Generation of Ultrashort Electron Bunches by cancellation of nonlinear distortions in the longitudinal phase space

K. Flöttmann, T. Limberg, Ph. Piot*

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Abstract

The recent injectors design studies performed in the frame of the TESLA X-FEL and TTF User Facility projects, achieve a low emittance by using a rather long photocathode drive-laser pulse duration (10−20 ps full width). Such a method has the disadvantage to generate nonlinear distortions in the longitudinal phase space, (s, δ) which limit the minimum achievable bunch length downstream of the subsequent bunch compressor(s). In this note we analyze how a higher frequency rf accelerating field can correct for these longitudinal phase space nonlinear distortions up to the third order (included) in s. We first consider the simple case of a single accelerating section followed by a magnetic bunch compressor and then extend the treatment to the case of a multi-stage compression scheme which might consist of a chain of bunch compressors interspaced by accelerating sections. We then apply our results to the special case of the TTF User Facility.

1 Introduction

In the TESLA-XFEL beamline the first stage bunch compression, composed of a four-dipole achromatic chicane, compresses the bunch down to 0.3 mm (rms). This compression is foreseen to take place after acceleration through a standard TESLA accelerating module which is located directly downstream of the RF-gun exit [1]. Since it is presently impossible to generate ultrashort high-charge bunches out of an RF-gun without impinging on the transverse emittance quality (because of the strong 3D space-charge coupling at low energy), we have relaxed the bunch length to preserve the required high quality emittance (achieved emittance is $\tilde{\varepsilon} \simeq 0.5$mm-mrd – without including thermal emittance). As a consequence, the subsequent acceleration of the rather long bunch [$\simeq$ 2mm (rms)], results in some distortion of the longitudinal phase space downstream of the cryomodule because of the RF-induced cos-like curvature. Such distortion, if not corrected, sets a lower limit on the compression process and can thus significantly decrease the available peak current which in turn will degrade the SASE gain mechanism. The use of an higher frequency accelerating section

*PIOT@MAIL.DESY.DE

to remove RF-induced curvature is then required; ideally one desires to "linearize" the longitudinal phase space (i.e $\delta(s) \propto s$). Such a method was first suggested in Reference [2] and then experimentally tested (see Reference [3]) at the BOEING FEL facility. The present work constitutes a generalization of the earlier work reported in Reference [3] where the full correction up to third order (i.e. $\delta(s) \propto s + \mathcal{O}(s^4)$ was not explored.

Let's consider an incoming electron bunch of initial energy E_o, and consider its acceleration in a chain of N accelerating cavities with spatial frequency k_{rf}, accelerating voltage $V_{i,1}$ and operating phase $\varphi_{i,1}$. The fractional energy offset, $\delta(s)$, of a relativistic electron (with initial energy offset δ_o and longitudinal coordinate s w.r.t. the bunch centroid) downstream of these cavities takes the form:

$$\delta(s) = \frac{1}{E_0 + \sum_{i=1}^{N} V_{i,1}\cos(\varphi_{i,1})} \times \left(E_0\delta_0(s) + \sum_{i=1}^{N} V_{i,1}\left[\cos(k_{rf}s + \varphi_{i,1}) - \cos(\varphi_{i,1})\right] \right). \tag{1}$$

The injection of an electron, with longitudinal coordinate (s_i, δ_i), into a longitudinally dispersive section, e.g. a magnetic bunch compressor, will map the coordinate to (s_f, δ_s) accordingly to:

$$s_f = s_i + R_{56}\delta_i + T_{566}\delta_i^2 + U_{5666}\delta_i^3 + \mathcal{O}(\delta_i^4), \text{ and } \delta_f = \delta_i. \tag{2}$$

We defined the Taylor expanded coefficient as: $R_{56} = \partial S^{i\to f}/\partial\delta$, $T_{566} = 1/2\partial^2 S^{i\to f}/\partial\delta^2$, and $U_{5666} = 1/6\partial^3 S^{i\to f}/\partial\delta^3$, defining $S^{i\to f}$ as the path length through the compressor. Note that the maximum compression occurs ideally (i.e. under single-particle dynamics approximation) when $s_f = 0$ in the first of Eqns (2).

Let's assume (a proof is forthcoming in the next section) that the incoming phase space described by Eqn (1) can be expanded as a third order polynomial form

$$\delta(s_i) = \alpha_1 s_i + \alpha_2 s_i^2 + \alpha_3 s_i^3 + \mathcal{O}(s_i^4). \tag{3}$$

Expliciting Eqn(3) in the first of Eqns(2), gives the expression for the final electron longitudinal position:

$$s_f = (1 + \alpha_1 R_{56})s_i + (\alpha_1^2 T_{566} + \alpha_2 R_{56})s_i^2 + \mathcal{O}(s_i^3) \tag{4}$$

requiring $s_f = 0$ results in the three longitudinal "matching conditions" (to obtain maximum compression):

$$\alpha_1 = \frac{-1}{R_{56}},\ \alpha_2 = -\alpha_1^2\frac{T_{566}}{R_{56}}, \text{ and } \alpha_3 = \frac{1}{R_{56}}\left(-\alpha_1^3 U_{5666} + 2\alpha_1\frac{T_{566}^2}{R_{56}}\right). \tag{5}$$

The coefficients α_i's are indeed the successive derivatives of $\delta(s)$ in s at the origin,

$$\alpha_i \stackrel{\text{def}}{=} \frac{1}{i!}\left[\frac{\partial^i \delta(s)}{\partial s^i}\right]_{s=0} \tag{6}$$

The matching conditions defined in Eqns (5) can generally not be fulfill simultaneously: since from Eqn (1) the α_i coefficients depend on each other, e.g.:

$$\begin{aligned}\frac{\alpha_2}{\alpha_1} &= \frac{k_{rf}}{2}\frac{\sum_{i=1}^{N} V_{i,1}\cos(\varphi_{i,1})}{\sum_{i=1}^{N} V_{i,1}\sin(\varphi_{i,1})} \\ &= \frac{k_{rf}}{2}\frac{1}{\tan(\varphi_1)}, \text{ assuming } V_{i,1} = V_1 \text{ and } \varphi_{i,1} = \varphi_1\ \forall \text{i}.\end{aligned} \tag{7}$$

A first method, that can correct distortions up to second order, would be to design a magnetic compressor section that has the proper linear, R_{56}, and quadratic, T_{566}, momentum compactions; from Eqn (4) these two parameters should be related via:

$$T_{566} = (-)\frac{\alpha_2}{\alpha_1^2}R_{56} = \frac{k_{rf}R_{56}^2}{2}\frac{1}{\tan\varphi_1}, \tag{8}$$

which reduce, assuming $\tan\varphi_1 \simeq \sin\varphi_1 = 1/(k_{rf}R_{56})$, to the relation:

$$T_{566} = \frac{2\pi^2 R_{56}^3}{\lambda_{rf}^2}, \tag{9}$$

Thus, the magnetic compressor is required to have its linear and quadratic momentum compaction of same signs. Figure 1 presents the operating points (for different bending angles) of three different compressors in the (R_{56}, T_{566})-space; it is seen that a F0D0 compressor (scaled from the one proposed to compress the beam after the damping ring for the TESLA linear collider – see Ref. [7]) can provide the proper relation between R_{56} and T_{566}. However there are problems inherent to the use of such a device:

- it forces to introduce a (linear) longitudinal phase space correlation which has an opposite sign to the one needed in the multi-stage compression scheme devised for TESLA XFEL (and TTF User Facility). This latter fact simply results from the property of chicanes-based compressor: they have negative R_{56} whereas F0D0 compressors have positive R_{56}.
- because of the previous point we have to over-compress in this F0D0 compressor to get the proper sign of slope for the subsequent compressors. This longitudinal envelope cross-over may result in a significant degradation of the phase space especially in view of the highly charged bunch we aim to transport.

A second approach, which is the one adopted and analyzed henceforth in the present paper, is to introduce a higher harmonic (third harmonic) RF-field to get enough variables to independently control the α_i coefficients. We now concentrate on this method.

2 Analytic formulation

2.1 Case of a single compressor

We first consider the case when the beamline just consist of an accelerating section followed by a single bunch compressor and assume we desire to have the minimum bunch length at this location. We start with Eqn (1), and introduce a third harmonic RF-section characterized by its accelerating voltage and phase, V_3 and φ_3, respectively. Downstream of this section (which follows the fundamental accelerating section) the fractional energy spread takes now the form:

$$\delta(s) = \frac{1}{E_0 + \sum_{i=1}^{N} V_{i,1}\cos(\varphi_{i,1}) + V_3\cos(\varphi_3)} \times \{E_0\delta_0(s) + \sum_{i=1}^{N} V_{i,1}\left[\cos(k_{rf}s + \varphi_{i,1}) - \cos(\varphi_{i,1})\right] + V_3\left[\cos(3k_{rf}s + \varphi_3) - \cos(\varphi_3)\right]\}, \tag{10}$$

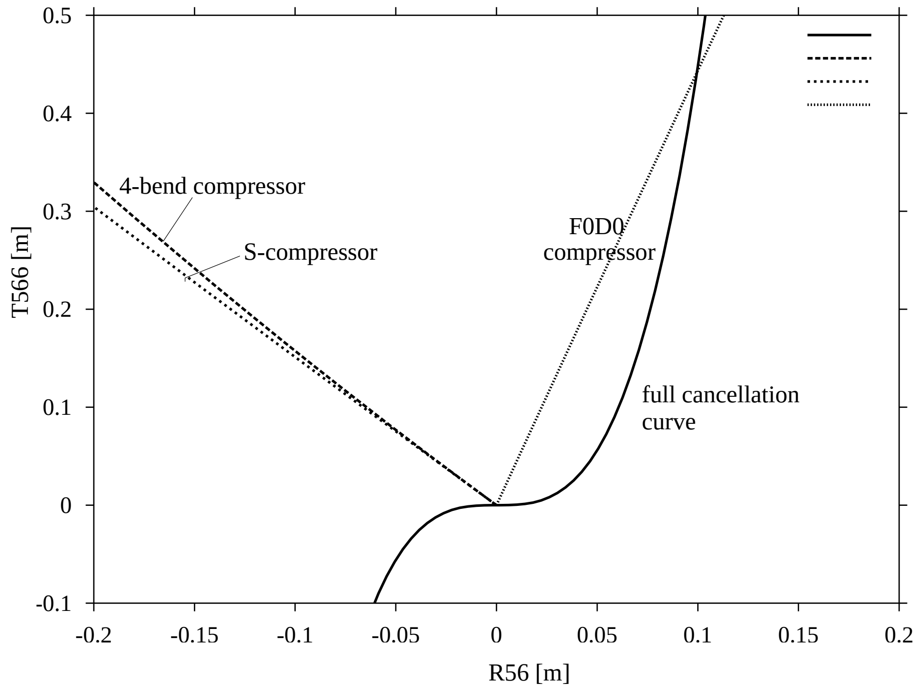

Figure 1: Comparison of the working points of three different types of compressors in the (R_{56}, T_{566})-space. The solid line is a graphical representation of Eqn (9). For a description of the S-compressor see Ref. [6]

Let's assume the incoming longitudinal phase space is issued from an RF-gun then it can also be expanded as a third order polynomial,

$$\delta_0 = \zeta_1 s + \zeta_2 s^2 + \zeta_3 s^3, \tag{11}$$

as a result of the effects of longitudinal space charge forces as introduced in Reference [4]. From the longitudinal matching equations Eqns (5) and the longitudinal phase space description Eqns (11), we can derive a system of three equations with four free-variables $(V_1, \varphi_1, V_3, \varphi_3)$:

$$\begin{aligned} V_1\sin(\varphi_1) + 3V_3\sin(\varphi_3) + \hat{\zeta}_1 &= \hat{A}\left(E_0 + V_1\cos(\varphi_1) + V_3\cos(\varphi_3)\right) \\ V_1\cos(\varphi_1) + 9V_3\cos(\varphi_3) + 2\hat{\zeta}_2 &= \hat{B}\left(E_0 + V_1\cos(\varphi_1) + V_3\cos(\varphi_3)\right) \\ V_1\sin(\varphi_1) + 27V_3\sin(\varphi_3) + 6\hat{\zeta}_3 &= \hat{C}\left(E_0 + V_1\cos(\varphi_1) + V_3\cos(\varphi_3)\right), \end{aligned} \tag{12}$$

where $\hat{\zeta}_i \stackrel{\text{def}}{=} \zeta_i / k_{rf}^i E_0$. The unknowns are φ_3, φ_1 and V_3, and V_1 is a free parameter determined from other consideration, e.g. one may desire the highest possible energy gain or instead may one may wish to operate the correlating section at zero crossing. In Eqn (12)we have defined:

$$\hat{A} = \frac{-1}{k_{rf}}\alpha_1 \stackrel{\text{max. compr.}}{=} \frac{1}{k_{rf}R_{56}},\ \hat{B} = \frac{-\alpha_1^2 T_{566}}{2k_{rf}^2 R_{56}},\ \text{and}$$

$$\hat{C} = \frac{-\alpha_1^3}{6k_{rf}^3 R_{56}}\left(U_{5666} - 2\frac{T_{566}^2}{R_{56}}\right) \tag{13}$$

From the latter equations, one can derive the accelerating voltage and operating phase of the third harmonic linearizer as a function of the (fundamental) accelerating section:

$$
\begin{aligned}
V_3 &= \left[\left(\frac{V_1\sin(\varphi_1)\left[1-\hat{A}/\hat{C}\right]-[6\hat{\zeta}_3\hat{A}/\hat{C}-\zeta_1]}{3\left[9\hat{A}/\hat{C}-1\right]}\right)^2+\right.\\
&\qquad \left.\left(\frac{BE_1+2\hat{\zeta}_2-(1-\hat{B})V_1\cos(\varphi_1)}{9-\hat{B}}\right)^2\right]^{1/2}, \qquad (14)\\
\tan(\varphi_3) &= -\frac{\left[V_1\sin(\varphi_1)(1-\hat{A}/\hat{C})-[6\hat{\zeta}_3\hat{A}/\hat{C}-\zeta_1]\right](9-\hat{B})}{3\left[\hat{B}E_1+2\hat{\zeta}_2-(1-\hat{B})V_1\cos(\varphi_1)\right](1-9\hat{A}/\hat{C})}. \qquad (15)
\end{aligned}
$$

The phase φ_1 is obtained, after elimination of V_3 and φ_3, by solving a second order equation of the form $\mu\xi_1+\nu\sqrt{1-\xi_1^2}=\kappa$ with $\xi_1=\sin(\varphi_1)$; the solution takes the form:

$$
\xi_1^{\pm}=\frac{\mu\kappa\pm\nu\sqrt{\mu^2+\nu^2-\kappa^2}}{\mu^2+\nu^2}, \qquad (16)
$$

where the coefficents are $\mu=(8\hat{A}V_1)/(9\hat{A}-\hat{C})$, $\nu=(8\hat{A}V_1)/(\hat{B}-9)$, and $\kappa=\hat{A}/(\hat{B}-9)\times(2\hat{\zeta}_3-9E_1)+\hat{A}/(9\hat{A}-\hat{C})\times(6\hat{\zeta}_3-9\hat{\zeta}_1)$. Because the fundamental mode accelerating section provides beam acceleration, we impose $\xi_1>0$ which implies that only the ξ_1^- solution is allowed (this latter solution is indeed the only one physically possible since $\xi_1^+>1$). So we finally have:

$$
\varphi_1=\arcsin\left(\frac{\mu\kappa-\nu\sqrt{\mu^2+\nu^2-\kappa^2}}{\mu^2+\nu^2}\right). \qquad (17)
$$

The Eqns (14) and (17) are used henceforth to find the optimum setup of the first and third harmonic section given an amplitude for the accelerating field V_1 and an incoming beam energy E_0. As an example of typical operating parameters for the 3^{rd} harmonic section, we present in Fig. 2 the required 3^{rd} harmonic "accelerating" voltage (V_3) along with the fundamental, φ_1, and 3^{rd} harmonic , φ_3, sections operating phase. For this particular case we have taken the small bending angle approximation for the higher order term of the Taylor expansion in Eqn (2) (i.e. $T_{566}=-3/2\times R_{56}$, and $U_{5666}=2\times R_{56}$ [5]). Taking the particular case of $R_{56}=18$ cm, we compare in Fig. 3 the impact of the 3^{rd} harmonic sections on the longitudinal phase space: it clearly demonstrates the benefits of using such a scheme for un-distorting the longitudinal phase space; as aforementioned the peak current is greatly enhanced (see Fig. 4).

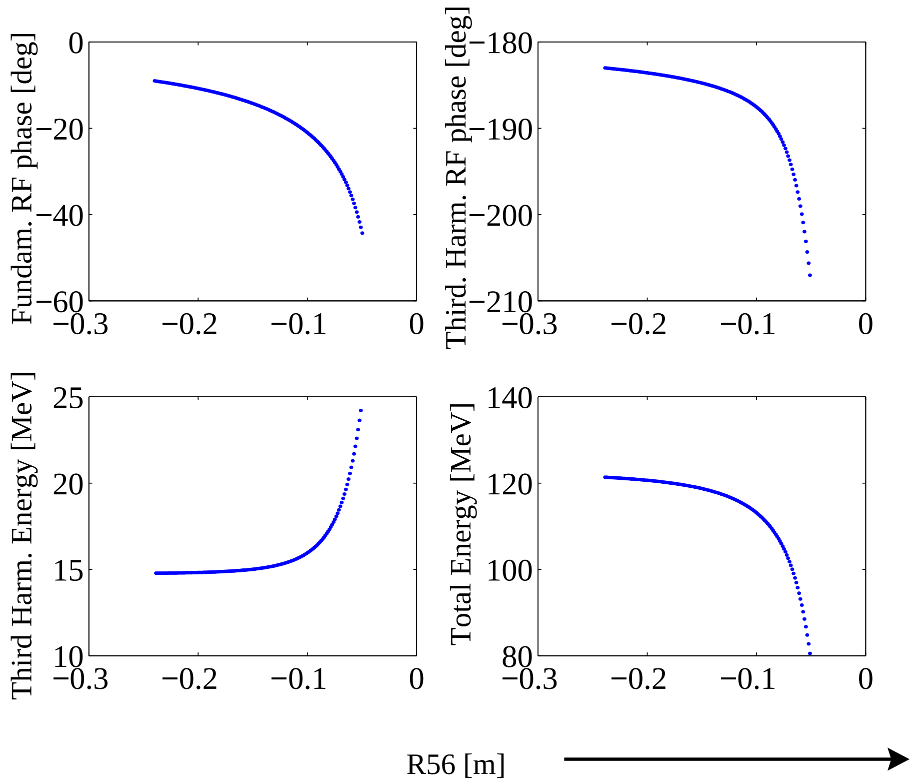

Figure 2: Required RF-parameters, for the fundamental (φ_1) and the 3^{rd} harmonic accelerating (V_3, φ_3) section, for different values of the momentum compaction, R_{56}, of a downstream magnetic chicane. Note that for these simulations the incoming energy spread was assumed to be zero (i.e. $\delta_o(s) = 0$).

2.2 Extension to the case of multi-stage compression scheme

We now turn to the case of a beamline which might consist of a series of compressors inter-spaced by accelerating sections. The use of a 3^{rd} harmonic linearizer at high energy is not practical: since during the correction process one has to remove a fraction of the incoming beam energy, a substantial part of the acceleration may be "wasted" at high energy. On the other hand the demand on the decelerating voltage to be provided by the third harmonic section will increase thereby requiring the cavity gradient or the active length of the section to be increased. Both of these requirements are not a fortiori easy to implement. Thus one has to locate the correction at low energy and tune it in such a way that anticipated distortion are corrected.

A prime thought would be to tune the correlating and third harmonic section to match the longitudinal phase space to the R_{56}, T_{566}, U_{5666} of the whole upstream beamline in such a way that the minimum bunch is obtained at the beamline end; this method is not possible since between the compressors, accelerating sections are located. These section can be used to (locally) impart the proper time-energy correlation. Instead we will assume that we use a single third harmonic section located, as before, prior to the first stage compression. We also do the following assumption: the bunch length downstream of the first compression stage is short enough that there is no RF-induced

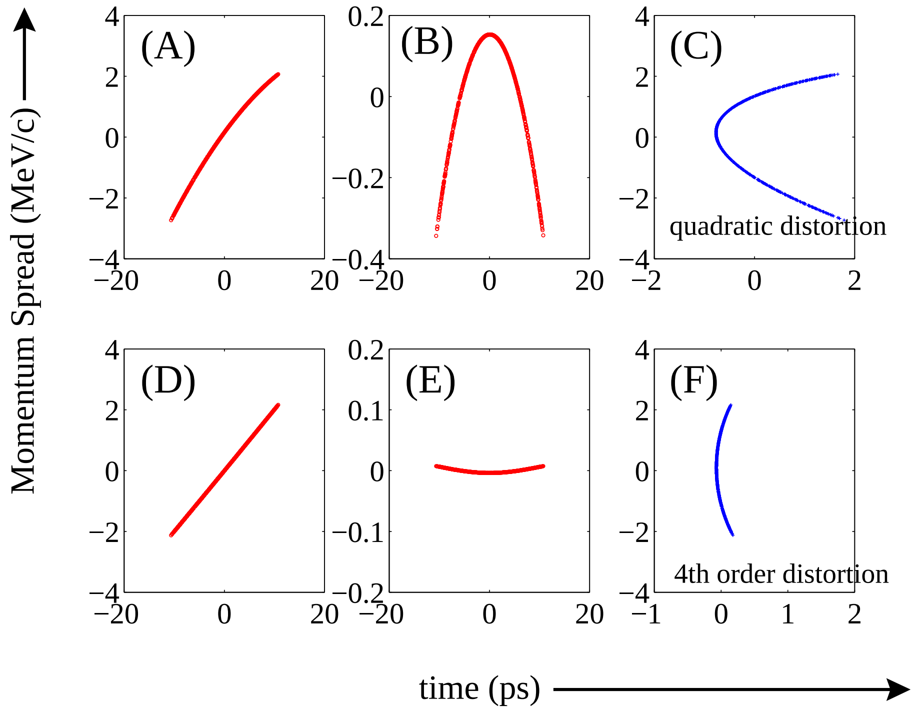

Figure 3: Comparison of the longitudinal phase space evolution with and without operating the 3^{rd} harmonic section. The longitudinal phase space are pictured downstream of the 3^{rd} harmonic sections, (plots (A) and (D)), and of the bunch compressor (plots (C) and (F)). The plots (B) (resp. (E)) correspond to the plots (A) (resp. (D)) minus the linear correlation between the momentum spread and time.

distortion in the subsequent acceleration modules. The latter assumption implies that the longitudinal transfer map of an accelerating section is well approximated by its linear transfer matrix. Under such an approximation, the transfer matrix of the jth "module" that consists of a compressor characterized by its momentum compaction, $R_{56,j}$, located downstream of an accelerating section parameterized by its accelerating voltage, $V_{1,j}$, and phase $\varphi_{1,j}$ takes the form:

$$\mathcal{L} = \begin{pmatrix} 1 & R_{56,j} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2\pi}{\lambda}\frac{V_{1,j}\sin\varphi_{1,j}}{E_j+V_{1,j}\cos\varphi_{1,j}} & 1 \end{pmatrix} = \tag{18}$$

$$\begin{pmatrix} 1-\frac{2\pi R_{56,j}}{\lambda}\frac{V_{1,j}\sin\varphi_{1,j}}{E_j+V_{1,j}\cos\varphi_{1,j}} & R_{56,j} \\ -\frac{2\pi}{\lambda_{rf}}\frac{V_{1,j}\sin\varphi_{1,j}}{E_j+V_{1,j}\cos\varphi_{1,j}} & 1 \end{pmatrix}, \tag{19}$$

Thus considering a beamline composed of N of these compressor-accelerator modules will results in a momentum compaction:

$$R_{56}^{total}(V_{1,1..N}, \varphi_{1,1..N}) = \left[\Pi_{j=1}^{N}\mathcal{L}_j\right]_{56}. \tag{20}$$

In this latter equation, we suppose that the $R_{56,j}$ are fixed by design (i.e. the bending angle of the bunch compressors are fixed); thus the total R_{56}^{total} is only a function of $V_{1,1..N}$, and $\varphi_{1,1..N}$.
Let's now simplify our problem by only considering the case of two compression stages. In such a

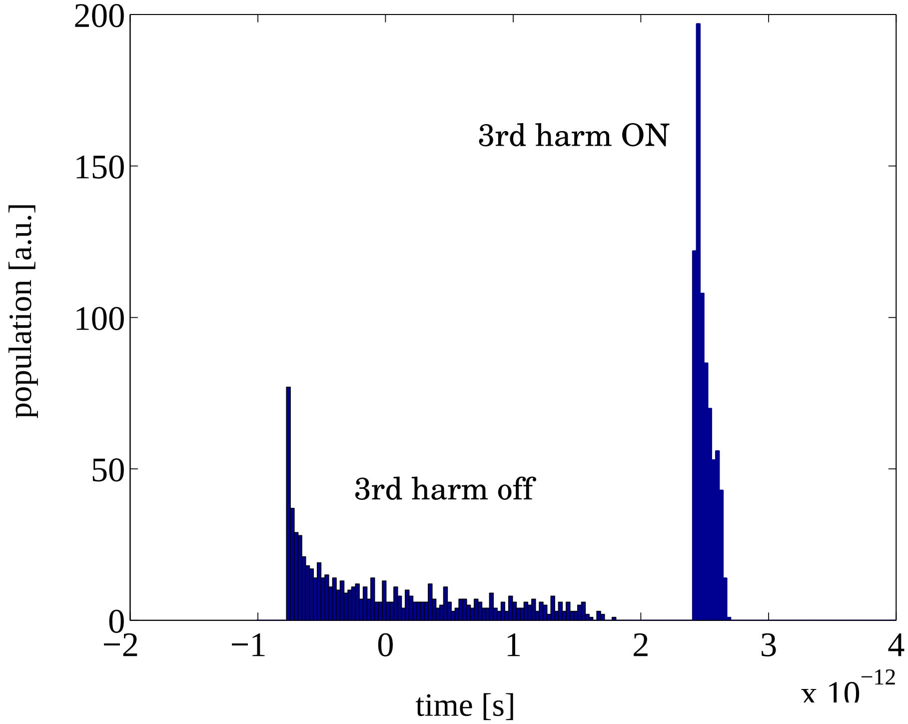

Figure 4: Comparison of the charge density profiles, downstream of the bunch compressor, when the 3^{rd} harmonic section is or not operated.

case, we have:

$$R_{56}^{total} = R_{56,1} + R_{56,2} - \frac{2\pi}{\lambda_{rf}} \times R_{56,1} R_{56,2} \sin \varphi_{1,2}. \tag{21}$$

The question is how do we setup the first accelerating section (that includes the 3^{rd} order harmonic) to achieve the minimum bunch length downstream of the second compression stage?

A first approach consists of requiring that only this first section is used to impart an energy correlation; i.e. all the subsequent accelerating sections are operated for maximum energy gain that is $\varphi_{1,2} = 0°$. In such a case, $R_{56}^{total} = R_{56,1} + R_{56,2}$, and the rate of compression downstream of the first section is given by: $s_f/s_i = R_{56,2}/R_{56,1}$. Such a ratio might not be favorable: e.g. in the case of TTF User Facility its value is $\sim$1/4, which results in a too long bunch at the exit of the first compressor. As a consequence it will accumulate further RF-curvature during its subsequent acceleration in the second accelerating section.

An alternative is to tune the first accelerating section in such a way to achieve the desired bunch length downstream of the first compressor. For such a purpose one needs to match the value of φ_1 so that $\alpha_1 = (s_f/s_i - 1)/R_{56,1}$. The second accelerating section is then operated off-crest to impart the proper time-energy correlation for maximum compression downstream of the second compressor.

3 Application of the technique to TTF User Facility

We apply the aforementioned analysis to the case of the TTF User Facility, which incorporate two bunch compressors. First we study the case where only one compressor, the first one (CS1), is operated to achieve maximum peak current; such a case is not purely academic: during the commissioning stage of the TTF User Facility we anticipate such type of operation for commissioning and learning how to setup the 3^{rd} harmonic section. Then we analyze the case where the bunch is compressed in two stages. The TTF User Facility longitudinal dynamics scheme is depicted in Fig. 5, and the properties of the two bunch compressors are detailed in Table 1.

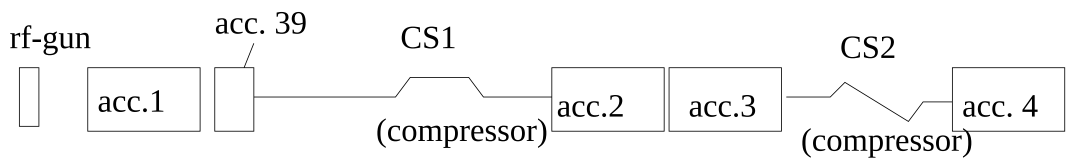

Figure 5: Overview of the compression scheme in TTF User Facility. The labels "acc" and "CS" stands respectively for accelerator module and compression stage. "acc. 39" refer to the 3.9 GHz "linearizing" section.

Compressor	R_{56} (cm)
CS1	-16.51
CS2	-6.38

Table 1: Parameters of the TTF User Facility bunch compressor.

3.1 Incoming phase space from the gun

The characterization of the initial phase space is of importance since it can be the essential contribution to the third order term in the longitudinal phase space distortion. An example of a third order polynomial fit of the incoming phase space (2 m downstream of the RF-gun) is pictured in Fig. 6. The operating condition of the gun are: a launch phase $\phi_o = 34$ deg, and a peak field of 40 MV/m. The third order fit of the phase space density gives the ζ_i coefficient (defined in Eqn (11)) gathered in Table 2. These coefficients are henceforth used for all the subsequent computations presented in this Section.

coefficent	value
ζ_1	2.50
ζ_2	-558.38
ζ_3	2.40×10^5

Table 2: Parameters of the TTF User Facility bunch compressor.

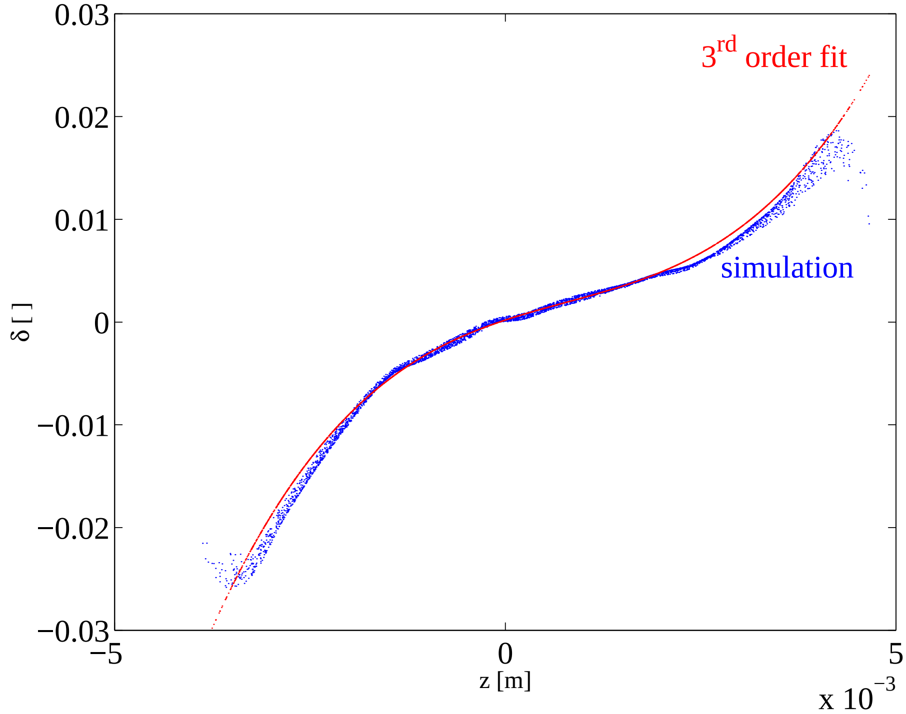

Figure 6: Comparison of the longitudinal phase space incoming from the rf-gun (at 2 m from the photocathode) with its third order polynomial fit.

3.2 Case of full compression during the first stage compression

Let's first consider the case of a single compressor following the accelerating and third harmonic sections, and assume we want to achieve the shortest possible bunch length. In such a case we require the longitudinal matching equations Eqns (5) to be satisfied for the parameter of compressor CS1. The achieved longitudinal phase space and charge density profile downstream of CS1 are picture in Fig. 7.

3.3 Nominal working point for two-stages compression scheme

Under nominal operation, the first stage compression is planned to compress the bunch down to $\sim$0.3 mm. This bunch length, downstream of the first compressor, was chosen as a compromise, to lessen space charge force and avoid rf-curvature effect in subsequent accelerating structures. Given the CS1 momentum compaction have to satisfy the longitudinal matching conditions, Eqns (12), for an effective $R_{56} = R_{56,1}/(1-\rho)$, where $\rho = .12$ is the compression ratio desired downstream of CS1. The higher order terms (T_{566}, and U_{5666} where taken to be those of CS1). The resulting longitudinal phase space and charge density downstream of the first accelerator section, and downstream of CS1 are pictured in Figs 8 and 9. Downstream of CS1 a peak current of the order of 400 Amps is achieved.

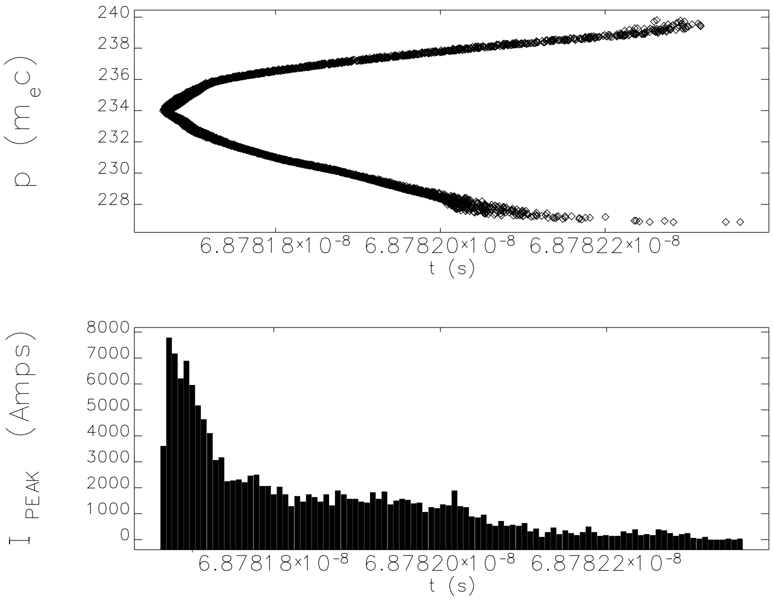

Figure 7: Longitudinal phase space and charge density profile downstream of the first bunch compressor. In this case the linearizer and the first accelerating section are operated such that the minimum bunch length is achieved downstream of the bunch compressor.

The second accelerating section (acc.2 and 3) is phased in a way to achieve the required peak current downstream of CS2: more than 2.5 kA. The resulting longitudinal phase space and charge density downstream of the CS2 are presented in Figs 10. The various phases and gradient used for this nominal operating point are presented in Table 3.

Parameters	value	units
Gradient acc.1 cavities	20.0 & 12.0 **a)**	MV/m
Phase acc.1	-11.49	°
Gradient acc.39 **b)**	12.15	MV/m
Phase acc.39	183.81	°
Gradient acc.2 & 3	20.0	MV/m
Phase acc.2 & 3	-5	°

Table 3: Parameters of the rf-elements for nominal operation of TTF User Facility compression scheme. **a)** the module acc.1 will have two set of four cavities operated at two different gradients. **b)** acc.39 consists of 4 TESLA-type cavity frequency-scaled to 3.9 GHz.

4 Conclusion

It has been shown that one can efficiently enhance the peak current by using a third harmonic accelerating section to correct for nonlinearities of the longitudinal phase space. The model used is

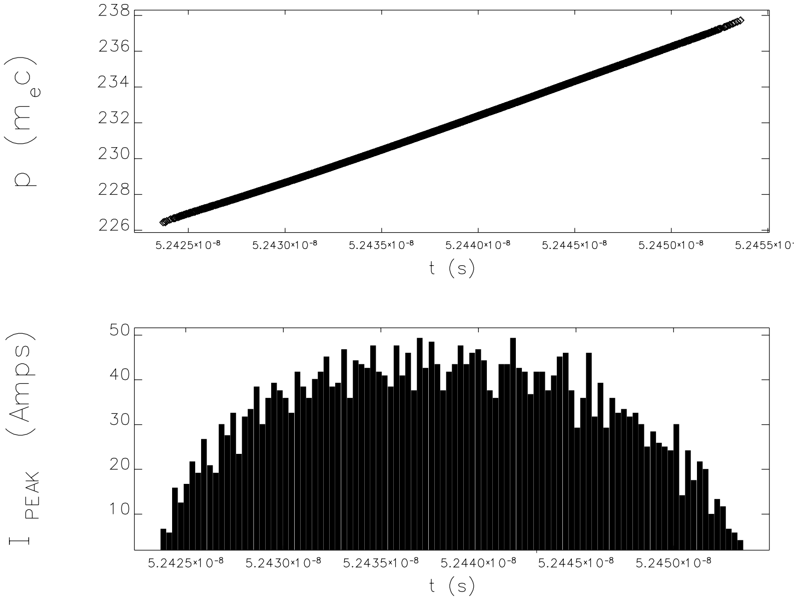

Figure 8: Longitudinal phase space and charge density profile downstream of the first accelerating and third harmonic section.

based on single-particle dynamics third order optics and does not take into account the distortion that occurs during transport (i.e. because of space charge or wake field) or during the compression mechanism (e.g. bunch self interaction via coherent synchrotron radiation): taking into account all these effects can be done only numerically and thus the optimization of the accelerating section phase and the third order section phase and gradient is the results of iterative simulation runs. In the particular case of the TTF User Facility, the analytic formulation presented in this note has been used as a starting point for numerical optimization, and a full integrated modeling, including various sources of collective effects, has been performed [8].

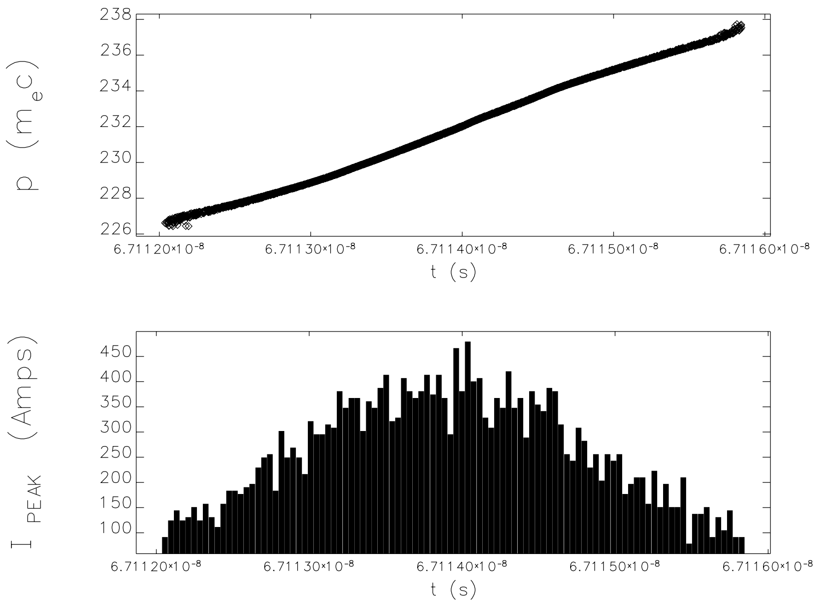

Figure 9: Longitudinal phase space and charge density profile downstream of the first bunch compressor.

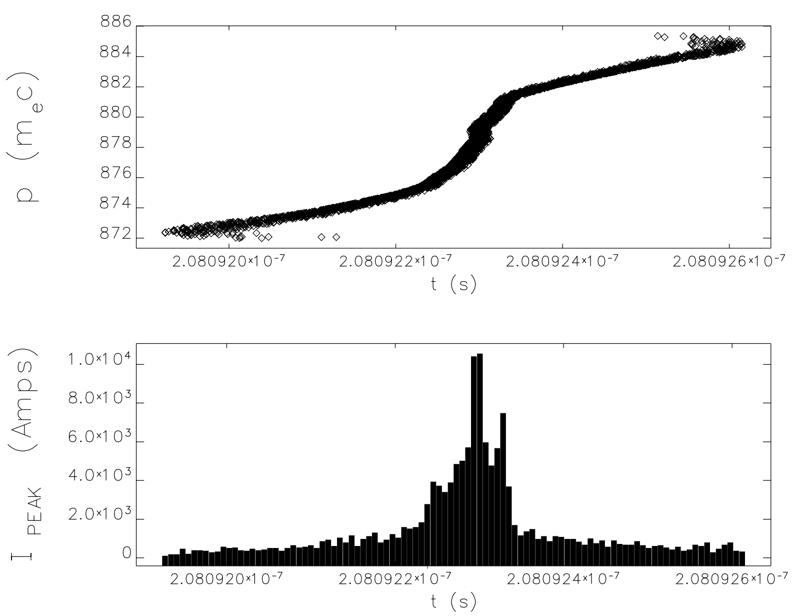

Figure 10: Longitudinal phase space and charge density profile downstream of the second bunch compressor.

A Why a 3^{rd} order harmonic RF-field?

In the derivation presented in this note, we *a fortiori* considered that the correction scheme would use the 3^{rd} harmonic rf-field. The reason for using a higher harmonic field is to linearize the longitudinal phase, that is to make the RF accelerating voltage seen by the bunch flat over the whole bunch length.
In the case one would desire to get an RF accelerating voltage flat over whole half RF-wavelength, one should use odd harmonic only. This comes from the Fourier series expansion of a "square" function,

$$\Pi_l(x) = \begin{cases} 1/(2l) & |x| \leq l \\ 0 & \text{elsewhere} \end{cases}, \tag{22}$$

which has the form:

$$\Pi_l(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi l} \sin(n\frac{\pi}{2}) \times \cos(nx\frac{2\pi}{l}). \tag{23}$$

Thus, in the case one would want a constant accelerating voltage over the whole half rf-period, the "first order" correction should be performed with the third harmonic RF-field.

In principle, under the assumption that the required accelerating voltage flatness length is small compared to the RF-wavelength (which is our case), one could use any field of higher frequency than the fundamental accelerating mode of the accelerator.

In the case of TESLA and TTF projects, because multi-bunch operation is required, the correcting field must be an harmonic of the fundamental accelerating field, i.e. $\omega_n = 2\pi n \times 1.3 \times 10^9$. At first order the energy that needs to be removed by the correcting field scales as the $1/n^2$. Thus this type of scaling favor high frequency. However the longitudinal, $\mathcal{W}_{||}$, and transverse wake field, $\mathcal{W}_{\perp}$, scale with the frequency as:

$$\mathcal{W}_{||} \propto \omega^2, \text{ and } \mathcal{W}_{\perp} \propto \omega^3. \tag{24}$$

This latter dependency would instead call for the lowest frequency. Our choice of 3.9 GHz was essentially motivated by the two aforementioned arguments.

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