting beam ellipse parameters can also be used as an input for calculations in the main linac.

Analytical expressions

The first order matrix we use was proposed by Chambers [1] and results from an approximate integration of the equations of transverse and longitudial motions in a pure standing wave field. He assumed a relativistic particle and an input energy much larger than the energy gain per wavelength.

In this approximation, the transfer matrix is given by :

$$\begin{pmatrix} \cos\alpha - \sqrt{2}\sin\alpha & \sqrt{8}\dfrac{E_i}{\Delta E} L \sin\alpha \\ -\dfrac{3}{\sqrt{8}}\dfrac{\Delta E}{E_f}\dfrac{1}{L}\sin\alpha & \dfrac{E_i}{E_f}(\cos\alpha + \sqrt{2}\sin\alpha) \end{pmatrix}$$

where $\alpha = \frac{1}{\sqrt{8}} \ln \frac{E_f}{E_i}$, E_i, E_f and ΔE being the input and output energies and the energy gain in a cavity of length L.

In the case of high input energy ($E_i >> \Delta E$), this matrix reduces to :

$$\begin{pmatrix} 1 - \frac{1}{2}\frac{E_{acc}}{E_i} L & L \\ -\frac{3}{8}\left(\frac{E_{acc}}{E_i}\right)^2 L & 1 - \frac{1}{2}\frac{E_{acc}}{E_i} L \end{pmatrix}$$

In this notation an equivalent focusing strength can be found to be $K = \frac{3}{8}\left(\frac{E_{acc}}{E_i}\right)^2$. In a recent paper, Rosenzweig [2] proposes another approach from which he obtains a focusing strength given by $K = \frac{1}{2}\left(\frac{E_{acc}}{E_i}\right)^2$ which is sligthly different but gives the same energy and field dependences.

Comparison with trajectory calculations

The following figures show the comparisons between computations of trajectories and the transport code using the Chambers matrices. The accelerating field is 15 MeV/m and the input energy is varied from 5 to 15 MeV. First a pure SW field was simulated to check the accuracy of the matrix approximation, then the effect of the fringing field in the beam tube was added to simulate the 9-cell cavity.

Fig 2 shows the beam envelopes for two energies in a pure SW field at a wavelength of 10 cm. The dotted curves are calculated by integrating the equations of motion, the solid ones represent the result of transport calculation (each cell was simulated by a matrix). On fig 2a the dashed curve represents the envelope without RF focusing. An excellent agreement between the matrix approximation and ray-tracing calculation appears, even in the low energy case, thus justifying the use of the Chambers approximation in pure SW field for the range of parameters of TTF.

In the case of the 9-cell cavities, the approximation of the actual field distribution by a pure SW field is valid in the inner cells where the field shows a sine-like variation (very low content of space harmonics). In the outer cells the field distribution is very dependant on the large beam tube opening which gives a far-extending fringing field. In this region, the beam is decelerated and defocused (see fig. 1). In the next simulations we have compared trajectory calculations in a true 9-cell cavity with transport results in which the cavity is replaced by matrices for the 9 cells with attached drift spaces at both ends, the overall length being the same in both cases.

Fig 3 shows the comparisons, again for 15 MeV/m and 15 and 5 MeV input energies. Of course the agreement is less spectacular than in the first case because the fringing field effect is ignored in the matrix representation. Nevertheless we find it satisfactory enough to use this formulation for beam optics studies in TTF without adding a modelisation for the fringing field region.

[1] E.E. Chambers HEPL TN-68-17

[2] S.C. Hartman and J.B. Rosenzweig "Ponderomotive focusing in axisymetric RF linacs",sub. to Physical Review A.

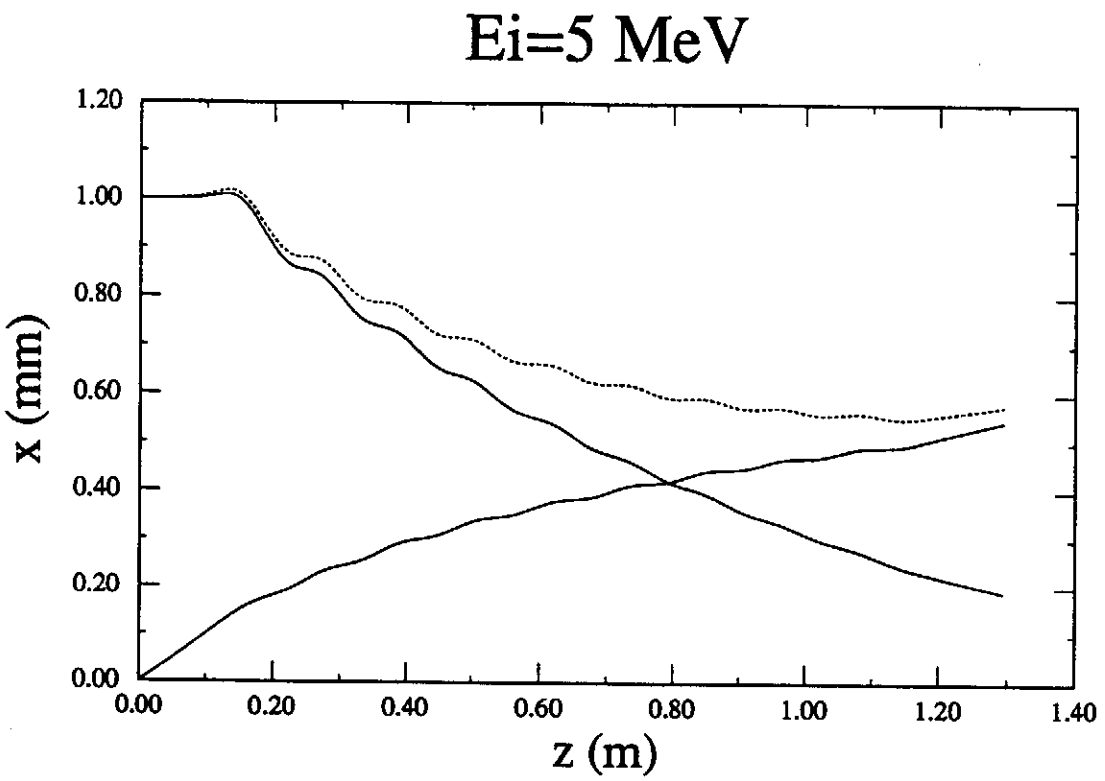
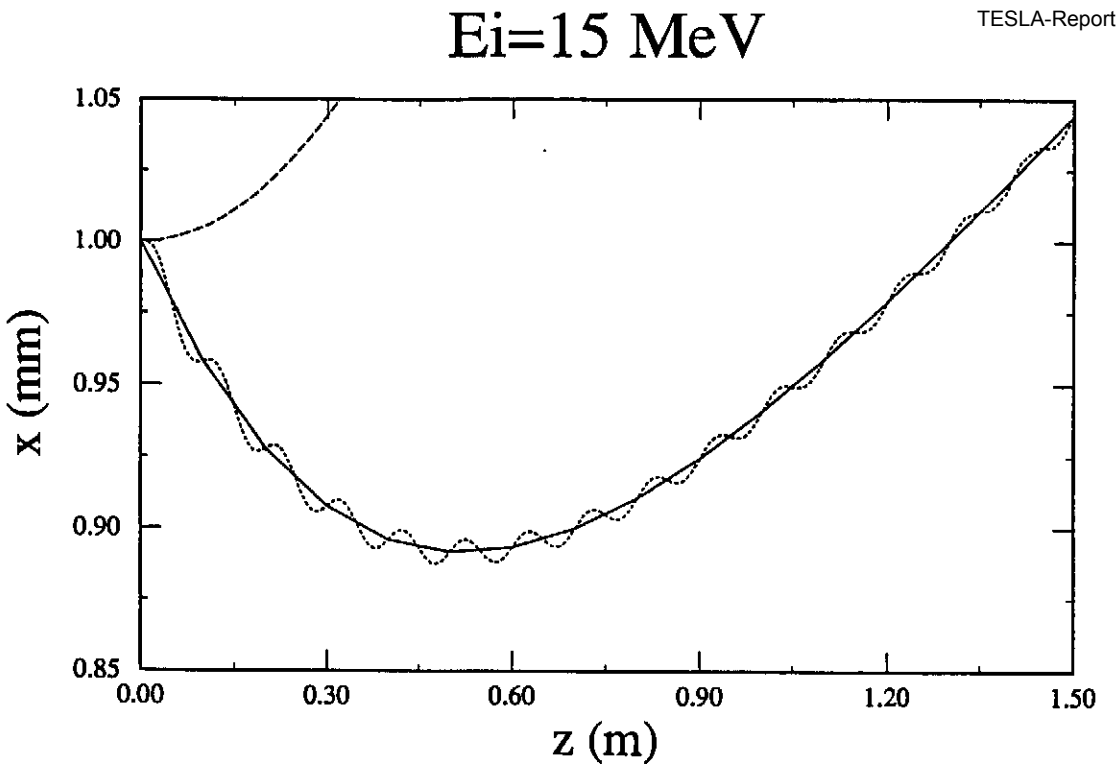
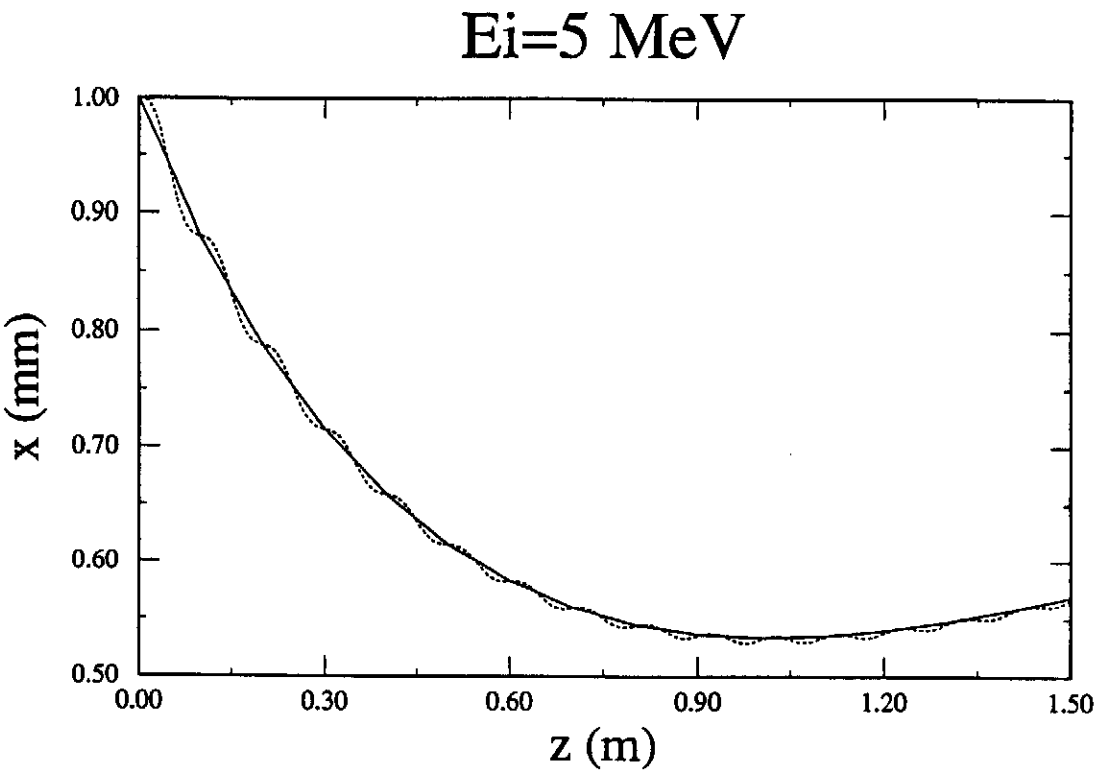


Fig. 1

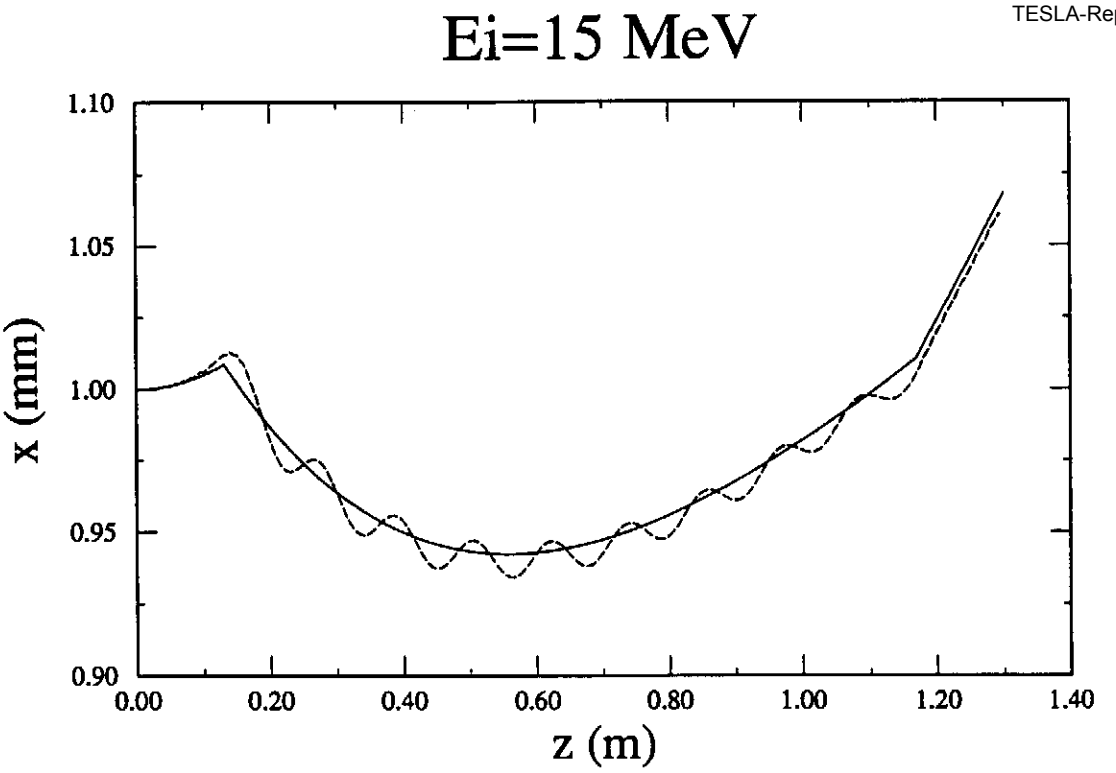

Ei=15 MeV
1.05
1.00
0.95
0.90
0.85
x (mm)
0.00
0.30
0.60
0.90
1.20
1.50
z (m)

a)


Ei=5 MeV
1.00
0.90
0.80
0.70
0.60
0.50
x (mm)
0.00
0.30
0.60
0.90
1.20
1.50
z (m)

b)

fig. 2


Ei=15 MeV
1.10
1.05
1.00
0.95
0.90
x (mm)
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
z (m)

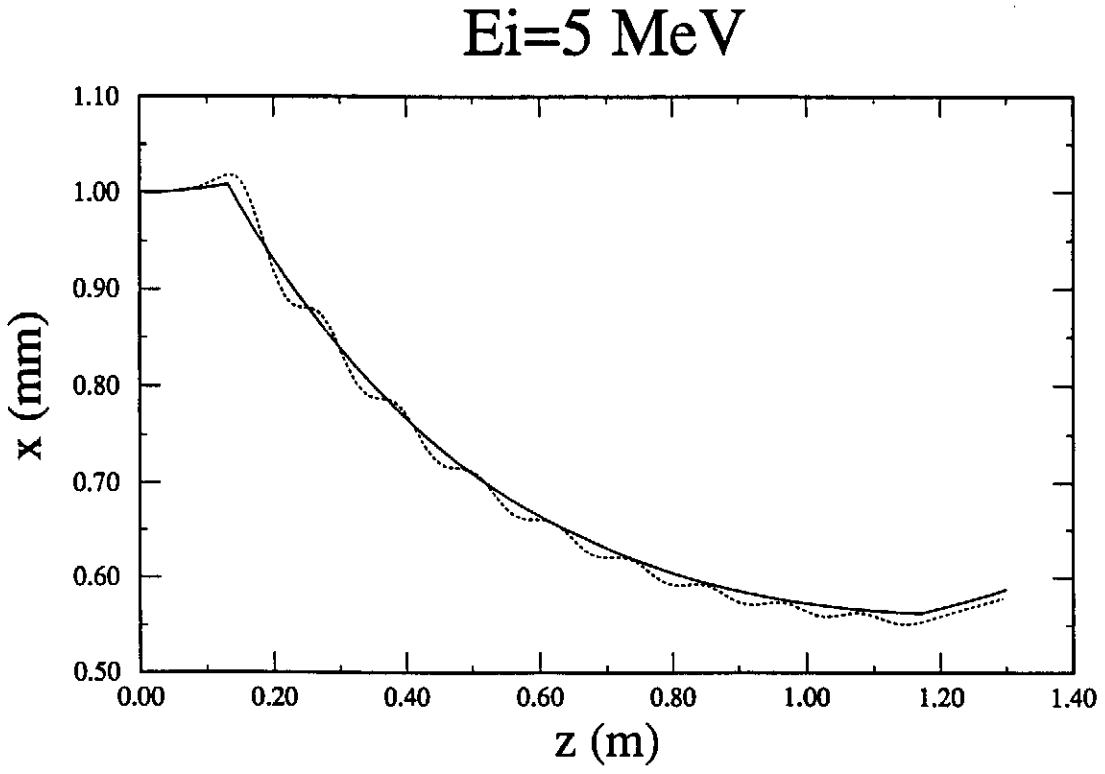

Ei=5 MeV
1.10
1.00
0.90
0.80
0.70
0.60
0.50
x (mm)
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
z (m)

fig. 3