Calculation of energy diffusion in an electron beam due to quantum fluctuations of undulator radiation

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Abstract

The paper presents calculations of the rate of energy diffusion in the electron beam due to quantum fluctuations of undulator radiation. The cases of a helical and a planar undulator are considered. Universal fitting formulae are obtained valid for an arbitrary value of the undulator parameter.

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1 Introduction

When relativistic electron beam passes through the undulator, it emits radiation. This process leads to the decrease of the mean energy of electrons and to the increase of energy spread in the beam due to quantum fluctuations of undulator radiation. The latter effect grows significantly with the increasing of energy of the electron beam, the field and the length of the undulator. This effect should be carefully taken into account when designing systems equipped with undulators. For instance, the effect of energy diffusion imposes a fundamental limit on a minimal achievable wavelength in an X-ray free electron laser [1]. There is a tendency to increase the number of undulators installed at storage rings and effect of energy diffusion in undulators could influence significantly on the parameters of the electron beam. There exists an idea that the fourth generation of synchrotron light sources will be based on linear accelerators and sequential use of electron beam in some number of undulators [2]. Peculiar feature of these sources is in high energy of electron beams (up to several tens of GeV) and long undulators. The effect of energy diffusion will be certainly significant in this case. This effect could be also important in the positron generation system of linear colliders, because of a large undulator length and high energy of electron beam [3,4]. So, we can conclude that the problem of precise calculation of energy diffusion in an undulator is of great practical importance.

For the first time coefficient of energy diffusion due to quantum fluctuations of synchrotron radiation has been calculated in ref. [5]. This expression is also valid for calculation of energy diffusion in the undulator at large values of the undulator parameter. At small values of undulator parameter the coefficient of energy diffusion could be simply derived using approximation of Thompson scattering of equivalent photons on electrons [6,7]. In practice, the parameters of devices are frequently lying in an intermediate region and in this paper we consider the case of arbitrary value of the undulator parameter. Using the results of numerical calculations, we have obtained universal fitting formulae for a helical and a planar undulator providing a high accuracy.

2 Basic equations

We consider ultrarelativistic electron beam propagating in an undulator. The mean energy loss of the electron is given by (in CGS units):

$$d\mathcal{E}/dt = -2r_e^2 c\gamma^2 H^2/3 \quad , \tag{1}$$

where $r_e = e^2/mc^2$ is classical radius of the electron, $\mathcal{E}$ is the energy of the electron, $\gamma = \mathcal{E}/mc^2$ is relativistic factor and $-e$ and m are the charge and the mass of the electron, respectively. Magnetic field $\vec{H}(z) = (H_\mathrm{x}, H_\mathrm{y}, H_\mathrm{z})$ at the undulator axis has the following form:

$$\vec{H}(z) = \begin{cases} (H_\mathrm{w}\cos(\kappa_\mathrm{w} z), -H_\mathrm{w}\sin(\kappa_\mathrm{w} z), 0) & \text{for a helical undulator} \\ (H_\mathrm{w}\cos(\kappa_\mathrm{w} z), 0, 0) & \text{for a planar undulator,} \end{cases}$$

where H_w is the amplitude of the magnetic field, $\kappa_\mathrm{w} = 2\pi/\lambda_\mathrm{w}$ and λ_w is the undulator period. The intensity of undulator radiation I is equal to the rate of mean energy losses of the electron:

$$I = \int d\omega \frac{dI}{d\omega} = -d\mathcal{E}/dt \ .$$

Quantum nature of radiation leads to the growth of energy spread in the electron beam. The rate of energy diffusion is given by the expression:

$$\frac{d < (\delta\mathcal{E})^2 >}{dt} = \int d\omega \hbar\omega \frac{dI}{d\omega} \ . \tag{2}$$

The spectral intensity of an undulator radiation $dI/d\omega$ was calculated in ref. [8]. To calculate the rate of the energy diffusion, we use expressions for $dI/d\omega$ obtained in ref. [8] at the following approximations:

i) Ultrarelativistic case is considered, $\gamma \gg 1$.

ii) An angle between the electron transverse velocity and undulator axis is small, i.e. $K/\gamma \ll 1$, where $K = eH_\mathrm{w}/\kappa_\mathrm{w} mc^2$ is the undulator parameter.

iii) Number of the undulator periods is large, so the spectrum broadening due to the finite number of periods is neglected.

3 Helical undulator

For the case of a helical undulator the expression for the rate of energy diffusion can be written in the following form:

$$\frac{d < (\delta\gamma)^2 >}{dt} = \frac{14}{15} c \lambdabar_{\mathrm{c}} r_{\mathrm{e}} \gamma^4 \kappa_{\mathrm{w}}^3 K^2 F(K) \ , \qquad (3)$$

where $\lambdabar_{\mathrm{c}} = \hbar/mc$. It is important to notice that the energy diffusion is expressed in terms of dimensionless function $F(K)$ of the only undulator parameter K which is consequence of the fact that the spectrum shape of the undulator radiation is defined with the only parameter K. Function $F(K)$ has the following form:

$$F(K) = \frac{60}{7} \frac{1}{(1+K^2)^3} \sum_{n=1}^{\infty} n^3 \int\limits_0^1 dy y^2 \left[J_n'^2(x) + \left(\frac{n^2}{x^2} - \frac{1+K^2}{K^2} \right) J_n^2(x) \right] , \qquad (4)$$

where J_n and J_n' are the Bessel function and its derivative, respectively and

$$x = \frac{2K}{\sqrt{1+K^2}} n \sqrt{y(1-y)} \ .$$

The plot of this function is presented in Fig.1. For the cases of small and large values of the undulator parameter K, there exist well known asymptotes [5–7]:

$$F(K) = \begin{cases} 1 & \text{at } K \ll 1 \\ \frac{275}{112\sqrt{3}} K \simeq 1.42K & \text{at } K \gg 1 \ . \end{cases} \qquad (5)$$

In the general case there is no possibility to obtain analytical expression and we write the following fitting formula:

$$F(K) = 1.42K + \frac{1}{1 + 1.50K + 0.95K^2} . \qquad (6)$$

This formula provides an accuracy better than 1 % in the whole range of parameter K (see Fig.1).

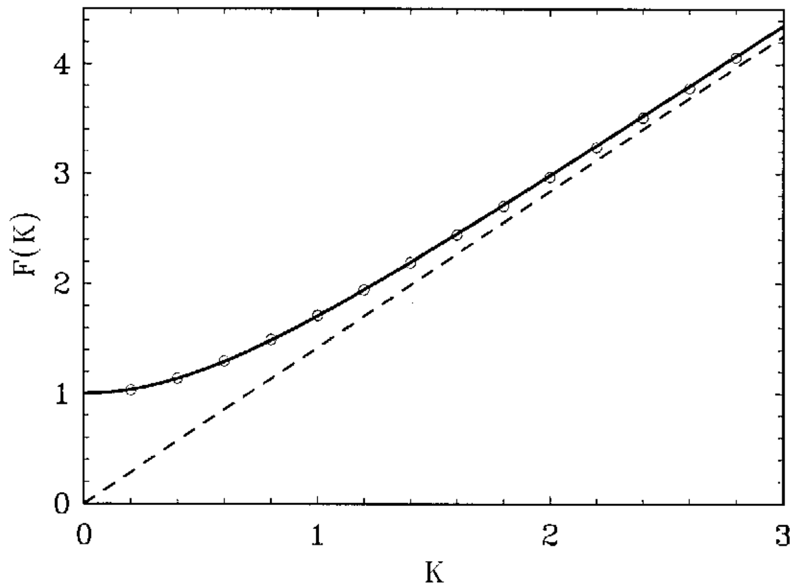

Fig. 1. Function $F(K)$ for helical undulator. Solid curve is calculated with formula (4), the circles are calculations with fitting formula (6) and dotted curve is calculated with asymptotic formula (5) for large values of K.

4 Planar undulator

For the case of a planar undulator the expression for the rate of energy diffusion can be written in the following form:

$$\frac{d < (\delta\gamma)^2 >}{dt} = \frac{7}{15} c \lambda\!\!\!^{-}_{\rm c} r_{\rm e} \gamma^4 \kappa_{\rm w}^3 K^2 F(K) \ . \qquad (7)$$

Universal function $F(K)$ given with the expression [1]:

$$F(K) = \frac{120}{7} \frac{1}{K^2(1+K^2/2)^2} \sum_1^\infty n^3 \int_0^1 dy y G_n \qquad (8)$$

where

$$G_n = \frac{1}{2\pi} \int_0^{2\pi} g_n d\phi, \qquad g_n = \frac{(S_1 + 2S_2/n)^2}{4(1-y) cos^2\phi} - y S_1^2 - \frac{2}{n} S_1 S_2,$$

[1] There is misprint in the corresponding expression (14) for $dI/d\omega$ in ref. [8]. There should be minus sign inside the square brackets in the expression for function $f(\varphi)$.

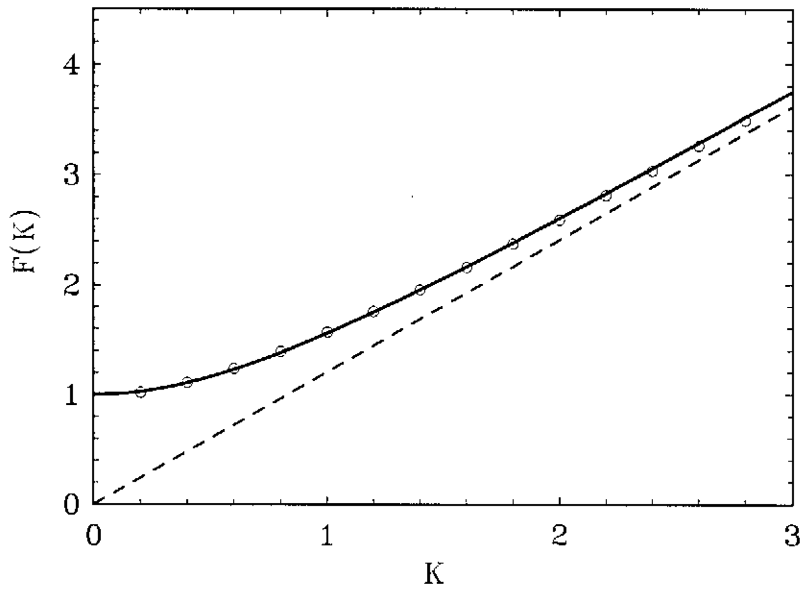

Fig. 2. Function $F(K)$ for planar undulator. Solid curve is calculated with formula (8), the circles are calculations with fitting formula (10) and dotted curve is calculated with asymptotic formula (9) for large values of K.

$$S_1 = \sum_{p=-\infty}^{\infty} J_p(z)J_{n+2p}(x), \qquad S_2 = \sum_{p=-\infty}^{\infty} pJ_p(z)J_{n+2p}(x),$$

$$x = \frac{2K}{\sqrt{1+K^2/2}} n\sqrt{y(1-y)} cos\phi, \qquad z = \frac{K^2}{4(1+K^2/2)} ny \ .$$

The plot of this function is presented in Fig.2. For the cases of small and large values of the undulator parameter K, there exist the following asymptotes:

$$F(K) = \begin{cases} 1 & \text{at } K \ll 1 \\ \frac{275}{42\pi\sqrt{3}} K \simeq 1.20K & \text{at } K \gg 1 \ . \end{cases} \tag{9}$$

In the general case function $F(K)$ is fitted with the following formula:

$$F(K) = 1.20K + \frac{1}{1+1.33K+0.40K^2} \ , \tag{10}$$

providing an accuracy better than 1 % in the whole range of parameter K (see Fig.2).

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References

[1] J. Rossbach, E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov, Interdependence of parameters of an X-ray FEL, DESY Print December 1995, TESLA-FEL 95-06, Hamburg, 1995. Nucl. Instrum. and Methods, in press.

[2] Proc. of the Tenth Advanced ICFA Beam Dynamics Workshop on 4th Generation Light Sources (Grenoble, January 1996), to be published.

[3] V.E. Balakin and A.M. Mikhailichenko, Proc. of the 12th Intern. Conf. on High Energy Accelerators (Batavia, 1983), p. 127.

[4] K. Flottmann and J. Rossbach, Proc. XVth Int. Conf. on High Energy Acc. (Hamburg, 1992), p. 142.

[5] M. Sands, Phys. Rev. **97**(1955)470.

[6] Ya.S. Derbenev, A.M. Kondratenko and E.L. Saldin, Nucl. Instrum. and Methods **193**(1982)415.

[7] S. Benson and M.J. Madey, Nucl. Instrum. and Methods, **A237**(1985)55.

[8] D.F. Alferov, Yu.A. Bashmakov, and E.G. Bessonov, Sov. Phys. Tech. Phys. **18**(1974)1336.