A New Limit to the Maximum RF Magnetic Field for Niobium†

J. Graber‡, N. Black, H. Padamsee, M. Pekeler‡
Laboratory of Nuclear Studies
Cornell University
Ithaca NY 14853

I. INTRODUCTION

What is the maximum RF magnetic field below which niobium will remain superconducting? The highest RF magnetic field at which superconductivity persists for niobium has been reported to be 160 mT for continuous wave (CW)[1] and 165 mT for pulsed operation,[2] both experimental results around 1.5 K. When extrapolated to $T = 0$ K with the usual parabolic dependence, the maximum $B_{RF\text{-critical}}$ ($T = 0$ K) to date was 170 mT. In this work we show that the RF critical magnetic field at $T = 0$ K must be no lower than a range from 231 to 245 mT. This value is also somewhat higher than the theoretical prediction.

II. THE CHALLENGE OF REACHING HIGH MAGNETIC FIELDS

The difficulty with trying to reach high RF magnetic fields in niobium microwave cavities is that one is usually stopped by phenomena such as thermal breakdown, originating at a small imperfection located in the high magnetic field regions of the cavity, or by field emission currents emanating from the high electric field regions. Both mechanisms absorb the available RF power, but thermal breakdown is more severe.

If thermal breakdown is reached at a spot in a high magnetic field, a fast rising pulse of high power makes it possible to continue raising the field, at the expense of an expanding normal conducting (NC) region. With this approach, it becomes possible to impose higher surface fields on other regions of the SC surface that are still superconducting. Hence the usual thermal breakdown field limiting mechanisms can be superseded by pulsed high power. Of course the $Q_0$ of the cavity at the high field is substantially lower, because of the growing NC region and increasing field emission currents.

Previous analysis[3] of the HPP phenomena has shown that thermal breakdown of the superconducting surface occurs in times which are slow enough that the field level at which thermal breakdown occurs in CW conditions was exceeded by up to 40% for times on the order of tens of microseconds.

Therefore, to reach high fields in spite of field emission and thermal breakdown, we use high power (typically 100 - 150 kilowatts). In order to surpass local thermal breakdown, the filling time of the cavity ($\tau = Q_L/\omega$) is shortened by lowering the loaded quality factor of the cavity ($Q_L$) defined by $1/Q_L = 1/Q_{ext} + 1/Q_0$, which is accomplished by lowering the quality factor ($Q_{ext}$) of the input coupler. $Q_{ext}$ is decreased by driving the coupling antenna closer to the iris opening of the cavity. $Q_{ext}$ values are adjustable from $10^{10}$ to $10^6$. To withstand the high power levels in the low temperature environment, we use short pulses (100 to 500 $\mu$s) at a low rep rate (1 Hz), so that the average power is low (tens of watts).

Experience with high pulsed power operation of SC cavities has shown that high fields can be reached in spite of field emission loading. If the high power is available it is possible to access high fields even though the unloaded quality factor ($Q_0$) of the cavity falls with the exponentially rising emission currents. More importantly, at the higher field levels, some sources emit so much current that they self destruct, or process. Sources of field emission are eliminated by an explosive mechanism. Once the emission current decreases, higher fields can be reached and more sites

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destroyed. A complete description of the processing phenomena can be found in the Ph.D. dissertation which covered HPP processing of 3 GHz RF cavities.[2]

III. The Method for Determining \( E_{\text{peak}} \)

The field level reached during pulsed operation depends on the power, the pulse length \( (t_{RF}) \), \( Q_{ext} \) and \( Q_0 \) as follows. At a given incident power level \( (P_{inc}) \), the equilibrium (CW) field reached would be:

\[
E_{\text{peak}}|_{t \to \infty} = \frac{E_{\text{peak}}}{E_{\text{acc}}} \sqrt{\left( \frac{4\beta}{(1+\beta)^2} \right) \left( \frac{P_{inc}}{Q} \right) \left( \frac{R}{Q} \right)} Q_0
\]

(1)

where \( E_{\text{peak}} \) is the peak electric field on the surface of the cavity, \( E_{\text{acc}} \) is the average accelerating gradient experienced by a particle traversing the cavity in phase with the cavity fields (equal to 0.5 * \( E_{\text{peak}} \) in the cavities tested here), \( Q_0 \) is defined above, \( \beta \) is the input coupling factor (equal to \( Q_0/Q_{ext} \)), \( P_{inc} \) is the power incident on the cavity, \( L \) is the active length of the cavity (5 cm for these cavities), and \( R/Q \) is the shunt impedance - a geometrical constant useful for defining the accelerating field (2100 Ohms/m in these cavities). This equation is valid for the cavity fields under steady state or CW conditions. A fixed output probe is calibrated for electric field level during CW measurement, in order to measure the fields under transient (pulsed) conditions.

In pulsed operation, the field at time \( t_{RF} \) is given by

\[
E_{\text{peak}}(t_{RF}) = E_{\text{peak}}|_{t \to \infty} \left(1 - \exp \left( -\frac{t_{RF}}{2\tau} \right) \right)
\]

(2)

where \( \tau \) is the cavity's characteristic filling time, as defined above. In the cavities studied here, the peak surface magnetic field is related to the peak electric field by the relationship:

\[
\frac{B_{\text{peak}}}{E_{\text{peak}}} = 2.3 \frac{\text{mT}}{\text{MV/m}}
\]

(3)

IV. The Method for Determining \( Q_0 \)

During the short RF pulse, the \( Q_0 \) cannot easily be determined, although the instantaneous field is known at all times. The extreme coupling of the input antenna to the cavity at the start of the RF pulse is the source of the difficulty. \( Q_0 \) measurement is also exacerbated by transient nature of the dissipative phenomenon such as field emission or thermal breakdown. However the peak field \( E_{\text{peak}}(t) \) reached is well determined from the transmitted power \( P(t) \) through the previously mentioned calibrated output probe.

In order to more fully investigate the transient \( Q_0 \) of the cavities, we developed an analysis which models the increasing field emission loading and the rapid growth of the NC region during the thermal breakdown process, which will be developed below. From this analysis we successfully reconstruct the measured instantaneous field \( E_{\text{peak}}(t) \) in the cavity. The reconstructed \( E_{\text{peak}}(t) \) gives us confidence that the \( Q_0(t) \) is properly modeled.

To obtain \( Q_0 \) as a function of total losses, we carried out the following analysis. The dissipated power is composed of the following terms:

\[
P_{\text{diss}} = P_{\text{SC}} + P_{\text{FE}} + P_{\text{NC}}
\]

(4)

The first term is the superconducting losses, composed of terms due to both the BCS and residual surface resistances. \( P_{\text{SC}} \) is derived from the CW \( Q_0 \) vs. \( E_{\text{peak}} \) curve at low fields. The second term in equation 4 is the field emission loss, which is modeled as
\[ P_{FE} = a\left(\frac{E_{peak}}{E_{peak}}\right)^2 \exp\left[-\frac{b}{E_{peak}}\right] \]  

(5)

where \( a \) and \( b \) are obtained by fitting the high field CW \( Q_0 \) vs. \( E_{peak} \) behavior. The final term in equation 4 is the NC or thermal breakdown loss, and it is modeled as

\[ P_{NC} = \frac{1}{2} R_{NC} \left(H_{peak}\right)^2 \pi (r_{NC})^2 \]  

(6)

We assume that the cavity has a single breakdown region which activates at a fixed field \( H_{BD} \). When \( H_{BD} \) is surpassed, a circular NC region begins to grow on the RF surface. We find the normal conducting radius \( r_{NC} \) via the following:

\[ r_{NC}(t) = \int_0^t dt \left[ v_{NC}\left[H_{peak}(t)\right]\right] \]  

(7)

While the fields are increasing, the expansion velocity of the NC region \( v_{NC} \) is given by

\[ v_{NC}\left[H_{peak}(t)\right] = A + B \left[H_{peak}(t) - H_{BD}\right]^2 \]  

(8)

Starting values for \( A \) and \( B \) are obtained from the simulative program Transient_HEAT\(^4\) and then are varied (over a small range) in order to self-consistently reproduce all data from a given experiment. When the fields begin to decrease due to the growth of the NC region, we maintain \( v_{NC} \) at the highest value reached until the surface field falls below 1/2 of the \( H_{BD} \). At this point \( v_{NC} \) is allowed to decrease exponentially. This relationship was determined empirically to obtain the best fit to \( P(t) \).

We emphasize that any other combination of loss mechanisms which sum up to the \( Q_0(t) \) used would be equally valid. The point here is that the trial solution \( Q_0(t) \) which we constructed does produce the experimentally measured \( P(t) \). We constructed \( Q_0(t) \) from plausible loss mechanisms to estimate how strong is the field emission loading and how large the normal conducting region grows during the high power pulse.

![Graph](image)

**Figure 1.** Comparison of the results listed here with the previous maximum measured field, and with the theoretical critical field behavior.
### Table 1. CW Performance of CAVITIES 1-9 AND 1-10.

<table>
<thead>
<tr>
<th>Cavity</th>
<th>maximum $E_{\text{peak}}$</th>
<th>$Q_0$ at max. $E_{\text{peak}}$</th>
<th>Limiting Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>52 MV/m</td>
<td>$8 \times 10^8$</td>
<td>Low Q, available power</td>
</tr>
<tr>
<td>1-10</td>
<td>58</td>
<td>$1 \times 10^9$</td>
<td>Field emission, available power</td>
</tr>
</tbody>
</table>

#### V. Results

In the experiments to be described, we tested two 1-cell 3 GHz cavities. The cavities were made from Russian high RRR Niobium (RRR = 700) in order that the purity, and therefore the thermal conductivity, be as high as possible. By maximizing the thermal conductivity, we delay the onset of thermal breakdown phenomena in the cavities. Table 1 summarizes the CW performance of the two cavities. Table 2 summarizes the pulsed high power performance. Figure 1 shows the theoretically expected parabolic dependence of critical magnetic field with temperature.

**Figure 2.** Photographs of oscilloscope traces showing forward and transmitted power during HPP. The left picture is from the experiment with cavity 1-9, with peak $P_{\text{inc}} = 170$ kW, and maximum $E_{\text{peak}} = 86$ MV/m. The right picture is from the experiment with cavity 1-10, with peak $P_{\text{inc}} = 120$ kW, and maximum $E_{\text{peak}} = 81.5$ MV/m.

**Figure 3.** Photographs of oscilloscope traces showing forward and transmitted power during HPP. The left picture is from the experiment with cavity 1-9, with peak $P_{\text{inc}} = 170$ kW, and maximum $E_{\text{peak}} = 86$ MV/m. The right picture is from the experiment with cavity 1-10, with peak $P_{\text{inc}} = 120$ kW, and maximum $E_{\text{peak}} = 81.5$ MV/m.
### Table 2. Pulsed (HPP) Performance of Cavities 1-9 and 1-10.

<table>
<thead>
<tr>
<th>Cavity</th>
<th>Temp.</th>
<th>$P_{RF}$</th>
<th>$t_{RF}$</th>
<th>$Q_{ext}$</th>
<th>max. $E_{peak}$</th>
<th>max. $B_{peak}$</th>
<th>$t_{(max\ E)}$</th>
<th>$Q_0$ @ max. $E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>1.5 K</td>
<td>150 kW</td>
<td>385 µs</td>
<td>$2.8 \times 10^6$</td>
<td>90.5 MV/m</td>
<td>208.6 mT</td>
<td>80 µs</td>
<td>8 x $10^6$</td>
</tr>
<tr>
<td>1-10</td>
<td>1.5</td>
<td>115</td>
<td>300</td>
<td>$5.6 \times 10^6$</td>
<td>82.3</td>
<td>189.3</td>
<td>110</td>
<td>9 x $10^6$</td>
</tr>
<tr>
<td>1-10</td>
<td>4.2</td>
<td>130</td>
<td>175</td>
<td>$7.0 \times 10^6$</td>
<td>82.3</td>
<td>189.3</td>
<td>110</td>
<td>9 x $10^6$</td>
</tr>
</tbody>
</table>

The dotted line shows the previous maximum measured magnetic field, and the plotted points are the results of cavities 1-9 and 1-10.

Figure 2 shows two photographs of the experimentally measured transmitted power in the pulsed mode, near the maximum fields obtained. Electric field is proportional to the square root of the transmitted power. The first photo is from the experiment of cavity 1-9, taken at $T = 1.5$ K, with maximum $E_{peak} = 86.0$ MV/m. The second photo is from the experiment of cavity 1-10, and was taken at $T = 4.2$ K, with $E_{peak} = 81.5$ MV/m.

![Graph showing incident power, $E_{peak}$, $Q_0$, and $P_{inc}$ over time](image)

**Figure 4.** Simulation values of $P_{inc}$, $E_{peak}$, and $Q_0$ shown as functions of time for the two simulations shown in Figure 3. The upper plot here corresponds to the left plot in Figure 3, and the lower plot to the right plot of Figure 3. Inspection shows that at the point of maximum $E_{peak}$, the $Q_0$ of the cavity has dropped to only slightly less than $1 \times 10^7$.  

TESLA-Report 1993-45
Figure 5. Plots of the simulated radius of normal conducting zone, as a function of time. The plots here correspond to the plots shown in Figures 3 and 4.

We obtained $Q_0$ at the maximum field in Table 2 by the methods described in the previous section. Figure 3 shows the calculated $P_r(t)$ for each of the photographs shown in Figure 2. Figure 4 shows the applied $P_{inc}(t)$, and the calculated $E_{peak}(t)$ and $Q_0(t)$ for each of the simulations shown in Figure 3.

Figure 5 shows the computed $r_{NC}(t)$ for the simulations shown in the previous two figures. We conclude that at the maximum $B_{peak}$ of Table 2, at least 95% of the cavity area is superconducting. We especially pay attention to the measurements on cavity 1-10 at $T = 4.2$ K. If we use the usual parabolic dependence of $B_{critical}$

$$B_{critical}(T) = B_{critical}(T = 0) \left[ 1 - \left( \frac{T}{T_{critical}} \right)^2 \right]$$  \hspace{1cm} (9)

then with maximum $B(T = 4.2K) = 189.3$ mT, the zero temperature $B_{RF-critical}$ for niobium is at least 238.5 mT. We estimate our experimental uncertainty to be approximately 3 MV/m in $E_{peak}$, corresponding to 7 mT in $B_{peak}$, therefore we expect the minimum value for $B_{RF-critical} (T = 0)$ to be between 231 and 245 mT.

We can further test the validity of our approach by modeling the change in maximum field reached and the time to this maximum. Once the onset of thermal breakdown is crossed, the time to reach the maximum field becomes shorter as the rise time of the fields is increased (in order to exceed the breakdown field). Our model successfully reproduces this behavior. We experimentally measured maximum $E_{peak}$ and $t(E_{peak})$ with $Q_{ext}$ and $t_{RF}$ fixed, and then varied the
Figure 6. Comparison of measured and simulated maximum $E_{\text{peak}}$, and the time to the maximum value of $E_{\text{peak}}$, as a function of incident power, with all other parameters held constant. The input coupler $Q_{\text{ext}}$'s were $4.7 \times 10^6$ and $1.0 \times 10^7$, for the upper and lower plots, respectively. The measurements were made during the experiments on cavity 1-9.

VI. DISCUSSION

Experimentally the RF critical field has been shown to be equal to the superheating critical field ($B_{sh}$) in Type I superconductors, such as indium, tin and lead. For example, $B_{RF}/B_{\text{critical}} = 3.5, 3.1$ and 1.5 have been measured near $T_c$ for these materials, respectively, in agreement with theoretical predictions for $B_{sh}/B_{\text{critical}}$. For Type II alloys such as Sn-In, In-Bi or Nb-Ti, $B_{RF}$ well above $H_{c1}$ has been measured. So it is clear that the field at which DC magnetic flux will first penetrate a Type II superconductor is not the limiting RF critical field. When compared with $B_{sh}$,
the data for the type II superconductors are not as definitive. The error bars are quite large and the measurements were done very near $T_c$. So the conclusion for type II superconductors that the critical RF field is equal to the superheating critical field is not as strong as for Type I.

The best estimate for Niobium for $B_{sh} \ (0 \text{ K})$ is 210 mT, from experimentally measured values for $B_{\text{critical}}$ ($0$) and $\kappa (T)$ as discussed in reference 6. Of course this estimate is subject to experimental uncertainties. Our experiments show that $B_{RF \text{ critical}}$ at $0 \text{ K}$ is larger than 231 mT, which is only 11% larger than the superheating field prediction. This is too small a margin to state that the new result is higher than $B_{sh}$. However the new result is a significant increase over the previous best experimental result $B_{RF \text{ (1.5 K)}} = 170 \text{ mT}$, and perhaps even slightly greater than $B_{sh}$.

VII. REFERENCES