Pepper pot design for space charge dominated high brightness beams

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Introduction

The Free Electron Laser currently under construction at the Tesla Test Facility (TTF) [1] at DESY requires an electron beam of 1nC charge and transverse and longitudinal emittances of 1π mrad mm and 20keVmm, respectively. First simulation results show that this beam parameters can be reached with an rf gun photo injector [2]. The injector consists of a 1½ cell rf gun, solenoids for compensation of space charge induced emittance growth and a booster cavity. Electrons are emitted from a photocathode driven by a laser pulse. In order to test the gun we will set up a low energy beam line parallel to the TTF linac consisting of the gun and a vacuum pipe with diagnostics only. The location close to the linac allows us to use the klystron and the laser that will be built up in the linac hall. A layout of the injector area of the TTF is shown in the appendix.
The gun works at a gradient of 50MV/m and will accelerate the electrons to an energy of about 5MeV. At this energy the beam dynamics is still influenced by space charge forces. Due to the compensation of the space charge induced emittance growth [3] the emittance is a function of time or longitudinal position, respectively. Figure 1 shows the beam emittance and related beam parameters as function of the position in the beam line z. This simulation results have been obtained after careful optimization of numerous parameters like the strength and the position of the focusing solenoid etc. The minimum emittance of 1.3π mrad mm is reached at about 2m behind the gun cavity. In order to verify our calculations we want to measure the phase space distribution of the beam at the position of the emittance minimum as function of various parameters. Table 1 summarizes the range of beam parameters that has to be expected during tuning procedures.

charge q	1nC
bunch length σ_L	1mm
current I	~100A
rms spot size σ_x	0.5 - 1.5mm
normalized emittance εn_{rms}	1 - 4π mrad mm
correlated beam divergence α/β	+...– 2 rad/m
uncorrelated beam divergence	0.2-0.8mrad

Table 1: Compilation of typical beam parameters.

It should be noted, that the predicted emittances for the test beam line are above 1π mrad mm for the following reasons:

1. At the time of the first tests the laser will work with a rise time of 5.8ps rather than 2.3ps as it is necessary for the smaller transverse emittances [2]. The emittance obtained with 5.8ps in simulations is 1.8π mrad mm.
2. The simulations show that the focusing of the fields in the booster cavity lead to a further reduction of the emittance. This effect is missing in the test beam line.

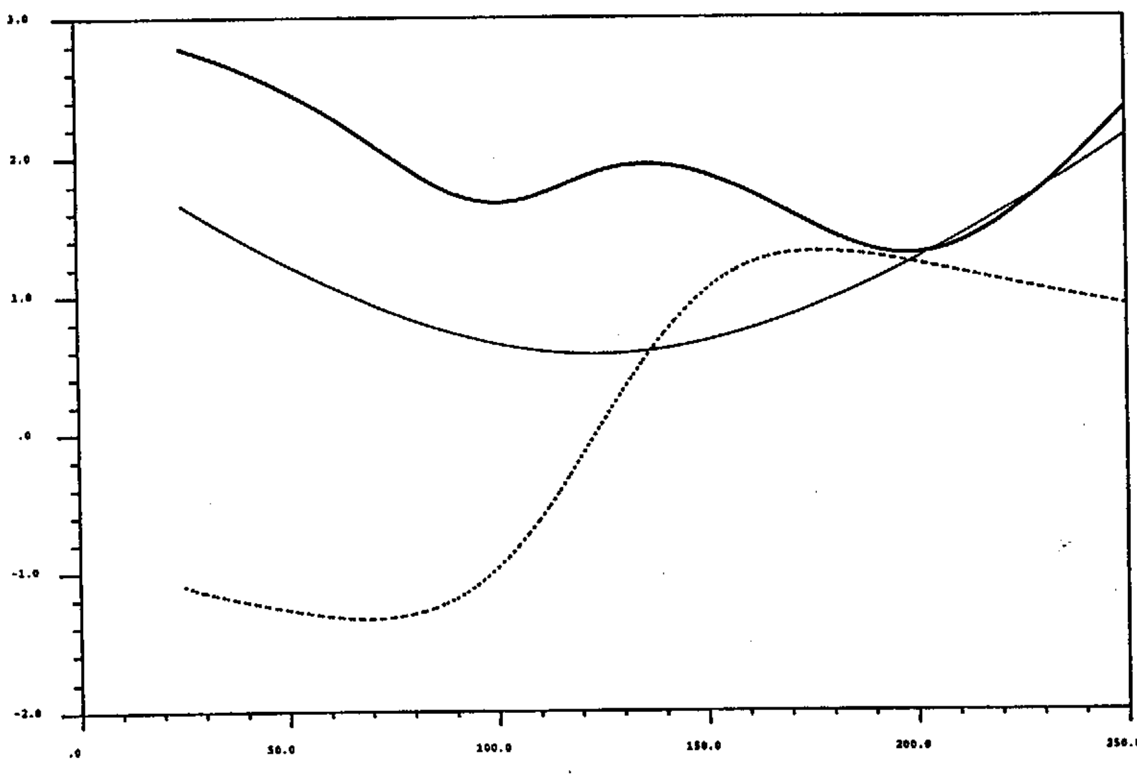

Figure 1 Beam parameters as function of the longitudinal coordinate z: Beam emittance π mrad mm (bold solid line); rms spot size mm (solid line); correlated beam divergence rad/m (dotted line)

Since the beam emittance in the test beam line will evolve under the influence of space charge forces, which in turn depend on the focusing that is applied, some commonly used emittance measurement techniques, like e.g. quadrupole scans and some tube and slit emittance techniques [4] can not be applied. In the following the design of a pepper pot will be discussed.

The formalism will also be applied to the case of a 20MeV beam, i.e. for the case of measurements in the FEL beam line behind the booster cavity.

The pepper pot

The pepper pot is a metal plate with a number of holes with a small radius compared to the size of the incoming beam. When this plate intercepts the beam path it cuts out of the beam a number of small beamlets. If the hole size is small enough the beamlets are no more space charge dominated and the spot size develops as function of the emittance only. The phase space area that will be accepted by a hole at position x_p with radius r and length s is a rhombic area described by the points: $(x_p - r, 0)$; $(x_p - r, \frac{2r}{s})$; $(x_p + r, 0)$; $(x_p + r, -\frac{2r}{s})$. If the hole is tilted by an angle φ with respect to the beam due to an alignment error the rhombic area is shifted to: $(x_p - r, \varphi)$; $(x_p - r, \frac{2r}{s} + \varphi)$; $(x_p + r, \varphi)$; $(x_p + r, \varphi - \frac{2r}{s})$ as sketched in Figure 2.

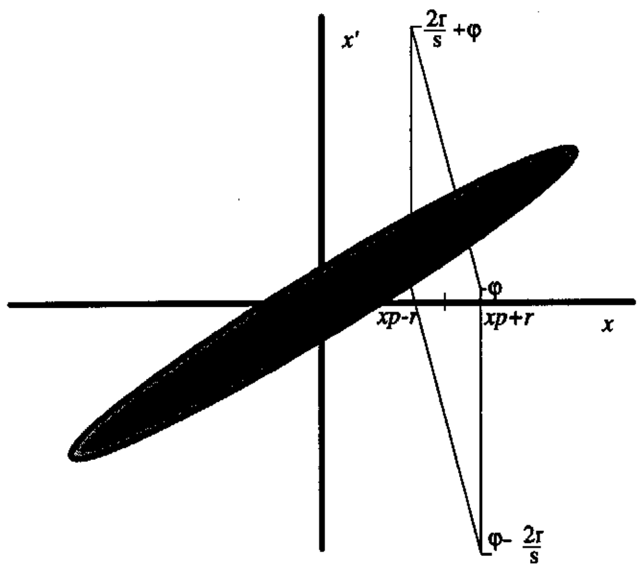

Figure 2 Phase space area cut out by a hole in the pepper pot.

In order to be negligible the alignment error has to be small compared to the opening angle of the hole $\frac{2r}{s}$.

The acceptance of the hole is reduced for a larger divergence due to the tilted cut in phase space. Only if the beam divergence is much smaller than the opening angle $\frac{2r}{s}$ the disturbance of the beam distribution can be neglected. If the divergence and the opening angle are comparable two effects can be distinguished: firstly the total intensity of particles passing trough holes positioned at large x is reduced and secondly the relative error due to the tilted cut off becomes more prominent. Both effects would lead to an underestimation of the beam emittance.

At a distance L behind the pepper pot the transverse distribution of the particles passing through the holes is observed on a screen.
The intensity distribution of a beamlet on the screen $I(\chi)$ is given by a convolution integral of the transverse beam distribution, denoted as $F(x)$, and the angular distribution $G(x')$ of the beamlet at the exit of the pepper pot hole:

$$
\begin{gathered}
I(\chi) = \int_{-r}^{r} F(x) \cdot G\left(\tfrac{\chi - x}{L}\right) dx \\
x' = \frac{\chi - x}{L} \\
\int G(x')dx' = 1
\end{gathered} \tag{1}
$$

If the spot size on the screen denoted as r_{screen} is much larger than the size r of the hole in the pepper pot, i.e. $r_{screen}/r \gg 1$, $F(x)$ can be treated as a δ-function and equation 1 reduces to:

$$
\begin{gathered}
I(\chi) = \frac{q}{r_0^{\,2}} G(x') \\
x' = \frac{\chi}{L}
\end{gathered} \tag{2}
$$

Equation 2 reveals directly the angular distribution of the beamlet, while the position of the particles is given by the position of the hole in the plate. If the distance between the holes is small so that several beamlets are generated the phase space of the incoming beam can be reconstructed along cuts parallel to the x'-axis.
The rms phase space ellipse of a beam can be written as:

$$
\begin{gathered}
\varepsilon_{rms} = \gamma x^2 + 2\alpha x x' + \beta x'^2 \\
x_{rms} = \sqrt{\varepsilon_{rms} \cdot \beta} \\
\beta\gamma - \alpha^2 = 1
\end{gathered} \tag{3}
$$

where α, β, γ are the twiss parameters. The phase space distribution of a space charge dominated beam is in general not an ellipse, but a rather bow tie like distribution. The rms emittance is defined in the simulations only as statistical quantity via rms values as:

$$
\begin{gathered}
\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x \cdot x' \rangle^2} \\
\langle\ \rangle = \text{second central moment of the distribution}
\end{gathered} \tag{4}
$$

Equation 4 can be applied directly to the phase space cuts that are reconstructed from the pepper pot measurements, thus yielding a quantity that can be compared to simulation results.
The phase space area cut out by a hole is not an ellipse, therefore it does not obey the standard envelope equation for a beam in a drift space. The beamlet distribution will be uniform in r. In order to estimate the spot size on the screen we will assume a uniform distribution also in r'. The spot size on the screen is then given as:

$$
r_{screen} = r + r' \cdot L \tag{5}
$$

The error that is introduced by assuming a δ-function in equation 1 the order $r/_{r_{screen}}$. This error leads to an overestimation of characteristic parameters, e.g. rms-values or FWHM values and hence to an overestimation of the emittance.
Assuming that the incoming beam is also uniform in r and r', the divergence of the beam can be estimated as:

$$r' = \frac{4\varepsilon n}{r_0 \cdot \gamma} \approx 0.4 \cdot 10^{-3}\,\text{rad} \qquad (6)$$

For the spot size of the beamlets only the uncorrelated beam divergence is of importance, while the correlated angular divergence leads to a transverse offset of the spot on the screen. Simulation results indicate that beamlets with smaller uncorrelated divergence as given by equation 6 can be generated if the hole is placed near the center of the distribution. Thus a minimum divergence of 0.2mrad will be assumed in the following. In order to resolve this divergence the condition:

$$0.2 \cdot 10^{-3}\,\text{rad} \cdot L >> r \qquad (7)$$

has to be fulfilled.

Space charge effects

If space charge forces are not sufficiently reduced in the drift space between the pepper pot and the screen the spot size of the beamlets is increased and the emittance will be overestimated. In order to estimate space charge effects a round beam with uniform distribution in r and r' will be assumed for the incoming beam and the outgoing beamlet in the following, thus $r = 2\sigma_r$ and $r' = 2\sigma_{r'}$ holds.
The differential equation for a cylindrical beam of uniform charge density under the condition of laminar flow is given as:

$$r'' = \frac{2 \cdot I}{I_A \cdot \gamma^3 \cdot r_{(z)}} \qquad (8)$$

$I =$ beamlet current

$r =$ beamlet radius

$I_A =$ Alfvén current $= 17$kA

$\gamma =$ Lorentz factor

An approximate solution of the differential equation is found by an integration of equation 8. In the limit that the beam expansion is dominated by the natural beam divergence the beam radius is given by:

$$r_{(z)} = r + r' \cdot z \qquad (9)$$

Including equation 9 into equation 8 a first order approximation of the beam radius including space charge effects is found:

$$r_{(z)} = \frac{2I}{I_A \cdot \gamma^3 \cdot r'}\left[\ln\left(1 + \frac{r' \cdot z}{r}\right)\left(\frac{r}{r'} + z\right) - z\right] + r + r' \cdot z \qquad (10)$$

The current of the beamlet can be expressed as:

$$I = I_0 \cdot \frac{r^2}{ro^2}$$

I_0, ro = current and radius of the incoming beam

Thus

$$r(z) = \frac{2I_0 \cdot r^2}{I_A \cdot \gamma^3 \cdot r' \cdot ro^2}\left[\ln\left(1+\frac{r' \cdot z}{r}\right)\left(\frac{r}{r'}+z\right)-z\right]+r+r' \cdot z \tag{11}$$

This expansion has to be compared with the beam expansion due to the natural beam divergence as given in equation 9. The ratio R of the spot size without space charge forces divided by the spot size as given by equation 9 describes the contribution of the space charge forces to the spot size. The effect is strongest for the smallest divergence to be measured, i.e. 0.2mrad. R depends only weakly on the distance to the screen if z is sufficiently large. For a distance of L=1m and a hole radius of r=7μm the ratio R is 1.035 and r/r_{screen} =0.035. A complete compilation of parameters is given below.

Signal to background ratio

Not all electrons hitting the pepper pot aside of a hole will be stopped in the metal due to the finite thickness of the plate. In order to reduce the background a thick plate would be desirable, however, angular acceptance, alignment tolerances and manufacturing considerations call for a thin plate.

The background is reduced by two effects: the stopping of electrons and the scattering of electrons. Denoting the number of electrons passing through a material of thickness s to the number of incoming electrons with $P_{Stop}(s)$ and the rms scattering angle of the electrons with $\theta_r(s)$, the particle density of background particles ρ_{back} can be estimated as:

$$\rho_{back}(s) \cong \frac{q \cdot P_{Stop}(s)}{\pi(\theta_r(s) \cdot L)^2} \quad \mathrm{C\,m^{-2}} \tag{12}$$

The particle density of the electrons passing through a hole ρ_{sig} is estimated as:

$$\rho_{sig} \cong \frac{q}{2\pi \cdot ro^2\left(1+\frac{r' \cdot L}{r}\right)^2} \quad \mathrm{C\,m^{-2}} \tag{13}$$

Typical particle densities on the screen will be about 100nC m^{-2}. The signal to background ratio SB is found as:

$$SB(s) = \frac{\rho_{sig}}{\rho_{back}} \cong \frac{(\theta_r(s) \cdot L)^2}{2 \cdot P_{Stop}(s) \cdot ro^2\left(1+\frac{r' \cdot L}{r}\right)^2} \tag{14}$$

Figure 3a and b shows $P_{Stop}(s)$ and $\theta_r(s)$ for 6MeV electrons versus the thickness s of a copper plate, respectively. Copper is the desired material for the pepper pot for its good heat conductivity and mechanical properties, a scaling for other materials will be given below. The calculations were performed with the Monte-Carlo code EGS4 [5] which simulates electromagnetic cascades. Figure 3a shows $P_{Stop}(s)$ and the production of photons in the metal plate. In general the screen will be insensitive for photons, hence they will not contribute to the background. Figure 3b shows the rms scattering angle $\theta_r(s)$ based on the EGS4 calculations and an analytical scattering angle based on a fit to Moliére's theory of multiple scattering given by [6]:

$$\theta_{Mol}(s) = \frac{19.2\ \mathrm{MeV}}{E} \cdot \sqrt{\frac{s}{X_0}} \cdot \left[1 + 0.2 \cdot \ln\left(\frac{s}{X_0}\right)\right] \tag{15}$$

E = beam energy

X_0 = radiation length = 1.43cm for copper

The analytical formula is not a fit to the scattering angle as calculated with EGS4 but a fit to the gaussian core of the distribution of scattering angles. Since it neglects the non-gaussian tails of hard scattered particles it leads to smaller scattering angles than the Monte-Carlo code. In addition the fit is valid only for small scattering angles and does not take into account the boundary conditions which is important especially for low energy particles, i.e. for a thicker target. This might explain the overestimation of the scattering angle for thicker targets. For the background calculation the gaussian core is more relevant than a simple rms angle, thus the Moliére fit will be used in the following.

Figure 4 shows the estimated signal to background ratio based on equation (14) and (15). A signal to background ratio above 100 is reached with a thickness of the plate of 1.5mm. The dominating effect is the scattering of the particles, while only ~40% of the electrons will be stopped in the plate.

Figure 5a, b and figure 6 show the results of EGS4 calculations and the signal to background ratio for a 20MeV beam, respectively. A distance of L=1.7m between the pepper pot and the screen is assumed.

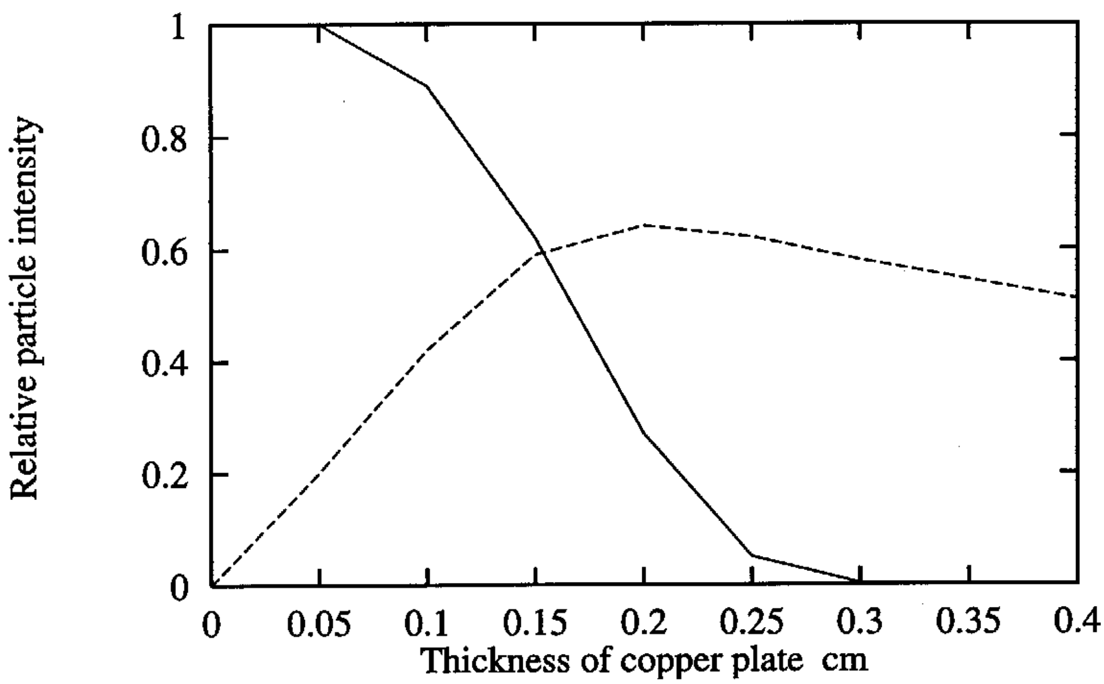

Figure 3a Relative electron intensity $P_{Stop}(s)$ (solid line) and photon production (dashed line) of 6MeV electrons in copper versus the thickness of the metal plate.

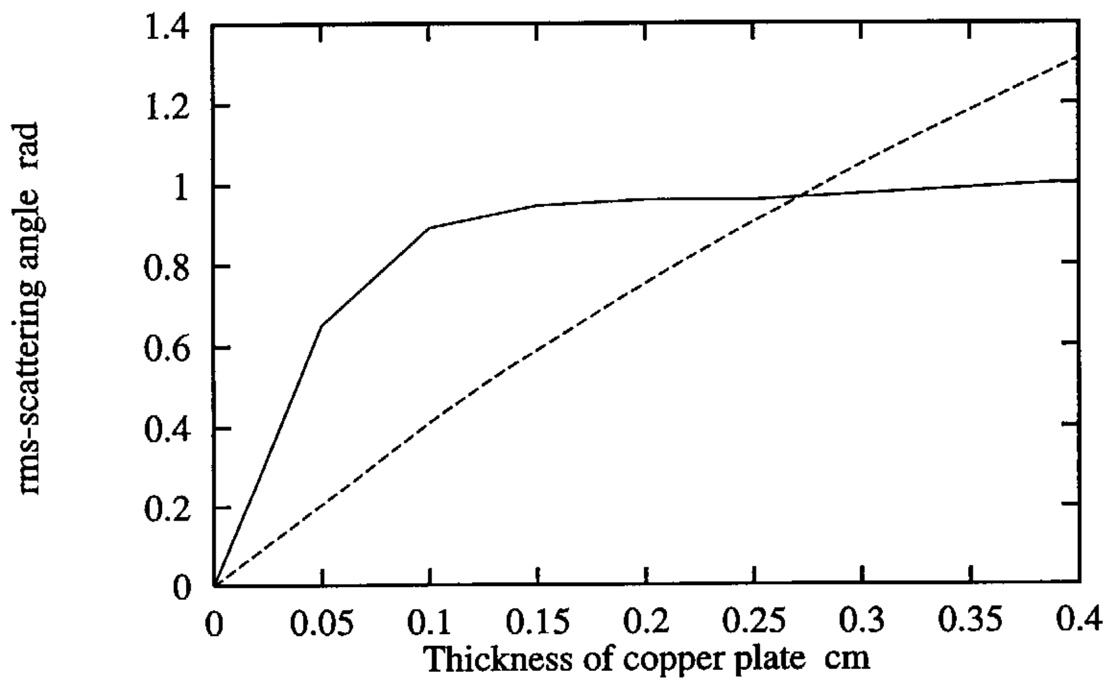

Figure 3b Comparison of rms-scattering angle $\theta_r(s)$ calculated by means of EGS4 (solid line) and by a Moliére fit (dashed line) as function of the thickness of a copper plate. Electron energy: 6MeV.

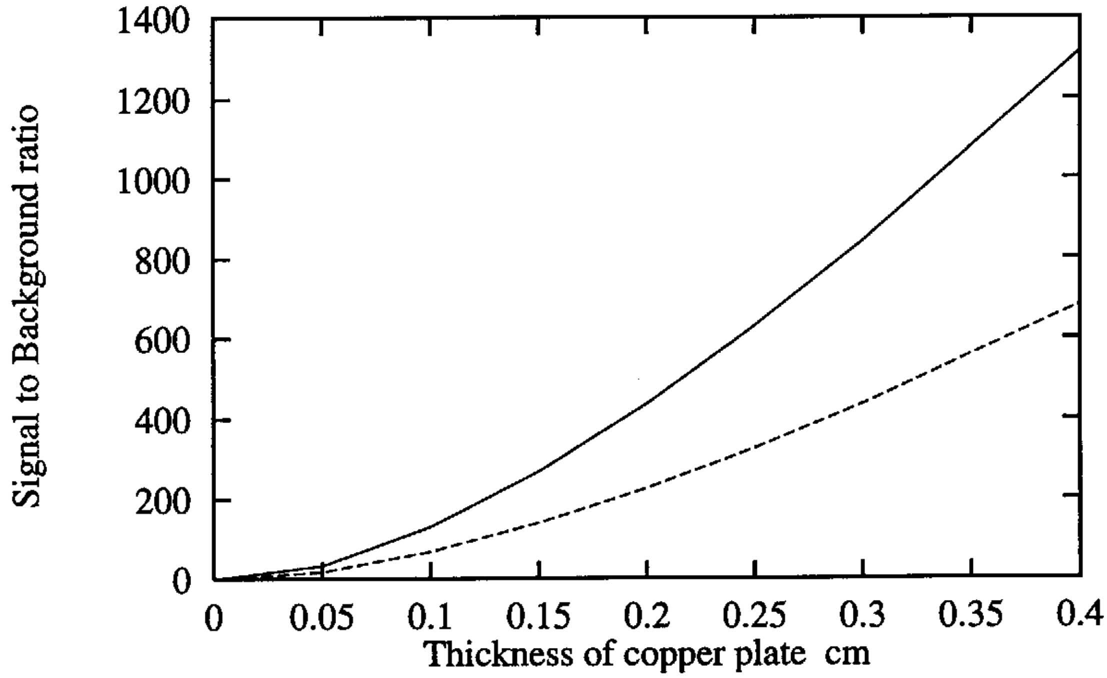

Figure 4 Estimated signal to background ratio for a pepper pot measurement versus thickness of the plate. Parameters of the beam according to table 2, hole radii of 7μm (solid line) and 5μm (dashed line), respectively.

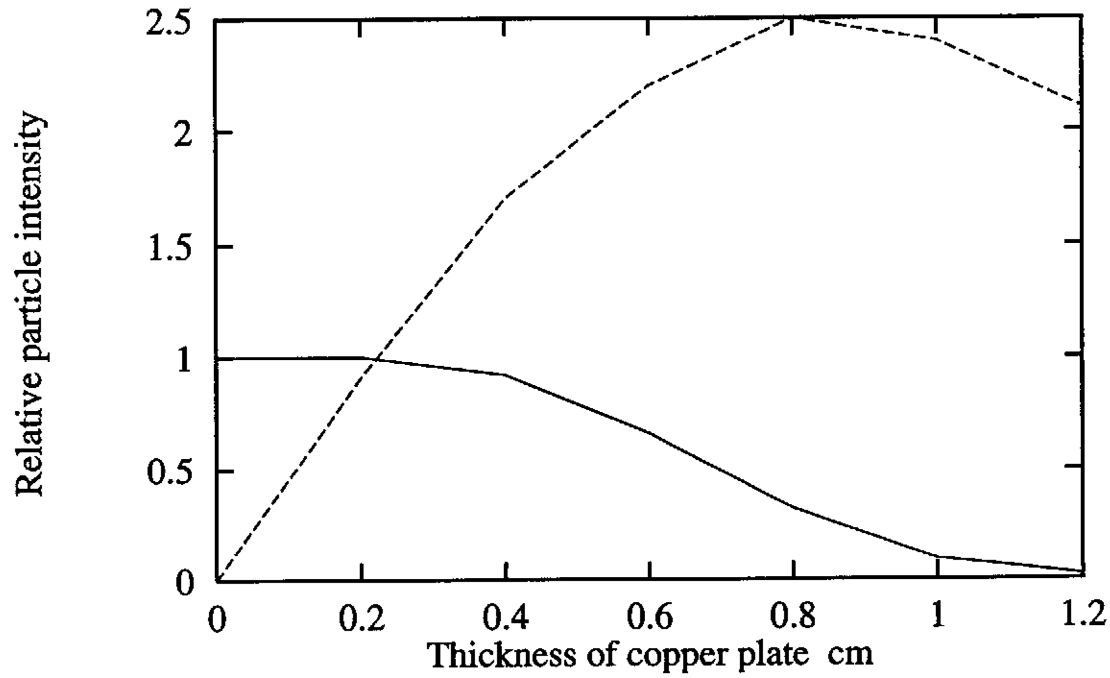

Figure 5a Relative electron intensity $P_{Stop}(s)$ (solid line) and photon production (dashed line) of 20MeV electrons in copper versus the thickness of the metal plate.

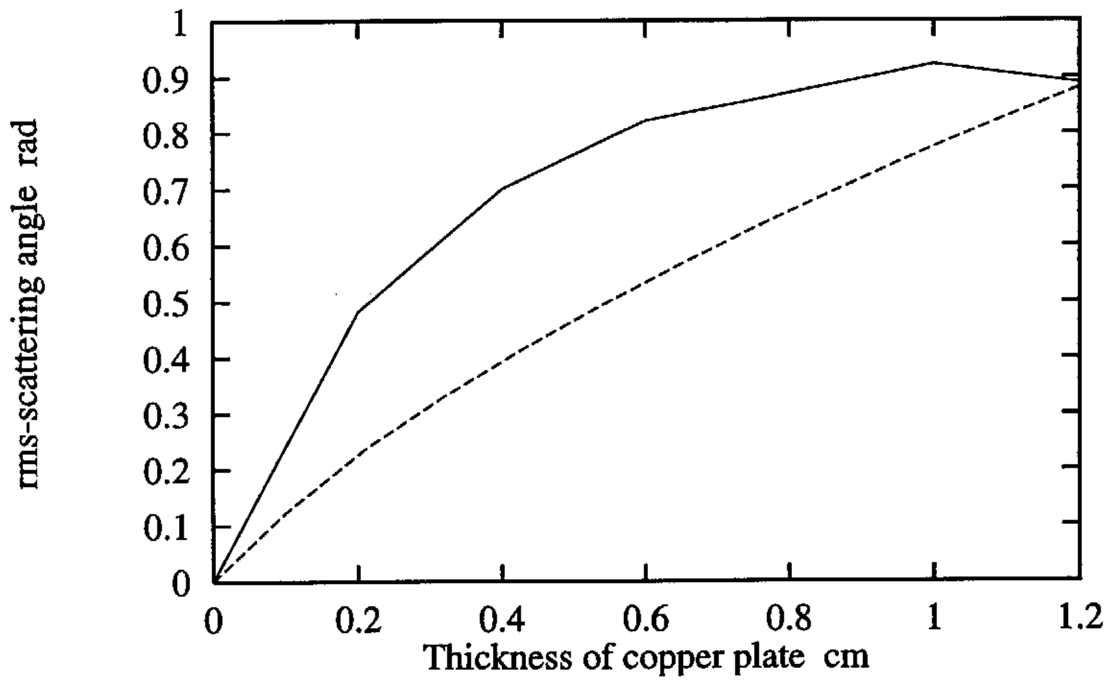

Figure 5b Comparison of rms-scattering angle $\theta_r(s)$ calculated by means of EGS4 (solid line) and by a Moliére fit (dashed line) as function of the thickness of a copper plate. Electron energy: 20MeV.

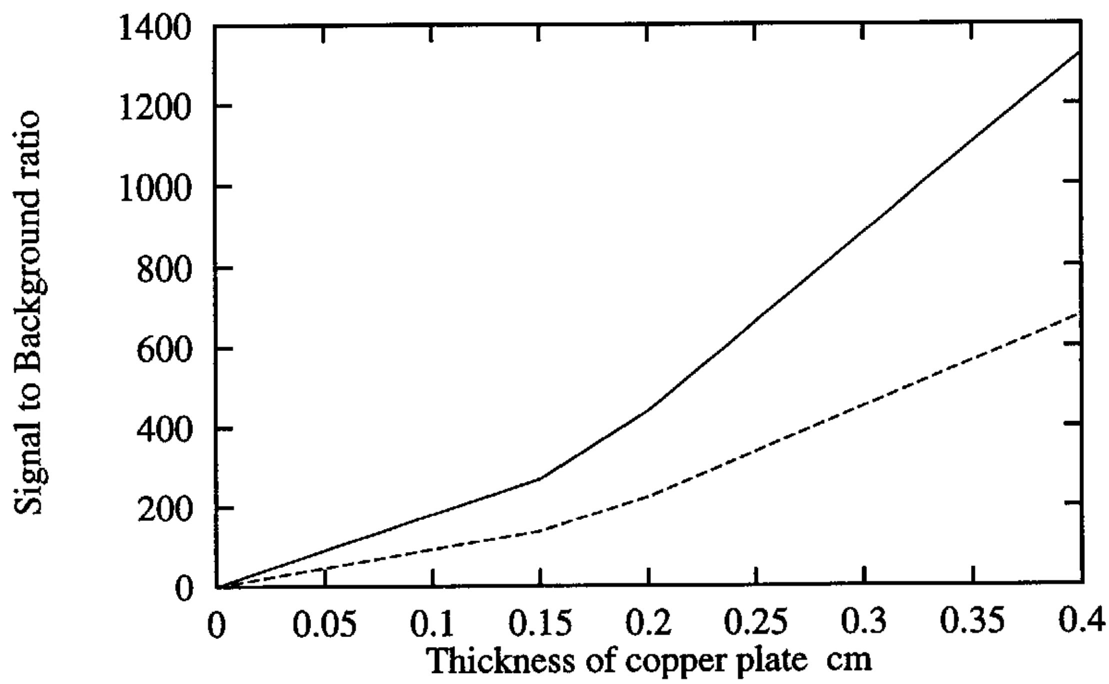

Figure 6 Estimated signal to background ratio for a pepper pot measurement versus thickness of the plate. Parameters of the beam according to table 3, hole radii of 7μm (solid line) and 5μm (dashed line), respectively.

Layout of the pepper pot plate

So far the dimensions of a single hole have been discussed only. In order to investigate the effect of the distance of the holes in the pepper pot plate on the phase space representation a simulation has been performed for different beam distributions:
a) A gaussian distribution in x and x', truncated in x at 2.5σ.
b) A uniform distribution in x and x'.
c) A uniform distribution in x, but fan-like opened in x'.
The rms emittance of all three distributions was 1π mrad mm. Figure 7a-c shows the calculated rms emittances for different numbers of holes and different distances between the holes. The holes have been equally spaced and the relative position of the beam to the hole array has been varied for each setting. The error bars indicate rms variations for the calculated emittances as the relative position of the beam to the hole array has been changed. Results that have been obtained with the same number of holes are connected with a dashed line. The line on the right side corresponds to 3 holes, the line on the left side corresponds to 6 holes. While good results can be obtained in case of the gaussian distribution (a) with only 4 holes, the emittance is overestimated by about 10-20% even with 6 holes in case of the distributions b and c. The large errors are mainly due to the unphysical sharp edges of the distributions in the calculations. The real beam distribution will be fan-like but with smother edges than in the calculations. Therefore a measurement with ~10% accuracy should be possible with a hole distance equal or smaller than the rms beam size.
The results can be improved if the rms spot size on the pepper pot plate is measured separately instead of deducing it from the beam let distribution behind the pepper pot. In this case the error in the calculation of the emittance of distribution c is reduced by more than a factor of 2, while it vanishes nearly completely in case of distributions a and b.
The distance of the holes in the pepper pot has to be small in order to obtain a good representation of the phase space, on the other hand it has to be large enough to avoid an overlap of the beam spots on the screen, thus small beam spots would be desirable. However, large spots on the screen are necessary for a precise measurement of the beam divergence. This contradicting requirements are somewhat reduced if the electron beam is divergent so that the spot distance on the screen is enlarged. Fortunately this is the case for a measurement at the emittance minimum (see Figure 1). Nevertheless two plates with different hole distances are required, i.e. one with ~0.6mm distance for low emittance beams and a second one with ~1.2mm distance for beams with larger emittance. Two orthogonal rows of holes in form of a cross, as sketched in Figure 8, with about 11 holes in a row are necessary to measure the emittance in both transversal planes.

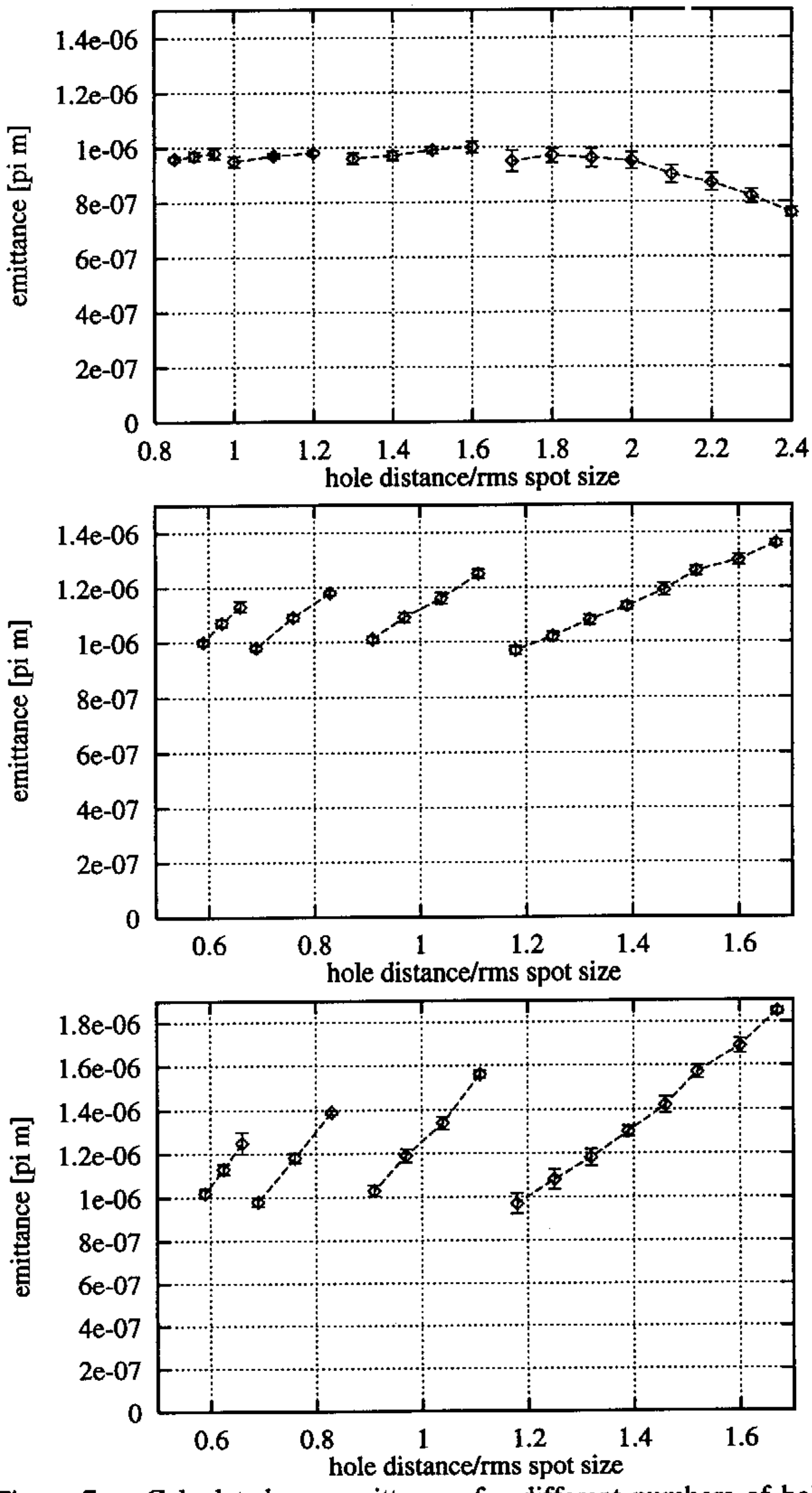

Figure 7a-c Calculated rms emittances for different numbers of holes and different hole distances. Gaussian distribution (top), uniform distribution (middle) and fan-like distribution (bottom).

In order to reconstruct the phase space at more locations than it can be done with the limited number of holes in the plate we will investigate the possibility to move the beam and reconstruct different parts of the phase space area with subsequent measurements. If the beam is moved by Δx_{beam} on the pepper pot the spots on the screen will move by:

$$\Delta x_{screen} = \Delta x_{beam} \cdot -\frac{\alpha}{\beta} \cdot L \qquad (16)$$

Hence the relative position of the beam on the pepper pot can be deduced from the measurements (if $\alpha/\beta \neq 0$) and a combination of subsequent measurements into one representation of the phase space is possible.

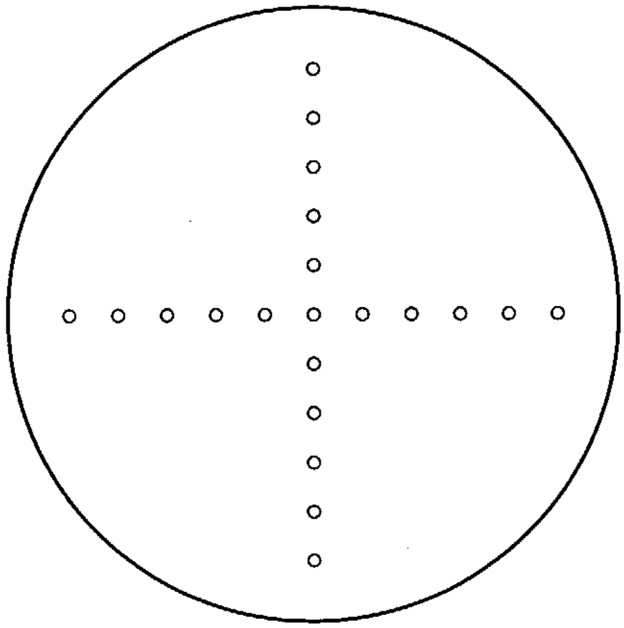

Figure 8 Schematic layout of a pepper pot.

Heat load of the pepper pot

The TTF FEL is supposed to run in a multi-bunch mode with up to 7200 bunches in one rf pulse and a repetition rate of 10Hz. While most of the tuning will be done in a single bunch mode one might also be interested in measurements of the emittance during multi-bunch operation. However, the energy deposited by the electron beam can easily destroy the pepper pot within a single pulse of a few hundred bunches. Thus multi-bunch measurements are not possible up to the full rate. The energy deposition of a 6MeV beam in a copper plate of 1.5mm thickness amounts to 62% of the energy of the incoming beam. For a 20MeV beam only 10% of the incoming beam energy is deposited in the plate. Thus the absolute energy deposition is about 0.03W per bunch for both energies. Radiation cooling is sufficient in case of single bunch measurements. Measurements at some hundred bunches per pulse require either water cooling or the reduction of the repetition rate. In any case the pepper pot should be made out of a material with good thermal properties like a high heat conductivity. Thus copper would be the desired material.

Materials with a higher atomic charge Z would allow to reduce the thickness of the plate. Since the signal to background ratio is dominated by the rms-scattering angle the thickness of the plate can be scaled simply with the radiation length. A copper plate of 1.5mm corresponds to a tungsten or tantalum plate of 0.40mm. Both materials have a poor heat conductivity but can be heated up to very high temperatures without melting or thermal damages. In addition the energy deposition is somewhat reduced in tungsten and tantalum.

Compilation

A compilation of pepper pot parameters for two different hole radii and parameters of the incoming electron beam is shown in table 2 for the case of a 5MeV electron beam. The accuracy that has to be achieved for the hole radii is rather loose. The plate has to be only 1.5mm thick to achieve a reasonable signal to background ratio. Therefore the opening angle is rather large and the alignment tolerance for the tilt of the holes with respect to the beam axis is about 1mrad. The accuracy of the emittance measurement is estimated to be better than 10% for 1π mrad mm.
Table 3 lists parameters for a 20MeV beam. The distance L to the screen has to be increased since the beam divergence is smaller. The larger distance helps to keep the signal to background ratio high. On the other hand the effect of the space charge forces is increased in the long drift thus compensating the decrease due to the higher gamma to some extend. The hole radius cannot be increased due to the limit of r/r_{screen}.

Beam parameter		
normalized emittance εn_{rms}	1π mrad mm	
current I_0	100A	
beam size r_0	$1\cdot10^{-3}$ m	
beam divergence	0.2 mrad	
Lorentz factor γ	10	
Pepper pot parameter		
radius of central hole r	$7\cdot10^{-6}$ m	$5\cdot10^{-6}$ m
distance to screen L	1.0m	1.0m
plate thickness s	1.5mm	1.5mm
R	1.035	1.020
r/r_{screen}	0.035	0.025
signal to background ratio SB	260	130
opening angle $\frac{2r}{s}$	9.3mrad	6.7mrad

Table 2 Parameters of a pepper pot for a 5MeV beam. Two hole radii are compared.

Beam parameter		
normalized emittance εn_{rms}	1π mrad mm	
current I_0	100A	
beam size r_0	$0.5 \cdot 10^{-3}$ m	
beam divergence	0.1 mrad	
Lorentz factor γ	40	
Pepper pot parameter		
radius of central hole r	$7 \cdot 10^{-6}$ m	$5 \cdot 10^{-6}$ m
distance to screen L	1.7m	1.7m
plate thickness s	1.5mm	1.5mm
R	1.008	1.005
r/r_{screen}	0.04	0.03
signal to background ratio SB	270	140
opening angle $\frac{2r}{s}$	9.3mrad	6.7mrad

Table 3 Parameters of a pepper pot for a 20MeV beam. Two hole radii are compared.

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Appendix

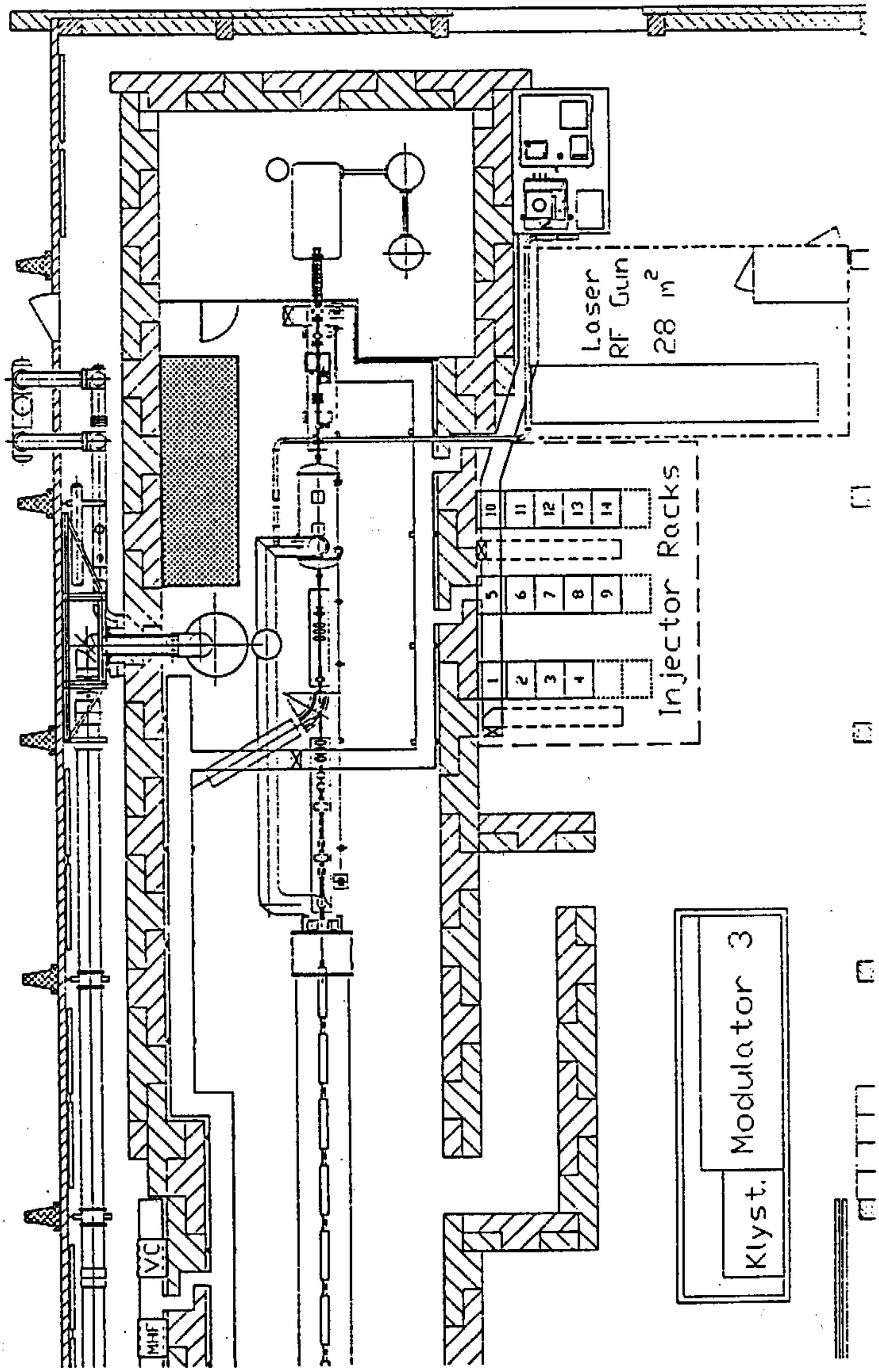

Figure A.1 Layout of the Injector area in the TTF hall with Injector I in place. The low energy beam line will be set up within the shaded area (approx. 4.5×1.5m^2).