Microphonics and RF Stabilization in Electron Linac Structures

H. A. Schwettman

W. W. Hansen Experimental Physics Laboratory
Stanford University
Stanford, CA 94305-4085

ABSTRACT

Although rf stabilization is a necessary condition to achieve beam stability in an electron linac, it is not a sufficient condition. Acoustic excitation of a longitudinal vibrational mode will lead to energy modulation of the accelerated electron beam despite rf stabilization. Acoustic excitation of a transverse mode will lead to pointing and position modulation of the electron beam in addition to energy modulation. In this paper, simple models of the longitudinal rf modes and the longitudinal vibrational modes of a uniform π-mode structure are used to provide an existence proof of this effect and to estimate its magnitude.

TESLA-Report 1993-42

I. INTRODUCTION

There is a general misconception in the community that rf stabilization is a necessary and sufficient condition to achieve beam stability in an electron linac. It is a necessary condition but it is not a sufficient condition. As shown in this paper, acoustic excitation of a longitudinal vibrational mode will lead to energy modulation of the electron beam despite rf stabilization. Similar arguments would indicate that acoustic excitation of a transverse mode will lead to pointing and position modulation of the electron beam in addition to energy modulation. These effects exist even if the structure frequency does not change as a result of the acoustic excitation.

2. RF AND MECHANICAL MODES OF THE STRUCTURE

We consider a uniform π-mode structure terminated, as illustrated in Fig. 1, by electrical shorts at the equatorial plane of the end cells. For small cell-to-cell coupling one can represent the "TM_{010}" modes of this structure by the electrical equivalent circuit[1] shown in the figure. If we assume the currents in the n^{th} cell, $I(n)$, are periodic in time ($e^{+i\omega t}$), then the circuit equations are:

$$(i\omega L + 1/2i\omega C + R/2)\, I(0) + i\omega\gamma L\, I(1) = 0$$

$$(2i\omega L + 1/i\omega C + R)\, I(n) + i\omega\gamma L\, [\, I(n-1) + I(n+1)\,] = 0$$

$$(i\omega L + 1/2i\omega C + R/2)\, I(N) + i\omega\gamma L\, I(N-1) = 0$$

where N is the number of cells and γ is the coupling coefficient. For high Q and small γ, the normal modes and frequencies are

$$I_q(n) = (2W_q/N)^{1/2}\, I_o \cos(n\beta_q) \qquad \text{Eqn. (1)}$$

$$\omega_q = \omega_c\, [1-(\gamma/2)\cos\beta_q]. \qquad \text{Eqn. (2)}$$

Here W_q is one half for q = 0,N and is unity otherwise. The quantity $\beta_q \equiv \pi q/N$ is the phase shift per cell, and $\omega_c^2 = 1/(2LC)$. The dispersion curve for the "TM_{010}" modes is shown in Fig. 2.

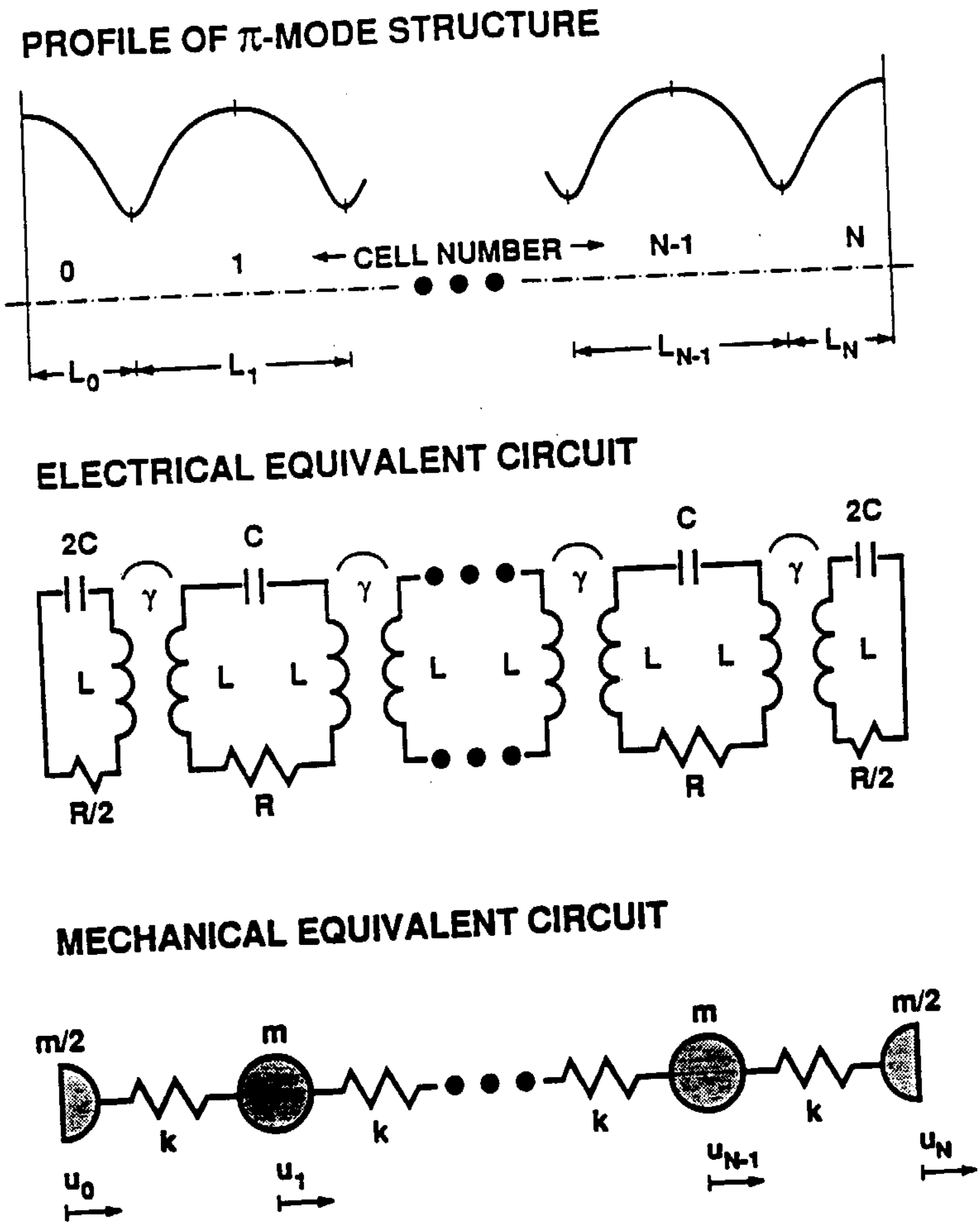

Figure 1: Profile of a uniform π-mode structure with equivalent circuits for the longitudinal rf modes and the longitudinal vibrational modes.

One can also represent the longitudinal vibrational modes by an equivalent circuit, as indicated in Fig. 1. The accelerator structure has flexibility in the iris region and thus is free to compress and stretch, much like an accordion. If we write the equations of motion for the individual masses and assume the displacements, u_q, are periodic in space:

$$u_q\,(n,t) = \mathbf{U}_q(t)\cos\,(n\beta_q), \qquad \text{Eqn. (3)}$$

we obtain a simple harmonic oscillator equation in $\mathbf{U}_q(t)$ which determines the frequency of the motion:

$$\omega_q = (4k/m)^{1/2}\sin\,(\beta q/2) \qquad \text{Eqn. (4)}$$

The quantities k and m are the spring constant and mass in the equivalent circuit, and the quantity β_q is again the phase shift per cell. The dispersion curve for the longitudinal vibrational modes is shown in Fig. 2.

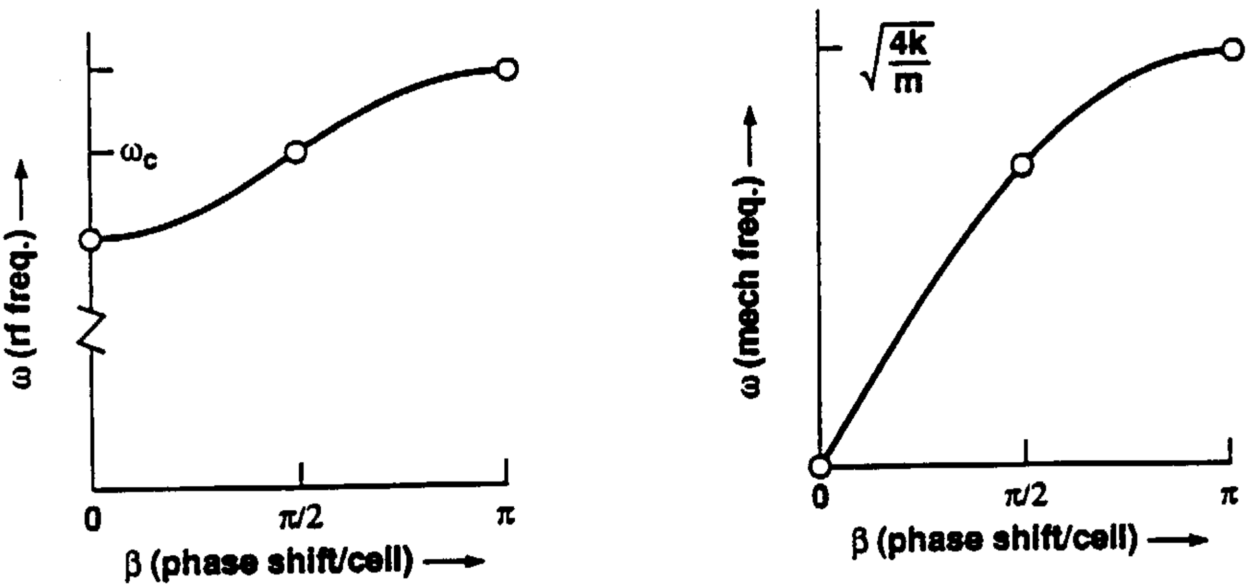

Figure 2a: Dispersion curve for longitudinal rf modes.

Figure 2b: Dispersion curve for longitudinal vibrational modes.

3. ADIABATIC MODULATION OF CELL FREQUENCIES

Acoustic excitation of longitudinal mechanical modes will result in a modulation in the length of individual cells. Stretching or compressing individual cells is well known to produce a change in the resonant frequencies of the cells and a change in the steady state field profile of the structure. For the CERN/DESY structure(2), the cell tuning rate at 1300 MHz is measured(3) to be: $df_c/dL_c = + 1.9$ kHz/μm.

Electrical disturbances will propagate along an accelerator structure at the group velocity. For a π-mode structure of infinite length the group velocity is zero, but for a structure of finite length the effective group velocity $v_g = \Delta\omega/\Delta k$ where $\Delta\omega$ and Δk are the frequency and wave vector differences between the π-mode and the nearest neighbor mode. For a structure with $\gamma = 10^{-2}$ and with less than ten cells, the group velocity is $> 10^{-3}$ c and the time for a disturbance to propagate from one end of the structure to the other is < 3 μsec. This time is very short and can be ignored in the mechanical vibration problem. Thus if there is a change in cell frequencies, a new steady state field profile will be established in the structure on a time scale governed by the electrical time constant: $\tau = Q_L/\omega$. For a loaded Q-value of 10^6 this time constant at 1300 MHz is 10^{-4} sec. Even for a longitudinal mode frequency of 1 kHz, the period of the vibrational motion is an order of magnitude longer than the electrical time constant. Thus it will be reasonable for most of the longitudinal vibrational modes to treat the cell length modulation as an adiabatic variation of the structure.

Let us consider a short π-mode structure consisting of one full cell with a half cell on either side. For this structure N = 2 and there are three longitudinal vibrational modes: q = 0, 1, or 2, corresponding to a phase shift per cell: $\beta_q = 0, \pi/2$ or π. The 0-mode corresponds to center-of-mass motion and is not interesting here. The π/2-mode and the π-mode, shown in Fig. 3a and 3b respectively, produce modulation of individual cell length and thus modulation of cell frequency. As indicated in Fig. 3a, the π/2-mode produces a change in cell frequency, $\delta f_c(n)$, that is equal in all cells. The change in the resonant frequency of the structure is equal to the cell frequency change. As indicated in Fig. 3b, the π-mode produces a change in cell frequency, $\delta f_c(n)$, that is zero in the center cell and is of equal magnitude but opposite sign in the two end cells. The change in the resonant frequency of the structure for this vibrational mode is zero. Excitation of the $\pi/2$ vibrational mode and the attendant frequency modulation of the structure would clearly generate a problem in operation as an accelerator. We will show in the next sections of this paper that excitation of the vibrational π-mode, where no structure frequency modulation occurs, also generates a problem.

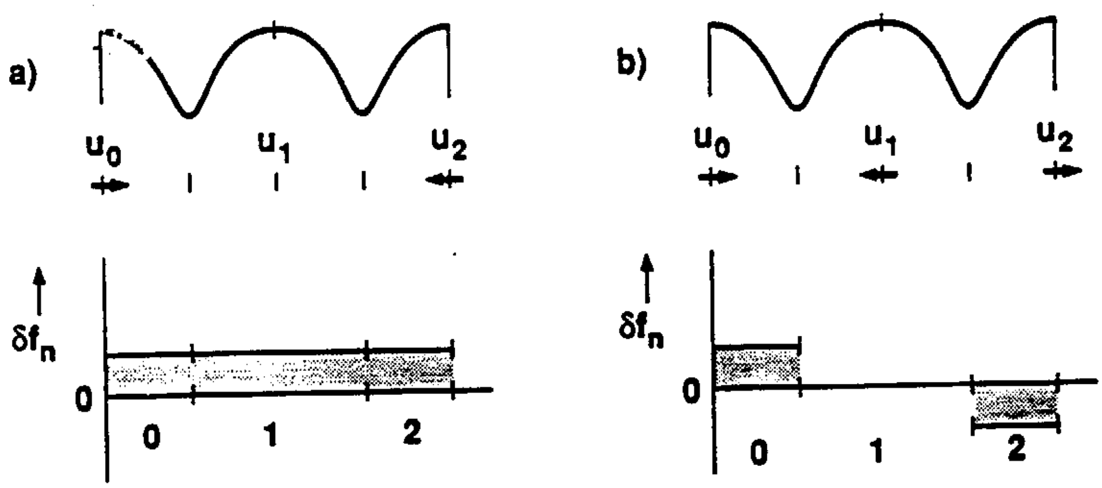

Figure 3a: Displacement u(n) for vibrational $\pi/2$-mode and resulting variation in cell frequencies, $\delta f_c(n)$.

Figure 3b: Displacement u(n) for vibrational π-mode and resulting variation in cell frequencies, $\delta f_c(n)$.

4. ADIABATIC MODULATION OF THE FIELD PROFILE

If the cell frequency changes induced by acoustic vibrations can be treated as adiabatic variations of the structure, then first order perturbation theory(4) can be used to calculate the perturbed field profile at each instant of time. The currents in the electrical equivalent circuit will be used to represent the fields:

$$\delta I_q(n) = \sum_{k \neq q} b_{q,k} I_k(n) \qquad \text{Eqn. (5)}$$

Let us consider again the case N = 2 and calculate the perturbation $\delta I_\pi(n)$ on the field profile of the rf π-mode. Assuming excitation in the vibrational π-mode (there is no structure frequency shift for this mode), we find:

$$b_{\pi,\pi/2} = \frac{\sqrt{2}}{\gamma}\left(\frac{\delta\omega_c}{\omega_c}\right), \qquad b_{\pi,0} = 0 \ .$$

For this vibrational mode the field profile in the rf π-mode is perturbed by the addition of a small $\pi/2$-mode contribution whose magnitude is modulated at the vibrational frequency, ω_π. For a 1300 MHz CERN/DESY structure $(\delta\omega_c/\omega_c) = 0.17\ (\delta L_c/L_c)$ and $\gamma = 0.017$. If we write $\delta L_c = A_o \cos \omega_\pi t$, then

$$\delta I_\pi\ (n,t) = (120\ A_o \cos \omega_\pi t)\ I_{\pi/2}\ (n).$$

5. ENERGY MODULATION IN AN RF STABILIZED STRUCTURE

In an rf stabilization system one detects the field in a single cell of the structure, usually the end cell, and stabilizes this field in amplitude and phase. For the special case considered in the previous section the total field in the end cell (n=0) is:

$$I_\pi\ (0,t) + (120\ A_o \cos \omega_\pi t)\ I_{\pi/2}\ (0).$$

If the rf stabilization system holds the total field constant, then the π-mode contribution must vary in time to compensate for changing

$\pi/2$-mode contribution. Since the electron beam gains energy only from the π-mode component, the rf stabilization system in fact produces a sinusoidally varying energy gain. For a vibration amplitude of 10 µm, the relative variation in energy gain is ± 1.7 x 10^{-3} (after proper normalization of I_π and $I_{\pi/2}$).

6. CONCLUSION

It has been shown that acoustic excitation of a longitudinal vibrational mode leads to energy modulation of the electron beam despite rf stabilization. Similar arguments would indicate that acoustic excitation of a transverse mode will lead to pointing and position modulation of the electron beam in addition to energy modulation. To achieve beam stability in an electron linac, rf stabilization is necessary but not sufficient. One must also minimize vibrational motion of the structure either by controlling vibration at its source, by isolation, or by damping.

7. ACKNOWLEDGEMENTS

This work was supported by the Office of Naval Research under Contract Number N00014-91-J-4152.

8. REFERENCES

1. D. E. Nagle, Coupled Resonator Model of Linear Accelerator Tanks, 1964 Linear Accelerator Conference, Midwestern Universities Research Association, pp. 21-30.
2. E. Häbel, P. Marchand, and J. Tückmantel, "An improved superconducting cavity design for LEP," Proc. Second Workshop on RF Superconductivity, Part 1, pp 281-297, Geneva, Switzerland, 1984.
   H. P. Vogel, DESY-M-85-09.
3. A. Marziali (private communication).
4. T. I. Smith, "Standing Wave Modes in a Superconducting Linear Accelerator," High Energy Physics Laboratory Report No. 437, Stanford University, Stanford, CA, 1966.