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Longitudinal and Transverse Wakes for the TESLA Cavity

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Introduction

While the R/Q and the Q of the higher order modes are used for multibunch instability studies, the longitudinal and transverse wake functions are needed for single bunch emittance growth and energy spread calculations. We report here the results of time domain computations carried out on the 9-cell TESLA cavity, whose geometry was proposed at the DESY january '92 meeting.

Time domain computations

For short bunch and long structures like the 9-cell TESLA cavity, the computation of wake potentials requires a large memory capability and is time consuming. Furthermore, as the beam tube diameter is larger than the in-cell iris apertures, the bunch cannot travel at a distance equal to the beam tube radius from the beam axis, method usually used because the computation can be stopped as soon as the bunch emerges from the cavity at the beam tube surface, where the forces are zero [1]. Long exit tubes are then required to be sure that all the downstream scattered wakefields interacting with the bunch have been taken into account. A total structure length of 2 meters, including cavity and beam tubes, is necessary for $\sigma = 1$ mm bunches, for which mesh sizes have to be lower than 0.2 mm. The code ABCI [2] was used because unequal mesh sizes in the axial and radial directions are allowed and the number of mesh points was fixed to 2 millions.

Longitudinal wakes

The voltage gain of a particle traversing a cavity depends on its longitudinal position z with respect to the bunch center and is the sum of the RF wave and the wake effects :

$$\Delta V\ (z) = E_{acc}L \cos\left(\phi_{RF} + \frac{2\pi}{\lambda_{RF}}\ z \right) + Q\, W\ (z)$$

where Q is the charge of the bunch and W (z) is the bunch wake, also called the bunch potential. For a gaussian distribution bunch, it can be expressed in terms of the delta wake function W_δ, defined as the voltage loss per unit charge of a test particle traveling with the distance s from a point-like bunch :

$$W(s) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^{\infty} W_\delta(s')\, e^{-\frac{(s-s')^2}{2\sigma^2}}\, ds'$$

This bunch wake, induced by a bunch of bunchlength σ, which is needed for energy spread calculations, is directly computed and given by the codes TBCI or ABCI and we have not consequently to worry about the delta wake.

The total loss factors for one cavity (total energy lost by a bunch of unit charge during one passage) are listed in table 1 for 3 bunchlengths. The plot of the bunch wake for σ = 1 mm is showed on figure 1 and the values are tabulated in table 2.

Transverse wakes

The transverse momentum kick experienced by a particle at longitudinal position z within the bunch during the traversal of a cavity depends on the displacement of all the previous charges

$$\Delta p_\perp = \frac{e}{c} \int_z^{\infty} \rho(z')\, W_{\delta\perp}(z'-z)\, x(z')\, dz'$$

where ρ (s) is the charge density and $W_{\delta\perp}$ (s) is the transverse delta wake, defined as the transverse impulse (times c/qQx) received by a test charge q traveling with the distance s from a point-like bunch of charge Q displaced by x from the beam axis. We need this time the delta wake, while the cavity codes give only the transverse bunch wake for a bunch of length σ, with offset x from beam axis

$$W_\perp(s) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^{\infty} W_{\delta\perp}(s')\, e^{-\frac{(s-s')^2}{2\sigma^2}}\, ds'$$

and a tranverse impulse factor for a bunch offset x, which is the total transverse momentum given to the bunch of charge Q by its own wake, times c/Q^2 x

$$k_\perp = \frac{c}{Q^2 x} \Delta p_\perp = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} W_\perp(s)\, e^{-\frac{s^2}{2\sigma^2}}\, ds'$$

If the emittance growth simulations are performed on slices from, say - 5 σ to + 5 σ, we need the delta wake function on the scale 0 - 10 σ.

The transverse impulse factor was computed again with the ABCI code for different bunchlengths and the results are showed on table 3.

We have first to determine the behaviour of the wake function $W_{\delta\perp}$ (s) at small s, where the high frequency part of the impedance dominates. Many authors have studied the asymptotic behavior of cavity impedance (see for example the review paper [3]), which exhibits a $\omega^{-1/2}$ or $\omega^{-3/2}$ rollof for the longitudinal case, according to wether the structure behaves like an isolated cavity or a periodic array. The transition between both regimes was also studied for a finite number of cells [4]. If the longitudinal decreases as $\omega^{-1/2}$, the transverse impedance decreases as $\omega^{-3/2}$ and thus the transverse impulse factor $k_{\perp\,(\sigma)}$ and delta wake $W_{\delta\perp}$ (s) exhibit asymptotic dependances of $\sigma^{1/2}$ and $s^{1/2}$.

Figure 2 shows the computed transverse impulse factor $k_\perp$ of the 9-cell TESLA cavity for small bunchlengths (sigma = 1, 1.5 and 2 mm) and a fit of $k_\perp$ suggests a $\sigma^{1/2}$ rollof.

We have now to find a fit of the delta wake $W_{\delta\perp}$ (s) which covers a larger range, say 0-10 mm. We propose the following transverse impulse factor fit :

$$k_{\perp\,(\sigma)} = \sum_n a_n \sigma^{\alpha_n} \qquad \text{with } \alpha_n = 0.5$$

The corresponding delta wake fit has then the following form :

$$W_{\delta\perp\,(s)} = \sum_n F(\alpha_n)\ a_n\, s^{\alpha_n} \qquad \text{with } \alpha_n = 0.5$$

where the function F (α_n) has been computed using the previously given definitions of $k_\perp$ and $W_{\delta\perp}$ and is plotted on figure 3 for α_n between 0 and 2.

A reasonable fit of the transverse impulse factor with only two terms can cover the whole range and is showed on figure 4. Using the function F (α_n) , we obtain finally the transverse delta wake fit, required for single bunch BBU calculations

$$W_{\delta\perp\,(s)} = 1294\ s^{0.5} - 7570\ s^{1.183} \qquad \text{in V/pC/m}$$

For example, the resulting transverse bunch wake, calculated from this delta wake fit for a bunch sigma of 1.5 mm, is showed on figure 5, along with the bunch wake given by the ABCI code for comparison.

[1] T. Weiland, Nuc. Inst. and Meth. 216 (1983), 31-34

[2] Y.H. Chin, CERN/LEP-TH/88-3

[3] J. Bisognano, CEBAF-PR-89-016, "Impedance and Wakefields beyond Cutoff"

[4] S.A. Heifeits and S.A. Kheifeits, SLAC PUB-4625, "High Frequency Limit of the Longitudinal Impedance of an Array of Cavities"

σ (mm)	1	1.5	2
total loss factor (V/pC)	10.6	9.0	8.0

Table 1 : Total loss factor (V/pC) for one cavity

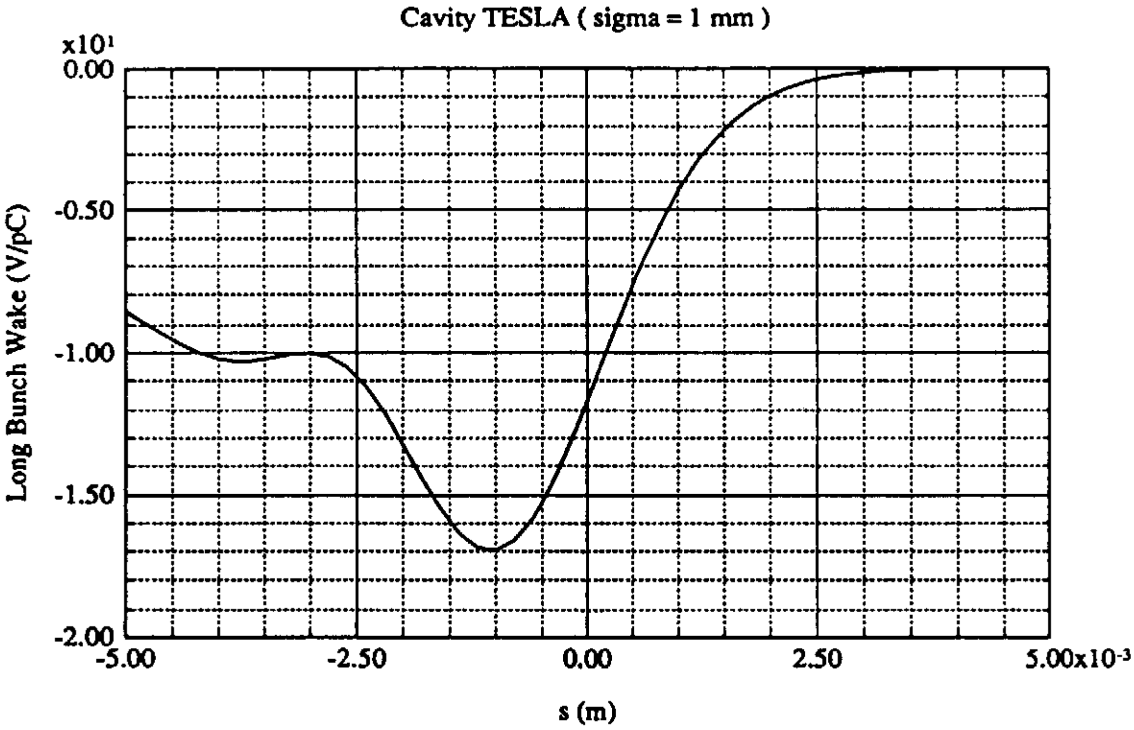


Figure 1 : Longitudinal bunch wake (V/pC) for one cavity and sigma = 1 mm
(head of the bunch to the right)

-0.500000E-02	-0.851970E+01
-0.480000E-02	-0.894200E+01
-0.460000E-02	-0.934880E+01
-0.440000E-02	-0.971730E+01
-0.420000E-02	-0.100180E+02
-0.400000E-02	-0.102200E+02
-0.380000E-02	-0.103060E+02
-0.360000E-02	-0.102790E+02
-0.340000E-02	-0.101730E+02
-0.320000E-02	-0.100550E+02
-0.300000E-02	-0.100150E+02
-0.280000E-02	-0.101460E+02
-0.260000E-02	-0.105240E+02
-0.240000E-02	-0.111840E+02
-0.220000E-02	-0.121050E+02
-0.200000E-02	-0.132110E+02
-0.180000E-02	-0.143830E+02
-0.160000E-02	-0.154750E+02
-0.140000E-02	-0.163410E+02
-0.120000E-02	-0.168610E+02
-0.100000E-02	-0.169590E+02
-0.800000E-03	-0.166050E+02
-0.600000E-03	-0.158240E+02
-0.400000E-03	-0.146810E+02
-0.200000E-03	-0.132660E+02
0.000000E+00	-0.116840E+02
0.200000E-03	-0.100370E+02
0.400000E-03	-0.841650E+01
0.600000E-03	-0.689490E+01
0.800000E-03	-0.552360E+01
0.100000E-02	-0.432920E+01
0.120000E-02	-0.332350E+01
0.140000E-02	-0.250100E+01
0.160000E-02	-0.184650E+01
0.180000E-02	-0.133910E+01
0.200000E-02	-0.954850E+00
0.220000E-02	-0.670370E+00
0.240000E-02	-0.464020E+00
0.260000E-02	-0.317080E+00
0.280000E-02	-0.214100E+00
0.300000E-02	-0.142890E+00
0.320000E-02	-0.941700E-01
0.340000E-02	-0.610900E-01
0.360000E-02	-0.387700E-01
0.380000E-02	-0.238270E-01
0.400000E-02	-0.139570E-01
0.420000E-02	-0.761200E-02
0.440000E-02	-0.373600E-02
0.460000E-02	-0.155860E-02
0.480000E-02	-0.491800E-03
0.500000E-02	-0.848300E-04

Table 2 : Bunch wake values for one cavity and sigma = 1 mm
from - 5 σ (rear of the bunch) to + 5 σ.(head of the bunch)

s (mm)	1	1.5	2	3	5	7.5	10
$k_\perp$ (V/pC/m)	18.08	22.63	25.72	30.26	35.59	40.49	43.45

Table 3 : Transverse impulse factor (V/pC/m) for one cavity

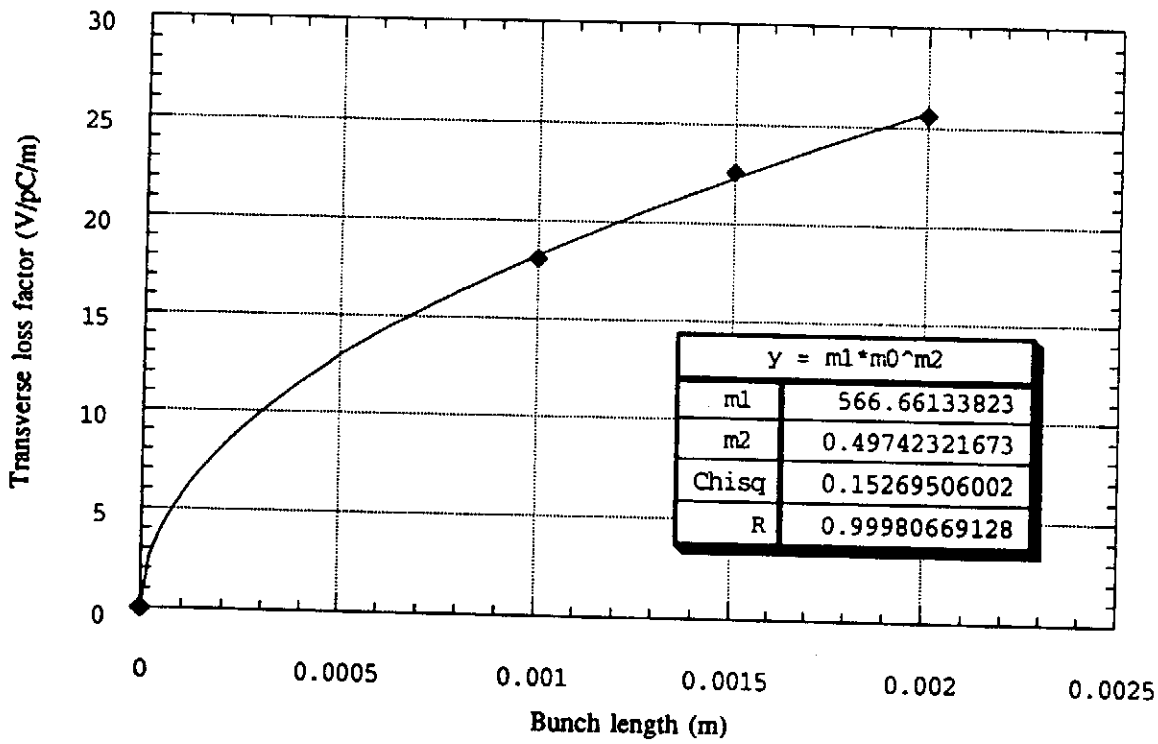


Figure 2 : Tranverse impulse factor (V/pC/m)
for the 9-cell TESLA cavity for small bunchlengths.

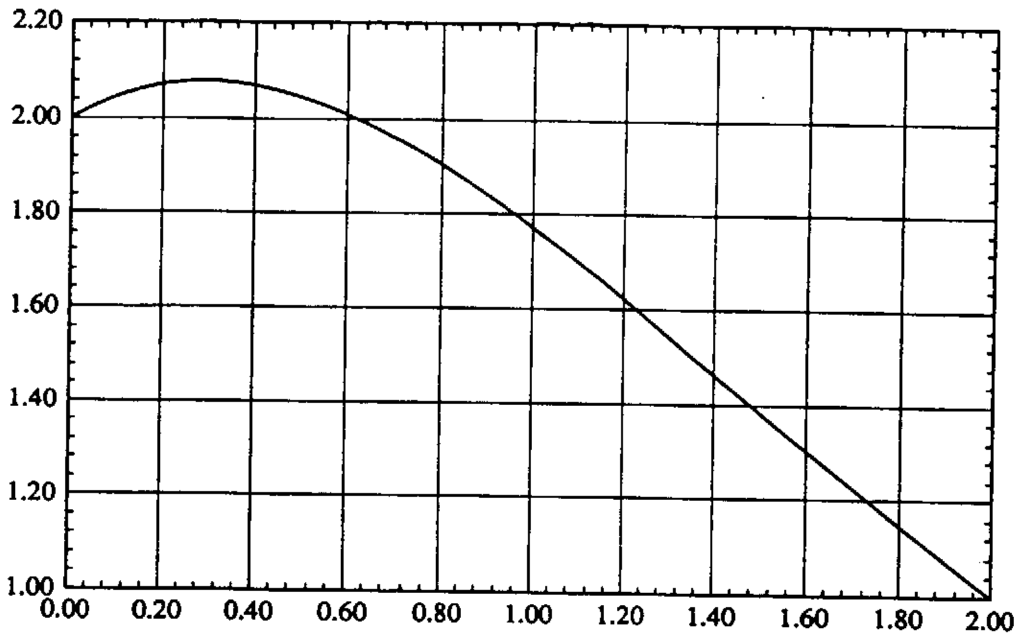


Figure 3 : Graph of the function F (α_n)

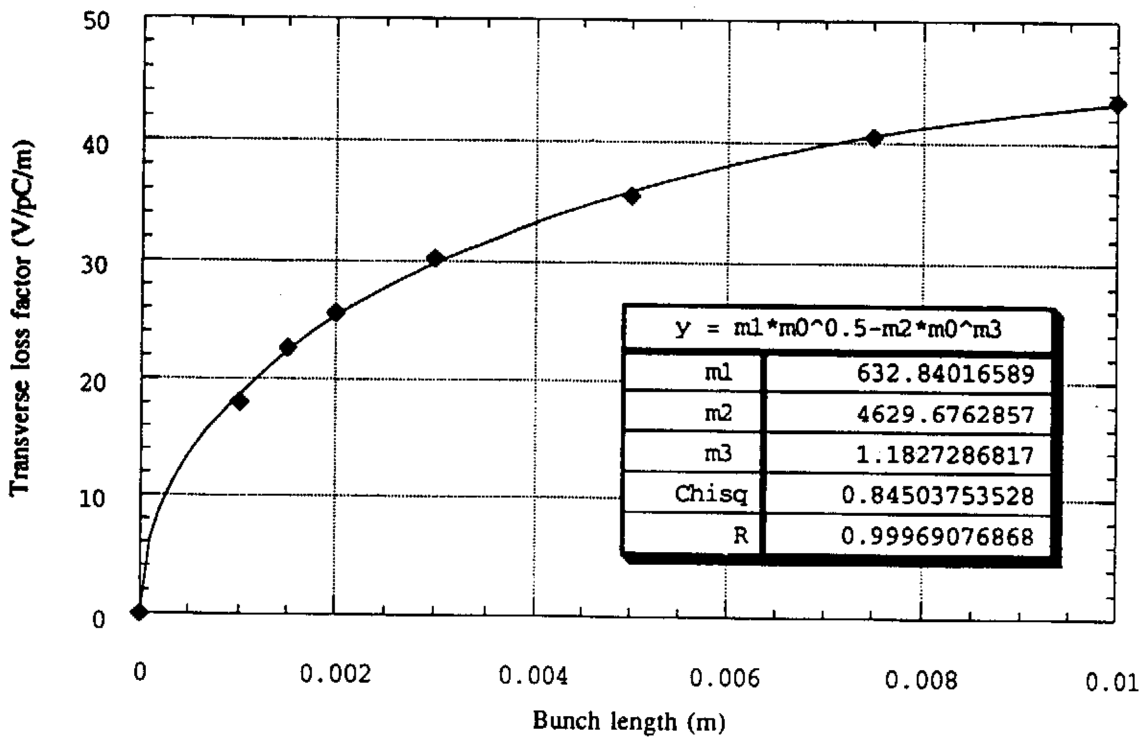


Figure 4 : Fit of the transverse impulse factor (V/pC/m)

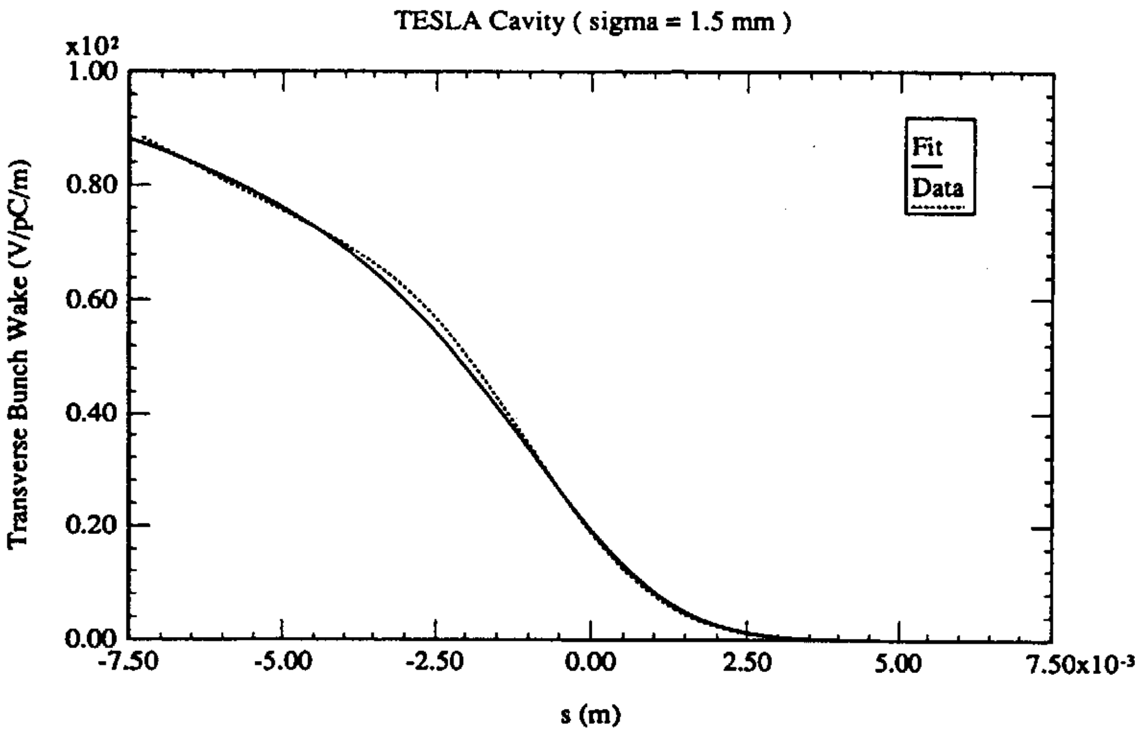
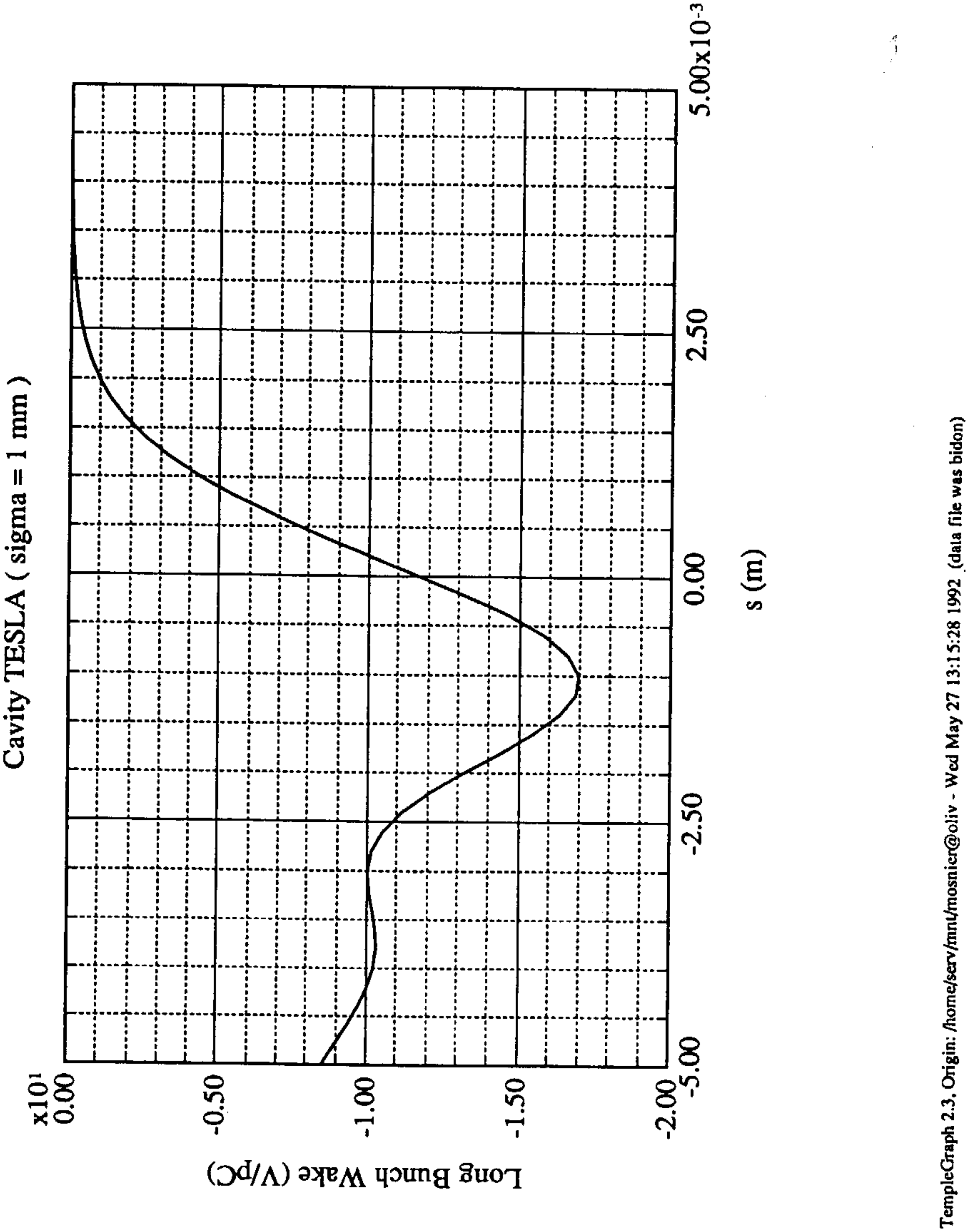
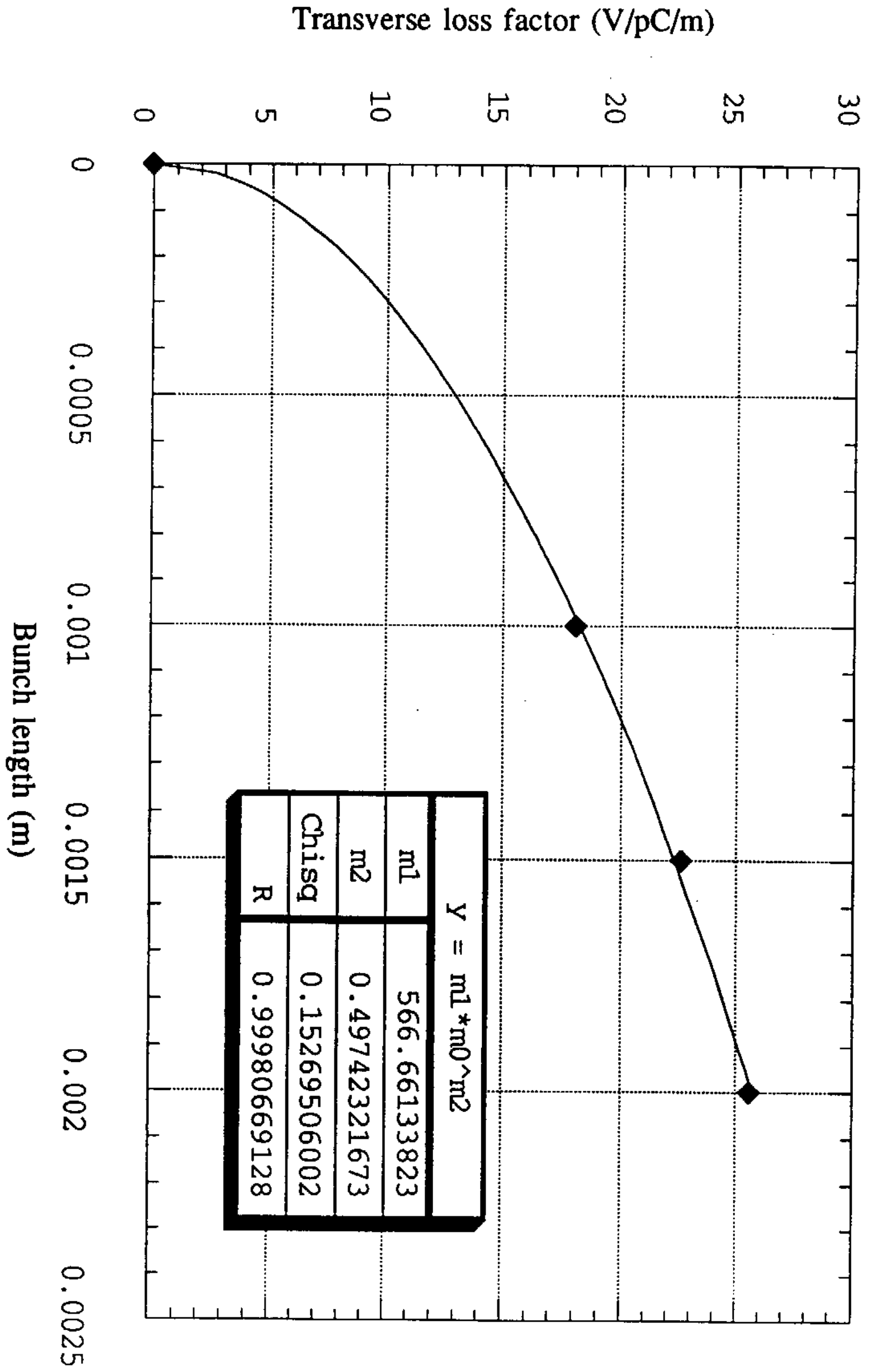
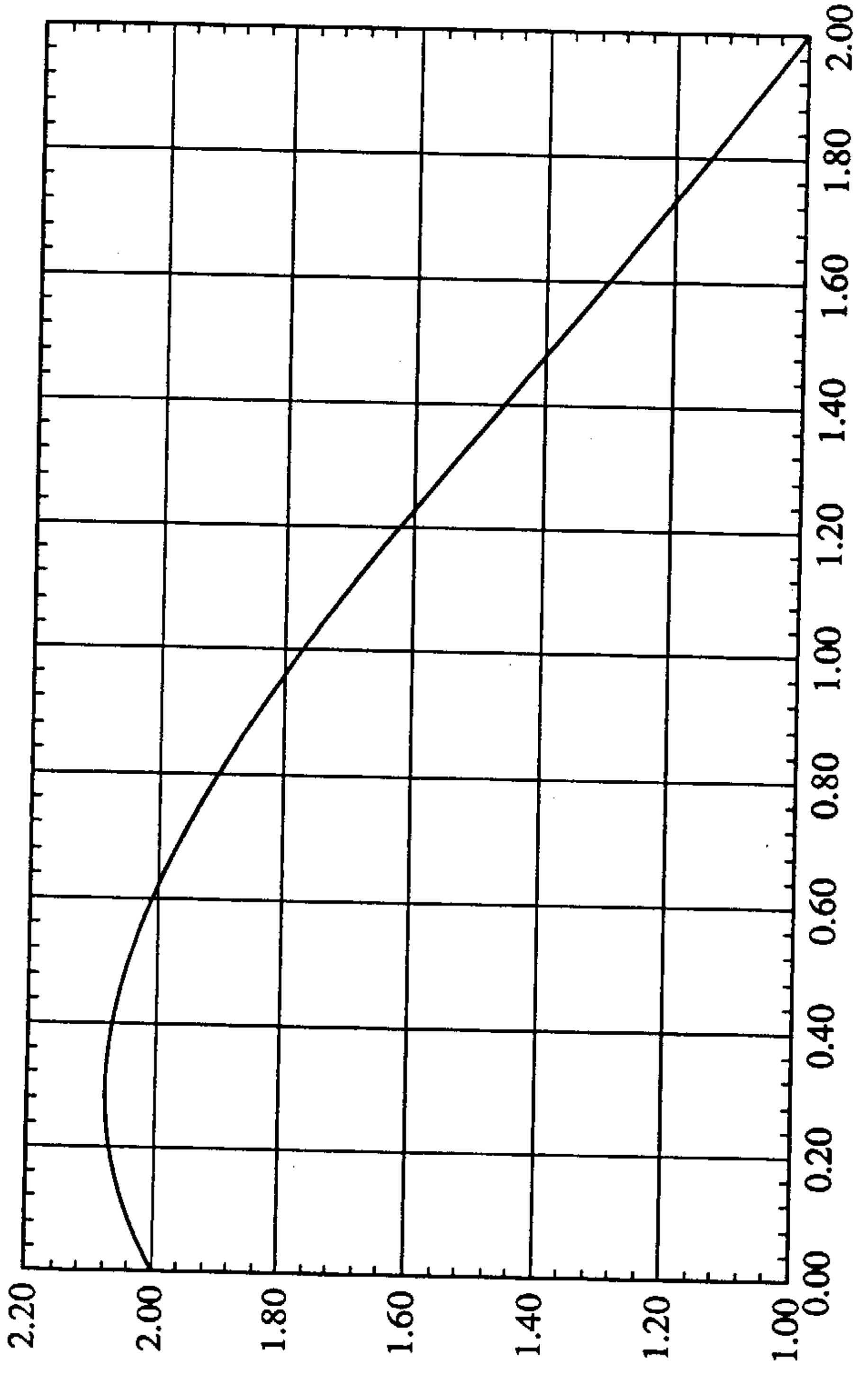


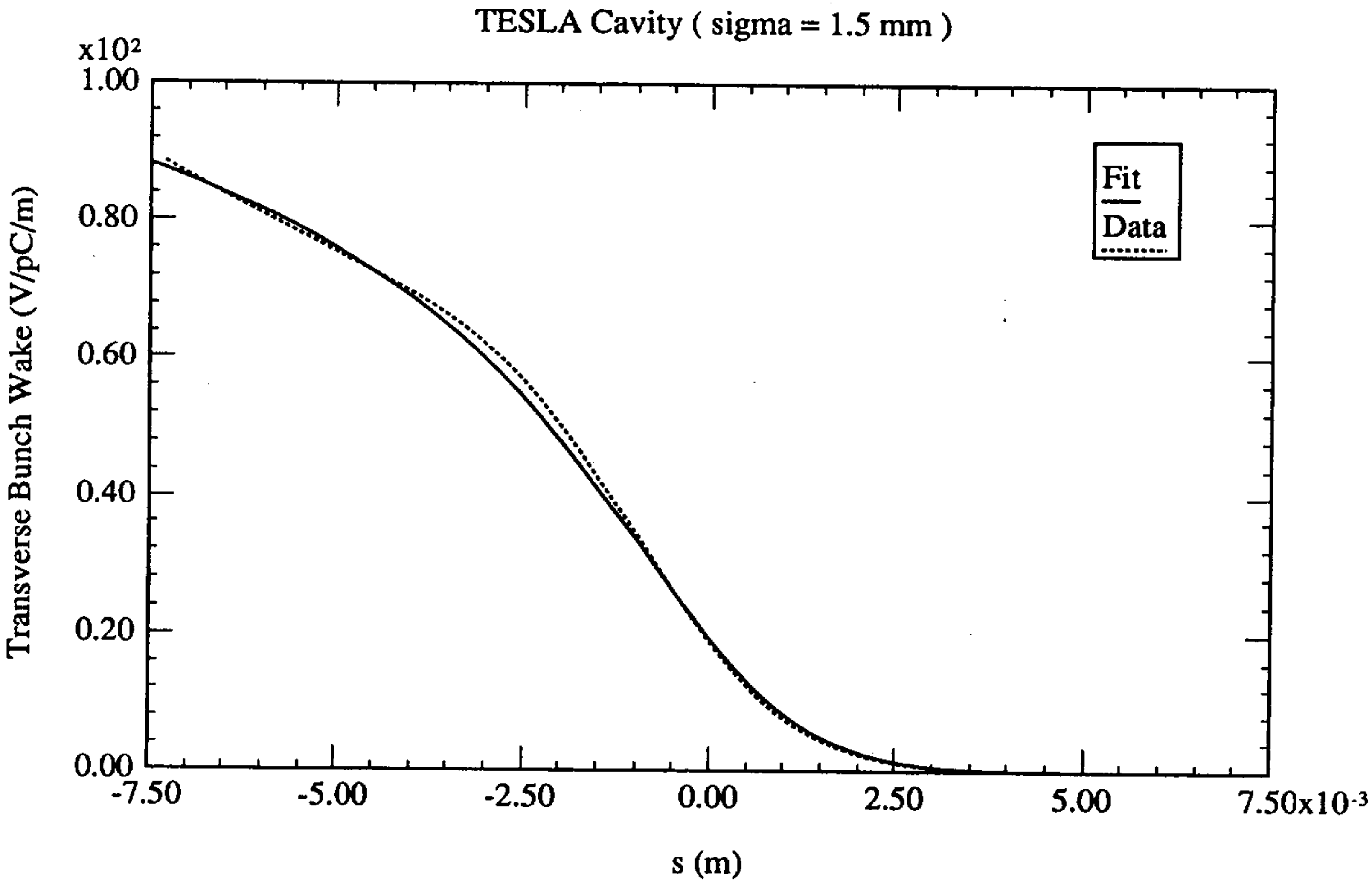
Figure 5 : transverse bunch wake with $\sigma = 1.5$ mm
calculated with the delta wake fit (solid) and computed by ABCI.


Cavity TESLA (sigma = 1 mm)
Long Bunch Wake (V/pC)
x10¹
0.00
-0.50
-1.00
-1.50
-2.00
-5.00
-2.50
0.00
2.50
5.00x10⁻³
s (m)
TempleGraph 2.3, Origin: /home/serv/mnt/mosnier@oliv - Wed May 27 13:15:28 1992 (data file was bidon)


Transverse loss factor (V/pC/m)
0
5
10
15
20
25
30
Bunch length (m)
0
0.0005
0.001
0.0015
0.002
0.0025
y = m1*m0^m2
m1 566.661333823
m2 0.497423321673
Chisq 0.152695506002
R 0.999806669128



TempleGraph 2.3, Origin: /home/serv/mnl/mosnier@sea2 - Wed Jun 3 16:04:06 1992 (data file was wakefl.res)



TempleGraph 2.3, Origin: /home/serv/mnt/mosnier@oliv - Tue Jun 2 12:36:55 1992 (data file was bidon)