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Estimation of field amplitudes during the operation of the 1.5 cell photoelectron RF gun of the PITZ collaboration

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Abstract
The present paper discusses the possibility to obtain information about the field flatness of a 1.5 cell normal conducting RF gun cavity during the running of the gun. By measurements of the microwave network parameters at room temperature and by measurement of the passband frequencies in the running regime of the gun it is possible to estimate the perturbation of field flatness, caused by an inhomogeneous temperature distribution.

1. Introduction

In the framework of the PITZ collaboration at DESY in Zeuthen a 1.5 cell normal conducting RF gun is under development [1)]. It is the main part of the injector for the x – ray FEL project of DESY and works at the standard frequency of 1.3 GHz. A special feature of this gun is the tuning procedure. The design frequency is obtained by variation of the cooling water temperature. The field distribution inside the gun can be measured and a mechanical tuning of the cells can be done only at room temperature. Because of that, a predefined field distribution will be conserved in the working regime only in this case, when we have a homogenous increasing of the temperature inside the whole gun. Numerical calculations show, that this is not the case and that the temperature at the end of the second cell differs essentially from the temperature of other cell parts [2)] (see Fig.1). Therefore the field distribution in the PITZ gun, which is very sensitive to the cavity shape and which is essential for the beam quality, is not well known. A possibility to estimate the field balance of an one-and-half cell RF gun by measuring the frequency difference between the two passband modes has been discussed in [5)]. In the present paper we will calculate the field distribution in the working regime by a simple analytical model using the measured field distribution and the frequencies at room temperature and the two frequencies of the passband mode measured during the operation of the gun.

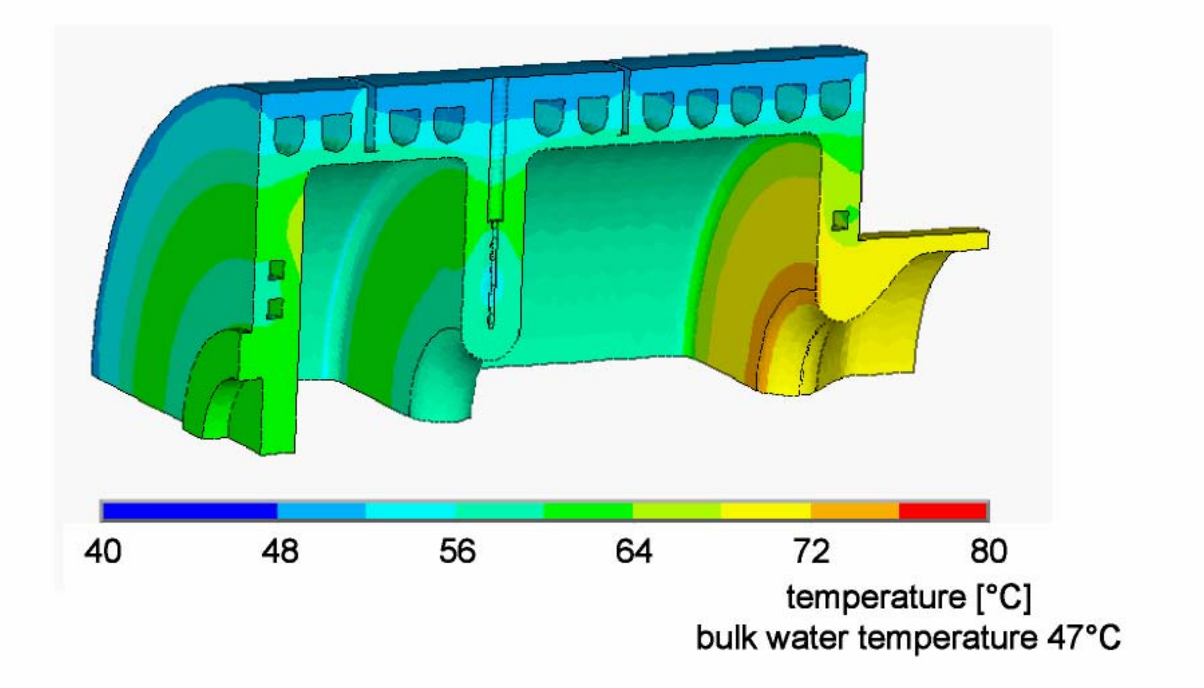

Fig. 1: Calculated temperature distribution of the PITZ gun in the working regime [2)]

2. Experimental determination of the microwave network elements

The microwave network of the 1.5 cell cavity can be represented by two coupled circuits as shown in Fig.2.

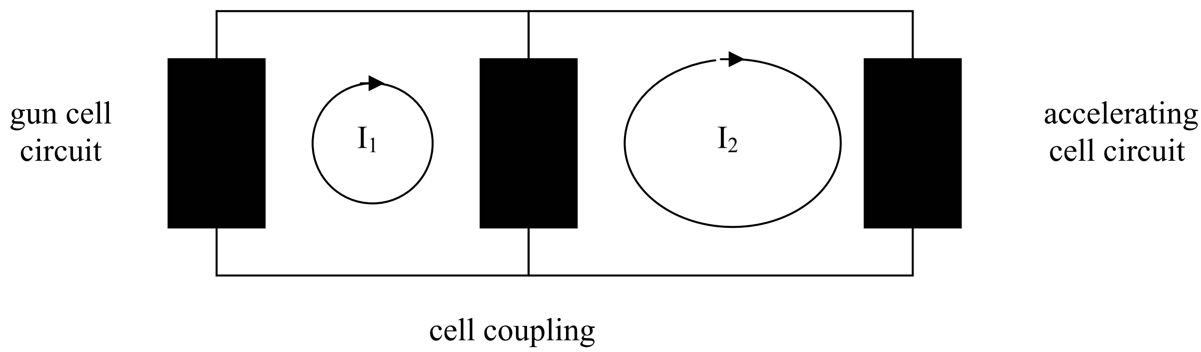

Fig. 2: Microwave network of the PITZ gun

In this simple model we have the RF mode of each separate circuit and the passband of the coupled system. This passband contains the π – and the 0 – mode and the Kirchhoff law determines the currents I_1 and I_2 of these modes by the following eigenvalue equation:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} * \left| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\rangle_\nu = e_\nu * \left| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\rangle_\nu \qquad (\nu = \pi, 0) \tag{1}$$

Here the matrix elements A_{11} and A_{22} characterize the separate circuits and the parameters A_{12} and A_{21} are responsible for the coupling between the two cells (circuits). If the cell coupling is capacitive and we neglect in the circuits the thermal losses (Q>>1), the eigenvalues e_ν are real and are simply the square of the passband frequencies ω_ν[3).]

$$e_\nu = (\omega_\nu)^2 \tag{2}$$

At room temperature it is possible to measure the frequencies and the field distribution of the passband by the bead pull method [4)]. The currents defined in Fig.2 and eq.(1) are proportional to the field amplitudes in the cells of the corresponding modes. If the relations between the field amplitudes of different cells in the π – and in the 0 - mode are ρ and λ respectively, the unnormalized eigenvectors of eq.(1) can be expressed as follows:

$$|\pi\rangle = \left| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\rangle_\pi = \begin{pmatrix} 1 \\ \rho \end{pmatrix} \qquad |0\rangle = \left| \begin{matrix} I_1 \\ I_2 \end{matrix} \right\rangle_0 = \begin{pmatrix} 1 \\ \lambda \end{pmatrix} \tag{3}$$

The network matrix A is in general not symmetric. We can determine the left hand eigenvectors by an orthogonalization procedure and obtain:

$$_\pi\left\langle \begin{matrix} I_1 \\ I_2 \end{matrix} \right| = \frac{1}{\rho - \lambda} * \begin{pmatrix} -\lambda \\ 1 \end{pmatrix} \qquad _0\left\langle \begin{matrix} I_1 \\ I_2 \end{matrix} \right| = \frac{1}{\rho - \lambda} * \begin{pmatrix} \rho \\ -1 \end{pmatrix} \tag{4}$$

The matrix A is completely defined by its eigenvalues and eigenvectors. A simple calculation gives:

$$A = \frac{1}{\rho - \lambda} * \left(\omega_\pi{}^2 * \begin{bmatrix} -\lambda & 1 \\ -\rho\lambda & \rho \end{bmatrix} + \omega_0{}^2 * \begin{bmatrix} \rho & -1 \\ \lambda\rho & -\lambda \end{bmatrix} \right) \tag{5}$$

In this way the matrix A of the microwave network, which simulates the behaviour of the 1.5 cell RF cavity of the PITZ gun at room temperature, can be calculated by the experimental results of the beat pull and frequency measurements of the passband modes.

2. Estimation of the field changes in dependence on perturbation

After the measurements at room temperature the cavity is heated up by the input of RF power. During the heating procedure the temperature is stabilized by a cooling system to a value, at which the frequency of the π mode has the design value of 1.3 GHz. If the temperature changes uniformly in all parts of the cavity and we have a homogenous temperature distribution also at the design frequency, the distribution of the RF field does not change. This type of cavity perturbation which is caused by homogenous heating can be described by the matrix $AW{=}A{*}(1{+}\varepsilon)$, where the parameter ε characterizes the RF heating. It has the same eigenvectors as A, but the frequencies of the cavity Ω_v are in first order perturbation $\Omega_v = \omega_v*(1+\varepsilon/2)$. A "homogenous" perturbation which is described by the matrix AW can be caused also by changing the pressure or the dielectric value inside the cavity.

In Fig.1 the calculated temperature distribution of the PITZ cavity in the working regime is shown [2)]. This distribution differs remarkably from a homogenous one. The temperature at the beam tube of the second cell has an about 20°C higher value then the surrounding parts. Therefore we introduce a second, non homogenous perturbation inside the second cell, which changes the matrix element A_{22} of eq.(1),(5) only. If the magnitude of this perturbation is characterized by the parameter η, the matrix AW, which characterizes the PITZ gun in the working regime can be represented as follows:

$$AW = A + A^{(1)} \ , \qquad A^{(1)} = \varepsilon * A + \begin{bmatrix} 0 & 0 \\ 0 & \eta \end{bmatrix} \tag{6}$$

The eigenvalues of AW, which are again the square of the passband frequencies Ω_υ, can be calculated in the first order perturbation theory:

$$\Omega_\nu{}^2 = \omega_\upsilon{}^2 + \langle \nu | A^{(1)} | \nu \rangle \qquad (\nu = \pi, 0) \tag{7}$$

Using eq.(4)-(7) one obtains for the perturbation parameter ε and η from the measured frequencies and field amplidudes:

$$\varepsilon = \left(\frac{\lambda \Omega_\pi{}^2 + \rho \Omega_0{}^2}{\lambda \omega_\pi{}^2 + \rho \omega_0{}^2} \right) - 1 \ , \ \eta = \frac{(\rho - \lambda)(\omega_\pi{}^2 \Omega_0{}^2 - \omega_0{}^2 \Omega_\pi{}^2)}{\lambda \omega_\pi{}^2 + \rho \omega_0{}^2} \tag{8}$$

In this way the matrix AW of the working regime is completely defined and we are able to calculate the corresponding eigenvectors $|W,\nu\rangle$, (ν = π,0) in the same order of perturbation theory.

$$|W,\pi\rangle = |\pi\rangle + \eta * \frac{\rho}{(\rho-\lambda)(\omega_\pi{}^2 - \omega_0{}^2)} * |0\rangle , \ |W,0\rangle = |0\rangle + \eta * \frac{\rho}{(\rho-\lambda)(\omega_\pi{}^2 - \omega_0{}^2)} * |\pi\rangle \tag{9}$$

As mentioned in the introduction, the disturbed eigenvectors are independent from the "homogenous" perturbation ε.

For a RF gun it is very essential to know the ratio of the field amplitudes R of the two cells during the working of the gun. A direct measurement of this ratio is not possible, but it is proportional to the ratio of the currents in the corresponding microwave circuits. The PITZ gun works in the π- mode. Therefore using eqs.(8) and eqs.(9) we obtain for R:

$$R = \frac{1}{\rho} * \left[\frac{(\omega_\pi^{\ 2} - \omega_0^{\ 2})(\lambda\omega_\pi^{\ 2} + \rho\omega_0^{\ 2}) + \rho(\omega_\pi^{\ 2}\Omega_0^{\ 2} - \omega_0^{\ 2}\Omega_\pi^{\ 2})}{(\omega_\pi^{\ 2} - \omega_0^{\ 2})(\lambda\omega_\pi^{\ 2} + \rho\omega_0^{\ 2}) + \lambda(\omega_\pi^{\ 2}\Omega_0^{\ 2} - \omega_0^{\ 2}\Omega_\pi^{\ 2})} \right] \tag{10}$$

If $\Omega_\pi / \Omega_0 = \omega_\pi / \omega_0$ we have a homogenous perturbation only and the ratio R is 1/ρ as in the case of room temperature (see eqs.3).

The previous calculation shows the possibility, to estimate the field ratio inside a 1.5 cell RF cavity during the running of the gun by measurement of the field amplitudes and frequencies at room temperature and of the frequencies during the working regime.

3. Numerical example of the network analysis

Now we will explain the discussed model by using room temperature parameters, which are calculated by the geometry of the PITZ gun [1)] given in Fig.3. The obtained room temperature values for the field ratios and the passband frequencies are given in tabl.1.

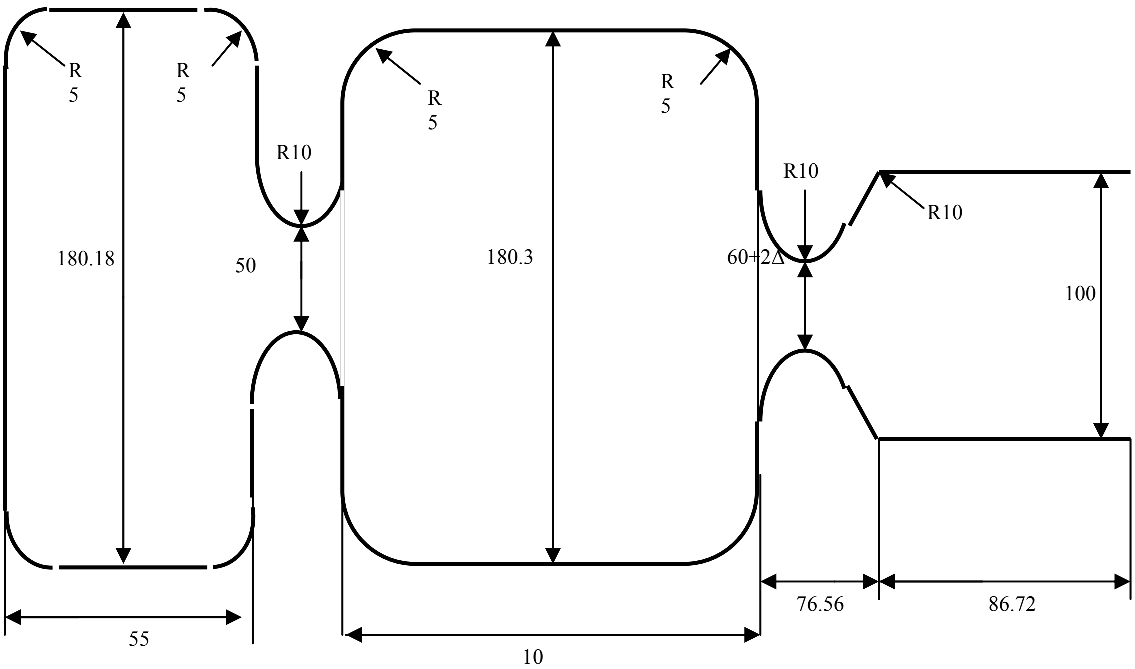

Fig.3 Geometry of the cavity

Values at room temperature	
$\omega_\pi/2\pi$	1303.464 MHz
$\omega_0/2\pi$	1298.109 MHz
ρ	-0.9704
λ	0.4979

Tab. 1: Frequency and field values, used in the following calculation, which corresponds to the calculated values of the PITZ gun [1)] for the geometry of Fig.3.

. The 0 – mode frequency $\omega_0/2\pi$ is about 5 MHz below the π - mode value. The field parameter ρ = -0.9704 is near to the ideal "flatness" of the π - mode distribution. The ideal field parameter λ can be estimated replacing a 1.5 cell cavity by a symmetric 3 cell cavity and applying the theory of the passband as described in [3)]. The calculated value of 0.4979 is near to the ideal value of 0.5.

In order to examine the field distribution in the working regime of the gun, we calculate now the field ratio R defined in eqs.(10) as function of the frequency Ω_0 , which has been not measured yet. The result is shown in Fig. 4. If the frequency $f_0 = \Omega_0/2\pi$ differs by 100 kHz from the unperturbed value of 1298.109 MHz the flatness of the field distribution in the π - mode changes by approximately 6% only.

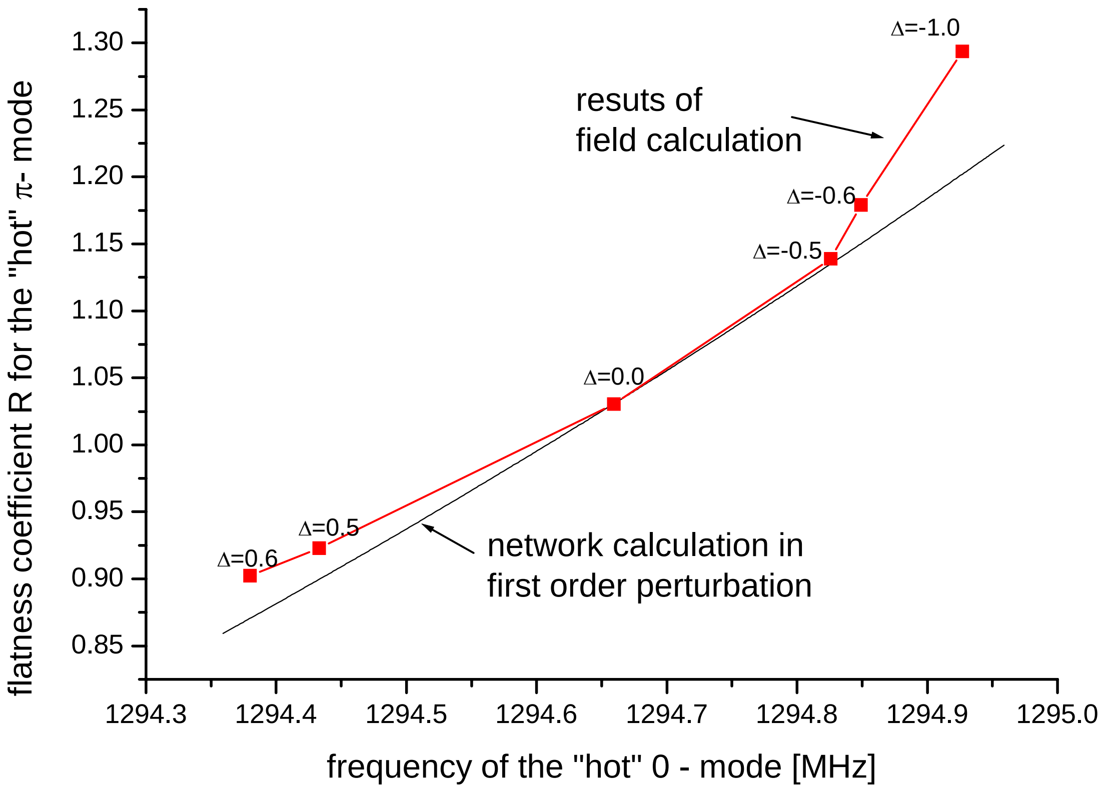

Fig. 4: Field flatness of the π- mode in dependence of the 0 – mode frequency, estimated by network analysis and by field calculation

4. Estimation of the flatness parameter R for special changes of the cavity geometry by field calculation

In order to check the approximations, which are contained in the derivation of eqs. (10), we calculate the RF field of the cavity for the geometry of Fig.3. We obtain the passband solutions and scale the two frequencies so, that the π-mode frequency is 1300 MHz. From the electric field amplitudes we estimate the flatness of the π-mode. After this we repeat the whole procedure for six different diameters of the iris at the end of the second cavity cell. Fig.4 shows the flatness as a function of the 0-mode frequency calculated in this way. The obtained values are in reasonable agreement with the result of the network calculation.

5. Conclusion

Describing the 1.5 cell RF cavity of the PITZ gun by a network of two coupled circuits it is possible to derive simple relations, which allow the estimation of the field flatness during the running of the gun. For this task one has to measure the field distribution and the passband frequencies of the cavity at room temperature and the passband frequencies during the running of the gun. In order to check the assumption of the network calculation and the first order perturbation theory, we made RF field calculation for special changes of the cavity geometry. This type of calculation doesn't contain the called approximation, but the result is less general. Nevertheless a good agreement between both methods is obtained

The sensitivity of the flatness with respect to the frequency Ω_0 makes a measurement of this value very useful, whereby an accuracy of 20kHz seems to be sufficient.

In the present calculation the field distribution of the gun cavity is disturbed by a perturbation inside the second cavity cell. This perturbation is predicted by the temperature distribution calculated in [2)]. The same method can be applied also to any other type of perturbation of the cavity field.

5. Acknowledgements

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6. References

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