Microbunch Radiative Tail-Head Interaction

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Abstract

In this article we study the effect of radiative interaction in a microbunch of charged particles following a curved trajectory. Due to the curvature, the cooperative electromagnetic fields radiated from back parts of the bunch can overtake the head particles. They produce substantial energy loss gradient along the bunch, that affects the beam dynamics in the microbunch.

1 Introduction

Several accelerator projects with high charge microbunches are under consideration now. Examples are modern linac-based FELs and damping rings and bunch compressors for future Linear Colliders. This article is devoted to the novel effect of the tail-head forces due to synchrotron radiation of the microbunches. Opposite to the well known collective effects in accelerators where the wake-fields produced by head particles act on the particles behind, the cooperative radiation fields generated at the tail overtake the head of the bunch if the bunch moves along a curved trajectory. Below we will call these fields as "overtaking" fields.

As is known [1], an ultrarelativistic microbunch moving in a magnetic field radiates electromagnetic energy in a wide frequency spectrum. The radiation is coherent in the characteristic frequency range of $\omega \ll c/l_b$ (where l_b is the bunch length), which is the tail of the single particle synchrotron radiation spectrum. The characteristic wavelength of the single particle radiation is $\lambda_{cr} \sim R/\gamma^3$ where R is the orbit curvature radius, γ is the relativistic factor. In the following we will assume that the bunch length is much larger then the incoherent radiation wavelength, $l_b \gg \lambda_{cr}$. The spectral coherent radiation intensity of the bunch with charge Ne at low frequencies is about:

$$\frac{dI}{d\omega} \sim \gamma \frac{(Ne)^2}{R} (\frac{\omega \lambda_{cr}}{c})^{1/3} \tag{1}$$

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Therefore, the energy loss per unit length can be estimated (see e.g. [2]) as:

$$\frac{d\mathcal{E}_{coh}}{cdt} \sim \int\limits_{0}^{c/l_b} \frac{dI}{d\omega}\frac{d\omega}{c} \sim \frac{(Ne)^2}{(Rl_b^2)^{2/3}} \qquad (2)$$

For smooth charge distribution function coherent radiation at frequencies $\omega \gg c/l_b$ is practically absent.

We are interested in the region of frequencies of $\omega \simeq c/l_b$. The radiation in this region is also a cooperative effect (coherent in parts of the bunch), but it can not be treated simply as a coherent radiation, since it depends on the shape of the bunch charge distribution. In particular, an important question arises – how the energy losses are distributed along the bunch.

As the energy is radiated mostly along the beam direction, one may expect different deceleration effect for the tail and the head of the bunch, and the energy loss difference for different particles inside the bunch can be roughly estimated to be the same value as the losses due to the coherent radiation, i.e.:

$$\Delta\mathcal{E}' \sim \mathcal{E}_{coh}/N. \qquad (3)$$

Besides this introduction, the article consists of four sections. Precise formulas for the overtaking longitudinal forces will be derived from the retarded potentials in Section 2. In Section 3 we suggest a simple physical explanation of the effects. Numerical estimations for the TESLA Test Facility Free Electron Laser [3] are presented in Section 4. Finally, in Section 5 we summarize the results and discuss their difference and essential novelty in comparison with some previous considerations of the beam dynamics in bends.

2 Radiative Force in a Bend

In the following we will derive formulae for the overtaking tail-head force. We assume that a bunch of charged particles moves along a curved trajectory with radius R – see Fig.1.

Let us introduce the four dimensional vector potential A in vacuum [4]:

$$A_0 = e \int \frac{d^3r' \cdot n(\vec{r'}, t - \tau)}{\tau c}, \; c\tau = |\vec{r} - \vec{r'}| \qquad (4)$$

$$\vec{A} = e \int \frac{d^3r' \vec{\beta'} \cdot n(\vec{r'}, t - \tau)}{\tau c}, \qquad (5)$$

where $n(\vec{r}, t)$ is the particle density.

Then the Hamiltonian $\mathcal{H}$ of the particle with kinetic energy $\mathcal{E}$ can be written as:

$$\mathcal{H} = c\sqrt{(\vec{P} - e\vec{A}/c)^2 + m^2c^2} + eA_0 = \mathcal{E} + eA_0 \qquad (6)$$

Taking into account the particle equation of motion, one gets $d\mathcal{H}/dt = \partial\mathcal{H}/\partial t$. The total derivation term dA_0/dt can be ignored and the energy loss is equal to:

$$\frac{d\mathcal{E}}{dt} = \partial\mathcal{H}/\partial t = e\frac{\partial[A_0 - (\vec{\beta}\vec{A})]}{\partial t} \equiv \frac{\partial V}{\partial t}, \quad \vec{\beta} = \vec{v}/c. \tag{7}$$

Thus, the *interaction Hamiltonian V* is :

$$V = e^2 \int \frac{d^3r'(1 - (\vec{\beta}\vec{\beta}'))}{c\tau} \cdot n(\vec{r}', t - \tau) \tag{8}$$

Let's present the retarding time τ as series:

$$c^2\tau^2 = (\vec{r} - \vec{r}')^2 = (y - y')^2 + \rho^2 + \rho'^2 - 2\rho\rho'\cos\theta =$$

$$= (y - y')^2 + (x - x')^2 + R^2\theta^2 - R^2\theta^4/12 + R\theta^2(x + x') \tag{9}$$

where $\rho = R + x$.

Let's call the main term in the series being the arc length between the the observation point B and the retarded position A, see Fig.1, $\xi = R\theta = z - z'$, then

$$c\tau = \left|\xi - \xi^3/24R^2 + \left[(y - y')^2 + (x - x')^2\right]/2\xi + \xi(x + x')/2R\right| \tag{10}$$

We consider the case when the transverse bunch size σ_r is small:

$$\frac{\sigma_r}{\sigma_s} \ll \left(\frac{R}{\sigma_s}\right)^{1/3} \tag{11}$$

Then, going back to the energy gradient, one may take into account only the first and the second term of τ:

$$\dot{\mathcal{E}} = Ne^2 \int \frac{d\xi(1/\gamma^2 + \xi^2/2R^2)}{\xi} \cdot \frac{\partial}{\partial t}\lambda(z - \beta ct - \xi + \beta|\xi - \xi^3/24R^2|) \tag{12}$$

where $N\lambda(s)$ is the linear bunch density along its trajectory ($\int \lambda(s)ds = 1$).

The equation can be further simplified if the Coulomb part is much smaller than the radiation part:

$$1/\gamma << \xi/R \simeq (\sigma_s/R)^{1/3}. \tag{13}$$

Then the parts of the integrand which are proportional to $1/\gamma^2$ can be neglected. The main contribution comes from $\xi > 0$ (i.e. from particles which are behind the test particle); making the integration with replacing the argument ξ by $\xi^3/24R^2$, one gets:

$$\frac{d\mathcal{E}}{cdt}(s) = -\frac{2Ne^2}{3^{1/3}R^{2/3}} \int_{-\infty}^{s} \frac{ds'}{(s - s')^{1/3}} \frac{\partial\lambda(s')}{\partial s'}, \tag{14}$$

here s, s' are coordinates along the moving bunch (e.g. $s = z - \beta ct$).

The total energy changes of the bunch as a whole is negative, i.e. there is loss of energy. Indeed, it is equal to

$$\frac{d\mathcal{E}_{coh}}{cdt} = -\frac{2N^2e^2}{3^{4/3}R^{2/3}} \int_{0}^{\infty} \frac{d\zeta}{\zeta^{4/3}}[g(0) - g(\zeta)], \tag{15}$$

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with the correlator

$$g(\zeta) = \int\limits_{-\infty}^{+\infty} ds \lambda(s)\lambda(s - \zeta). \tag{16}$$

With the use of the Fourier transformation one gets the total bunch coherent energy loss rate:

$$\mathcal{P}_{tot} = \frac{d\mathcal{E}_{coh}}{cdt} = -\frac{8\pi C_0 N^2 e^2}{3^{4/3}R^{2/3}} \int\limits_0^{\infty} |\lambda_k|^2 k^{1/3} dk < 0, \tag{17}$$

with

$$C_0 = \int\limits_0^{\infty} (1 - \cos x)x^{-4/3} dx = \frac{3\sqrt{3}}{2}\Gamma(2/3)$$

and

$$\lambda_k = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} \lambda(s)e^{-iks} ds.$$

For a bunch with a linear Gaussian distribution $\lambda(s) = (1/\sqrt{2\pi}\sigma_s)e^{-s^2/2\sigma_s^2}$, the energy loss gradient along the bunch (14) is equal to:

$$\frac{d\mathcal{E}}{cdt}(s) = -\frac{2Ne^2}{\sqrt{2\pi}3^{1/3}R^{2/3}\sigma_s^{4/3}}F_0(s/\sigma_s). \tag{18}$$

where

$$F(x) = \int\limits_{-\infty}^{x} \frac{dx'}{(x - x')^{1/3}}\frac{\partial}{\partial x'}e^{-x'^2/2}, \tag{19}$$

The overtaking potential function $F_0(x)$ is presented in Fig.2. The total energy loss (17) of the bunch is consistent with the previously known result [2]:

$$\mathcal{P}_{tot} = -\frac{N^2 e^2}{R^{2/3}\sigma_s^{4/3}}\frac{2^{4/3}3^{1/6}[\Gamma(2/3)]^2}{\pi}. \tag{20}$$

For a uniform bunch with length of l_b the losses are:

$$\frac{d\mathcal{E}}{cdt}(s) = -\frac{2Ne^2}{l_b 3^{1/3}R^{2/3}s^{1/3}}, \tag{21}$$

here s is the distance from the bunch tail end.

The total energy loss (17) of the bunch is equal to:

$$\mathcal{P}_{tot} = -\frac{3^{2/3}N^2 e^2}{R^{2/3}l_b^{4/3}}. \tag{22}$$

again, in a full agreement with [2].

3 Discussion

The following consideration suggests a simple physical explanation of the radiative microbunch tail-head effects and allows to derive approximate formulae.

We consider two segments of the ultrarelativistic bunch following the curved trajectory with radius R. Let the segments be spaced by the distance s that is of the order of the bunch length. For simplicity we denote one segment as a "tail" and another as a "head" of the bunch.

Consider the electromagnetic field of the tail of the bunch that is radiated when the bunch is at the point A – see again Fig.1. The condition, that the radiated field will overtake the head of the bunch in the point B is:

$$s = arc(AB) - |AB| = R\theta - 2R\sin(\theta/2) \approx \frac{1}{24}\theta^3 R, \quad if \ \theta \ll 1. \tag{23}$$

This condition determines three other important geometrical parameters which are mentioned in Fig.1:

$$\theta = 2(\frac{3s}{R})^{1/3}, \quad L_o = |AB| = \theta R = 2(3sR^2)^{1/3}, \quad r = |DB| = L_o\theta/2 = 2(9s^2R)^{1/3}. \tag{24}$$

The magnitudes of the overtaking transverse electric and magnetic fields which act on the head particle can be estimated as the fields of a charged line at the characteristic transverse distance of r:

$$E_\perp = H_\perp \simeq \frac{2Ne\lambda}{r}, \tag{25}$$

here Ne is the total bunch charge, λ is the function of linear density. For simplicity the density will be considered to be a constant within the bunch length l_b, i.e. $\lambda = 1/l_b$.

Taking into account that the directions of the electric and magnetic field are perpendicular to the line AC (because the fields were radiated at the point A), one gets the characteristic longitudinal force at the position of the head particle:

$$F_\parallel = eE_\perp \cdot \theta = \frac{2Ne^2\lambda\theta}{r} = \frac{2Ne^2\lambda}{(3sR^2)^{1/3}}. \tag{26}$$

– in agreement with the precise result (21). The force leads to a redistribution of the radiative energy losses inside the bunch, namely, to the relative increase of the energy of the head particles with respect to the tail. Nevertheless, the total energy change is negative. Note, that the effect takes place only for the limited charge distribution, i.e. the derivation of the distribution function along the bunch is not identical zero. In contrast, the coherent radiation of uniform current ring is absent and, of course, the longitudinal radiative tail-head force does not take place.

This physical explanation allows to make two useful comments which deal with geometrical limitations. First, let us assume that the "overtaking" length of L_o is smaller than the length of the bending magnet L_d. It easy to see, that the overtaking will take place only for fields which are radiated by small portion of particles $N_{eff} \sim Ns_{eff}/l_b$.

Here the length of effective cooperative interaction corresponds to overtaking inside the magnet length, i.e. $s_{eff} \simeq L_d^3/(24R^2)$. For 0.5 m long magnets with $R \sim 2$ m the value of s_{eff} is about 1 mm. Note, the effective longitudinal force (26) scales as $s^{-1/3}$, consequently, if the bunch length is more than s_{eff}, then with fixed number of particles one can expect reducing the cooperative energy loss gradient:

$$F_\| \sim F_\|\Big|_{L_d \geq L} \cdot \left(\frac{L_d^3}{24 l_b R^2}\right)^{2/3} \propto L_d^2. \tag{27}$$

Thus, for short curved track length, one can expect also some decrease of the total bunch energy loss due to coherent synchrotron radiation, as it was found in Ref.[6].

The second geometrical effect concerns radial shielding of the radiation. It is easy to see from Fig.1 that the effect takes place if the straight line AB of the radiation propagation does not fit into the vacuum chamber. For a pipe with radial size of $2b$ the condition is $b \leq r/4$, where r is defined in (24). Therefore, the effective intra-bunch distance of the radiative interaction is about $s_{eff} \sim \sqrt{b^3/R}$ and one can expect the corresponding decrease of the energy losses:

$$F_\| \sim F_\|\Big|_{b \geq r/4} \cdot \left(\sqrt{\frac{b^3}{l_b^2 R}}\right)^{2/3} \propto b. \tag{28}$$

Let us summarize the picture. The particles of the microbunch following a curved path "feel" the electromagnetic fields which were radiated by the particles behind. It leads to non-uniform energy losses along the bunch. As we will see in Section 4 below, beam dynamics at the microbunch facilities can be considerably affected by this effect.

The electric and magnetic forces also contain components which are transverse to the particle velocity. First results show that the total transverse force is about $\theta \ll 1$ times the longitudinal one. Detailed study of the transverse radiative effects is under way.

The used picture of electric and magnetic fields is somewhat rough, but reflects the main features of synchrotron radiation process. More detailed discussion on space properties of the radiation fields can be found in Ref.[7].

4   Radiative Microbunch Effects at the TTF FEL

Let us apply the obtained results to the TESLA Test Facility Free Electron Laser [3]. The main parameters of the FEL at the TTF are as follows:

energy in the undulator $E_{fin} = 1$ GeV,
electrons/bunch $N = 6.2 \cdot 10^9$,
normalized transverse emittance $\epsilon = 2$ μm.

We will consider the longitudinal effects at the bunch Compressors 2 and 3. It is found that in the main undulator, the characteristic overtaking length (the path for the tail radiation to take over the head – see Eq.(24)) $L_o = 2(3R^2\sigma_s)^{1/3}$ is about 0.2 m, that is much bigger than the undulator period of 2.7 cm. Therefore, our consideration is not applicable for the undulator.

The estimations of the effect of the tail-head radiative interaction are presented in Table 1. Vertical rows are marked according to the compressor number. The input parameters of the compressors are: energy E, length of each of four magnets L_d, dipole magnetic field B, curvature radius in the magnets R, horizontal beta-function β_x, and the bunch length σ_s.

Longitudinal effects are characterized by the rms induced energy difference in the bunch:

$$\Delta E = \frac{2Nr_0L_d}{3^{1/3}\sqrt{2\pi}(R\sigma_s^2)^{2/3}} \cdot mc^2\sqrt{<F_0^2(x)> - <F_0(x)>^2} \approx 0.22\frac{Nr_0L_dmc^2}{(R\sigma_s^2)^{2/3}}. \qquad (29)$$

There are also calculated the rms values of $\Delta E/E$ and $\Delta E/E_{fin}$. The change of particle angle due to induced energy loss $\Delta E/E$ in the bend is equal to:

$$\Delta x' = \frac{L_d}{R}\frac{\Delta E}{E}. \qquad (30)$$

Besides increasing both the correlated and uncorrelated energy spread of the beam, the most serious consequence is that the dispersion match of the compressor is distorted, because energy spread is generated inside the compressor. Using the equation (30), the rms emittance increase was calculated as the difference of emittance at the exit and at the entrance of the compressor:

$$\Delta\epsilon_{dE} = \gamma\left(\sqrt{<x^2>_f<x'^2>_f - <xx'>_f^2} - \sqrt{<x^2>_i<x'^2>_i - <xx'>_i^2}\right). \qquad (31)$$

As it is seen from the Table 1, the total emittance increase is of the order of 26 μm which is many times more than the design value of 2 μm. Eq. (31) describes the projected emittance of the whole beam. What matters for the FEL performance is the emittance of bunch segments no longer than the cooperation length L_c. Since for the TTF FEL, L_c is much smaller than the bunch length, our estimation of the emittance increase might be very conservative. Nevertheless, the effect is so big that a modification of the bunch compression layout may turn out to be necessary after more detailed analysis of beam dynamics.

5 Conclusions

We have analyzed the cooperative synchrotron radiation fields in a bend and found that the longitudinal force redistributes radiative energy losses along the bunch. The head particles are somewhat accelerated by the field radiated by tail particles. That effect can be described by longitudinal *overtake-function* $\mathcal{W}_0'(s)$:

$$\mathcal{W}_0'(s) = \frac{2}{3^{1/3}R^{2/3}}\frac{1}{s^{1/3}}\frac{\partial}{\partial s}, \qquad (32)$$

Table 1: Radiative Tail-Head Effects at the TTF FEL

	C2	C3	C2+C3
E,MeV	144	516	
L_d,m	0.5	0.5	
R,m	1.3	1.3	
β_x,m	11	11	
B,kGs	3.7	13.2	
σ_z, mm	0.8→0.25	0.25→0.05	
L,m	0.32→0.22	0.22→0.13	
ΔE,MeV	0.09	0.63	0.74
$\Delta E/E$,%	0.06	0.12	
$\Delta E/E_{fin}$,%	0.009	0.063	0.074
$\Delta\epsilon_{dE}$,μm	7	25	26

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here $\partial/\partial s$ is an operator of derivative along the longitudinal coordinate s ($s = -\infty$ corresponds to the bunch tail). The energy loss is the convolution of W_0' with the linear charge density:

$$\frac{d\mathcal{E}}{cdt} = \int_{-\infty}^{s} W_0'(s - s')\lambda(s')ds'. \tag{33}$$

Therefore, the energy losses originate from the derivative of the linear charge density. This is a characteristic feature of the effect. Besides the energy spread along the bunch, the effect leads to radial emittance increase when the bunch moves inside a bend.

As we mentioned above, there are also non-negligible transverse radiative forces, detailed studies of which are now under way.

We would like to make remarks on some previous work concerning forces acting on a bunch in a bend. In a recent publication of Carlsten and Raubenheimer [8] there was shown that the longitudinal Coulomb repulsion proportional to the line density derivative $\partial\lambda(s)/\partial s$ can play a role in magnetic bunch compressors. The effects we are discussing in this paper have no direct connection to the effects which come from the Coulomb fields. The Coulomb fields propagate *together* with the bunch particles, while the radiative forces are produced by electromagnetic waves due to cooperative radiation of tail particles that propagate independently. This difference becomes evident in realizing that Coulomb forces are proportional to the local charge density and its local derivative, while the radiative forces are a convolution of the linear density derivative with *overtake functions*. Another difference is that the longitudinal Coulomb force calculated in [8] leads to zero total energy loss of the bunch, while the integral of the radiative longitudinal force (33) over the bunch distribution gives the total energy loss in a remarkable agreement with the result of Nodvick and Saxon [2].

We also note that our consideration does not take into account the shielding effect of the metallic vacuum chamber, which is known to be essential for bunch lengths of the same order as the beam pipe radius [2, 9, 10].

Acknowledgments

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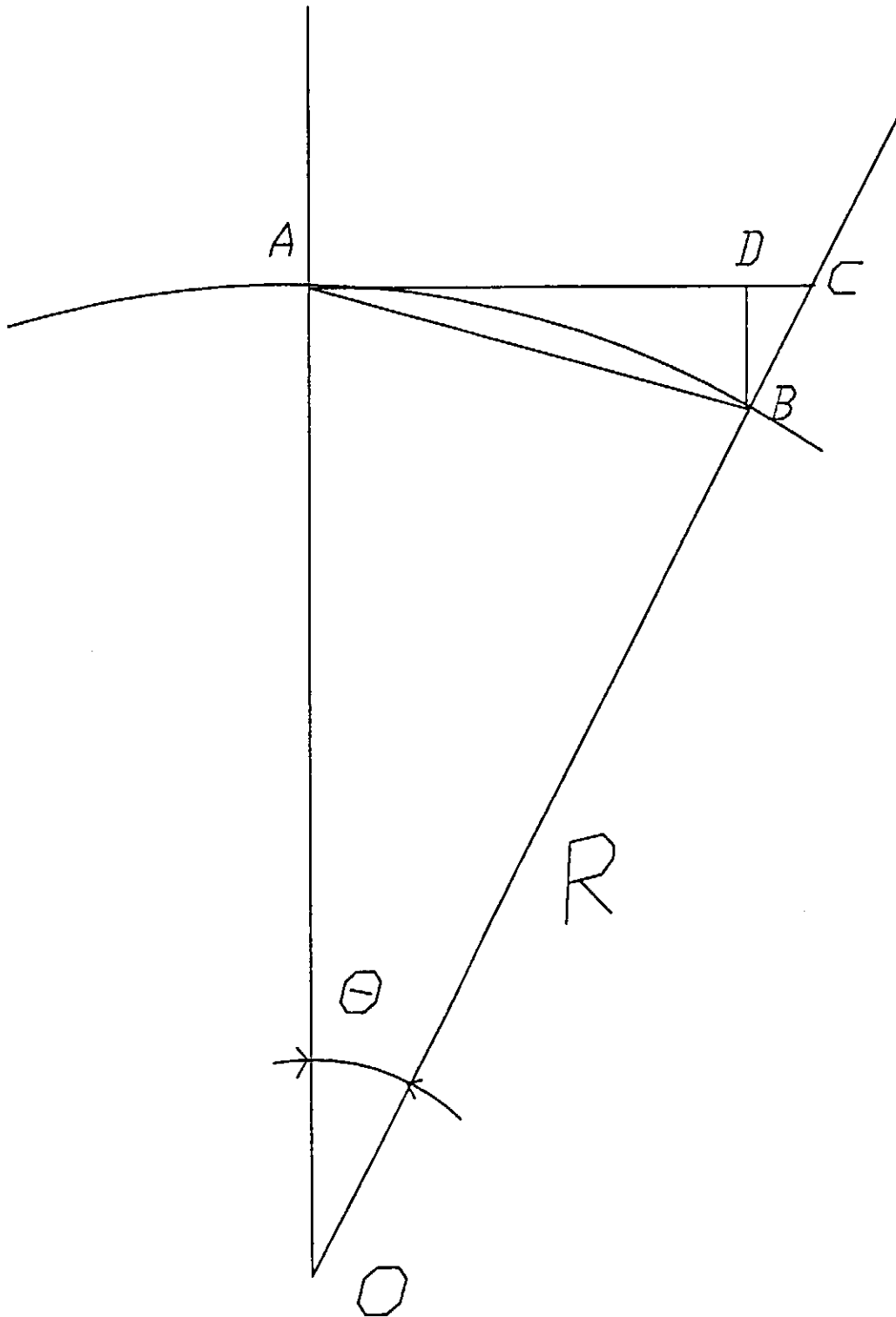[10] R.L.Warnock, P.Morton, *Particle Accelerators* **25**, 113 (1990).

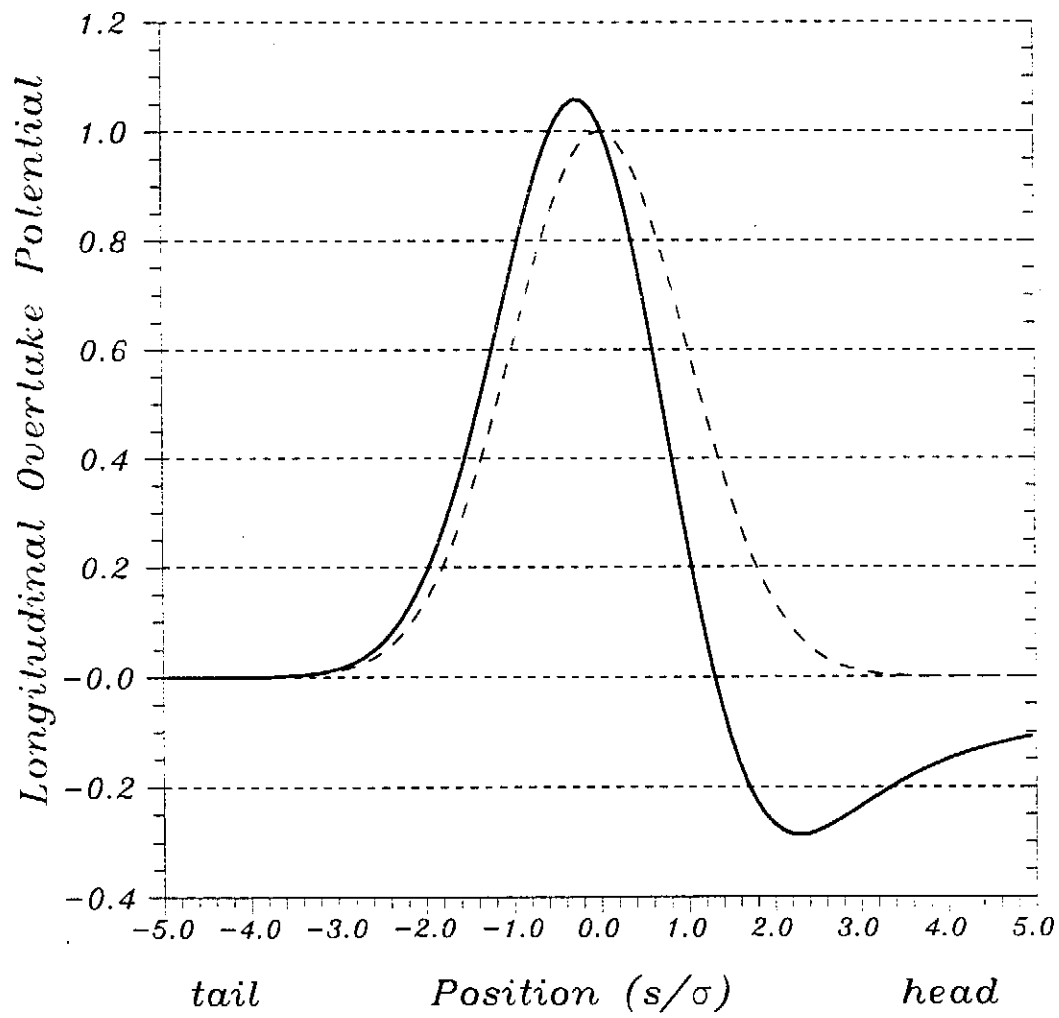Fig.1: Geometrical diagram of the retarding process.

Fig.2: Longitudinal overtake potential $F_0(s/\sigma_s)$ (solid line) for the Gaussian charge distribution (dashed line).