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TH Darmstadt

September 1999, TESLA 99-17

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contributed Paper to the
International Computational Accelerator Physics Conference,
ICAP'98,
September 14-18, 1998, Monterey, California, USA

THE SURFACE ROUGHNESS WAKEFIELD EFFECT

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Abstract

In the Linear Colliders FEL projects ultra short bunches are foreseen to be used. In addition to usual wakefields, coming from irregularities in the chamber, these bunches excite fields in transporting lines and undulators due to the surface roughness. This effect can be large for the extruded tubes, usually used in accelerators. Based on computer results it is shown, that the roughness wakefield effect can be described by a simple model for the monopole and dipole wakefields of a tube with thin dielectric coating.

1 INTRODUCTION

The surface roughness wakefield is the field, excited by a bunch traveling at the speed of light in a vacuum chamber with a rough wall surface. These wakefields might play a crucial role for the Linear Colliders and Free Electron Lasers(FEL). Since it is required, that the longitudinal and the transverse emittance is kept small, every additional contribution has to be studied carefully. The roughness depth of an extruded aluminum pipe is in the order of 0.5 μm in average or even 3 μm peak to peak. In FEL operation the bunchlength is below 25 μm. The bunch samples the surface structure of the tube. Wakefields due to the rough surface of the vacuum chamber influence the longitudinal as well as the transverse beam dynamics.

It is shown that the phase velocity of the fundamental tube mode is decreased by the disturbance of a manufacture roughness to the speed of light. Accordingly there is a synchronous wave accompanying the Bunch, which is called the rough tube mode.

2 ANALYTICAL APPROACH

To approach a description of the surface roughness effect a cylindrical tube with radius a is assumed. The boundary of this tube is disturbed by a surface structure with the depth δ (Fig. 1). For structures as shown in Fig. 1.I) and Fig. 1.II) the dispersion curve can be calculated easily.

2.1 *Dispersion of the Fundamental Rough Tube Mode*

The surface roughness of the tube decreases the phase velocity of the fundamental mode. The speed of light curve and the dispersion curve are nearly parallel for a wide range of phase advances per cell (Fig. 2). The first higher mode, with a radial dependency of 1, behaves nearly like a mode in a smooth tube. The phase velocity approaches the speed of light curve, but does not cross it. Consequently only the fundamental mode contributes to the wakefields.

* Work supported in part by DESY, Hamburg, Germany

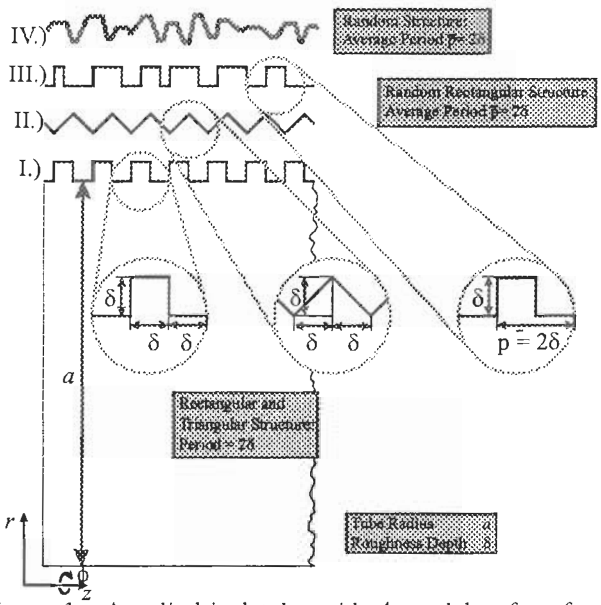

Figure 1: A cylindrical tube with 4 models of surface roughness: I.) periodically rectangular,II.) periodically triangular, III.) random with rectangular shape, IV.) random in longitudinal and radial direction. The tube radius is a, the depth of the roughness is δ, the period (I.), II.)) or average period (III.),IV.)) respectively is $2 \cdot \delta$

The diagram of phase and group velocity Fig. 3 shows, that the curve of the rough surface mode crosses the speed of light line at a specific single frequency. The group velocity does not reach this line. The relative difference between group velocity v_{gr} and speed of light c determines the length of the wake field pulse

$$\Delta t = \frac{c - v_{gr}}{c} \cdot L \qquad (1)$$

where L is the length of the vacuum chamber.

2.2 *Dielectric Layer Model*

To calculate the wakefields inside a tube the model of a wave guide covered with a thin dielectric layer is used. The applicability of this model to tubes with corrugations was demonstrated in [1]. It has been shown that this approach is extendable to the transverse wakefields created by a rough surface [2].

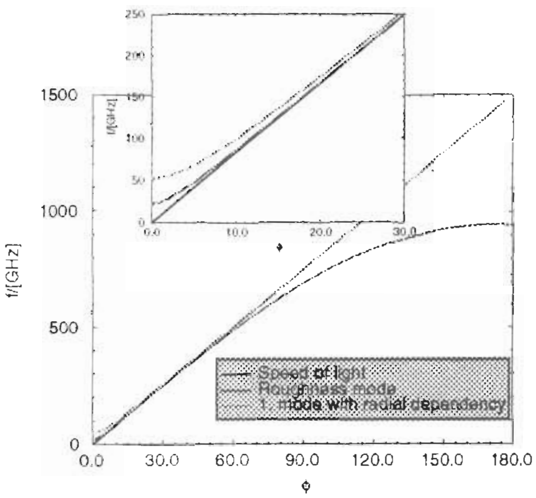

Figure 2: Dispersion diagram of the rough tube mode (structure as in Fig. 1.I). The radius of the tube is $a = 5$ mm. The period of the roughness is 100μm, the roughness depth is $\delta = 50\mu$m. The frequency is plotted against the phase advance ϕ per cell. The roughness mode curve crosses the speed of light curve at ≈ 200 GHz.

2.2.1 Monopole Case

In the monopole case ($m = 0$) the wave number of a wave guide covered with a thin dielectric layer is given by

$$k_0^2 = \frac{2\varepsilon}{(\varepsilon - 1)a\delta} \tag{2}$$

where ε is the relative permittivity, a the tube radius and δ the thickness of the dielectric layer. The longitudinal wakefunction is

$$W_0^{\|}(s) = \frac{Z_0 c}{\pi a^2}\cos(k_0 s) \tag{3}$$

using the impedance of free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$. c denotes the speed of light.

2.2.2 Dipole Case

In the dipole case the wave number is the same,

$$k_1^2 = \frac{2\varepsilon}{(\varepsilon - 1)a\delta} \tag{4}$$

as in the monopole case. The longitudinal wakefunction is given by:

$$W_1^{\|}(s) = 2\left(\frac{r_0}{a}\right)\left(\frac{r_1}{a}\right)\frac{Z_0 c}{\pi a^2}\cos(k_1 s) \tag{5}$$

where r_0 and r_1 are the offset of the driving charge and the witness respectively. Note that for $r_0 = r_1 = a$ the amplitude of the longitudinal dipole wakefield is twice as large as in the monopole case. Finally the transverse dipole wakefunction reads:

$$W_1^{\perp}(s) = 2\left(\frac{r_0}{a}\right)\frac{Z_0 c}{\pi a^3 k_1}\sin(k_1 s). \tag{6}$$

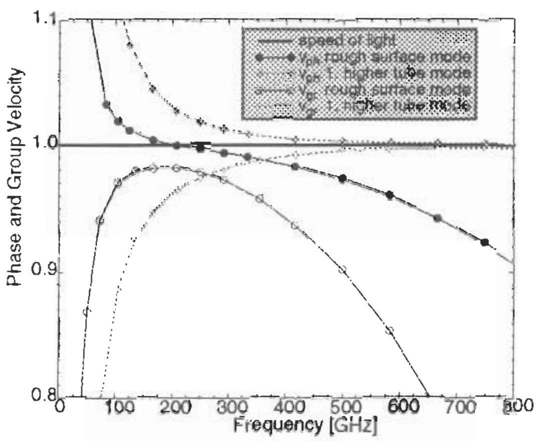

Figure 3: Dispersion diagram of the rough tube mode (structure as in Fig. 1.I). The radius of the tube is $a = 5$ mm. The period of the roughness is 100μm, the roughness depth is $\delta = 50\mu$m. The phase velocity $v_{ph} = \omega/\beta$ and the group velocity $v_{gr} = \partial\omega/\partial\beta$ are plotted against the frequency. The roughness mode curve crosses the speed of light curve at ≈ 200 GHz.

To describe the consequences of the transverse wakefields on the bunch in the transfer line the gradient $G(s)$ of the wakefunction as the quotient of the transverse dipole wakefunction and the offset to the axis is introduced:

$$G(s) = \frac{W_1^{\perp}(s)}{r_0} \tag{7}$$

Assuming that a bunch of an energy E enters the tube at an offset r_0 it will double its offset after a certain distance, to which in this paper is referred to as the instability length:

$$z_{inst} = \sqrt{\frac{E}{G(2\sigma_z)}} \tag{8}$$

2.2.3 Application of the Dielectric Layer Model to Surface Roughness

The applicability of the dielectric layer model to the longitudinal and transverse wakefields of a vacuum chamber with a rough surface modeled as shown in Fig. 1 has been demonstrated in [1, 2]. It is important to note, that neither the appearing of the rf-pulse nor its frequency depend on a strict periodicity of the structure [2].

2.2.4 Validity in Three Dimensions

There are many uncertainties in the transformation of this model to three dimensional problems, but the effective roughness depth in 3D is expected to be 3 times less than in 2D.

2.3 *Normalized Description*

The surface roughness wakefield, the loss parameter and the energy spread are given as functions of $k_0\sigma_z$. The frequency k_0 can be derived from the model above and is essentially a property of the tube and σ_z of course a bunch property.

The normalization is chosen in this way, that the maximum loss factor equals 1. The wakefunction is:

$$w_0^{||}(z) = 2\cos(k_0\sigma_z z). \quad (9)$$

The wakepotential derived from this wake is

$$W_0^{||}(z) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{z} e^{-\frac{y^2}{2}} \cos(k_0\sigma_z(z-y))dy. \quad (10)$$

Thus the normalized lossfactor is:

$$\mathrm{H}(k_0\sigma_z) = e^{-(k_0\sigma_z)^2}. \quad (11)$$

It gives the energy transported by the rf-wave traveling along the tube. The normalized energy spread is:

$$\Delta(k_0\sigma_z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W_0^{||2}(z) e^{-\frac{z^2}{2}} dz - e^{-2(k_0\sigma_z)^2} \quad (12)$$

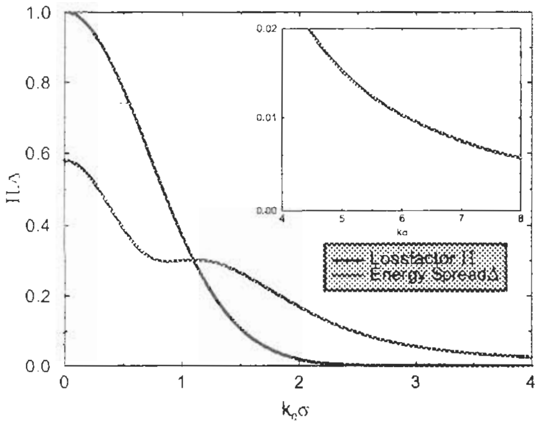

Figure 4: Loss factor H and Energyspread Δ due to the rough tube mode in normalized description.

Depending on the value of $k_0\sigma_z$ the wakefields act capacitive $k_0\sigma_z \approx 0$ or inductive $k_0\sigma_z >> 1$. Fig. 4 shows a flat top of the Energyspread in the region $0.75 < k_0\sigma_z < 1.25$. The energy spread stays the same while the loss factor, and thus the energy of the rf-pulse is decreasing.

The amplitude of the wakefield decreases in this region of $k_0\sigma_z$, It is the transition from capacitive to inductive wake characteristic. The tail of the bunch is now accelerated. Therefore the energy spread is not changing.

2.4 *Dielectric Layer Model and Normalized Description*

To calculate the lossfactor and the energyspread from the normalized description the frequency k_0 has to be determined according to eq. 2 Assuming a radius a = 2mm and a roughness depth $\delta = 100\mu$m, the permittivity of the dielectric layer is found to be around $\varepsilon \approx 1.9$ in cases Fig. 1.I.) and Fig. 1.III.) and $\varepsilon \approx 1.4$ in case Fig. 1.II.). Note that the equivalent permittivity depends on different parameters as e.g. the roughness shape.

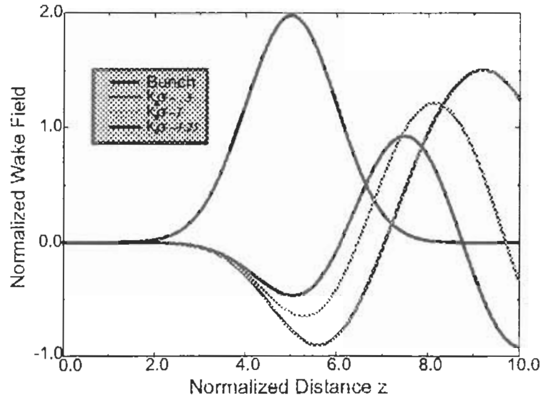

Figure 5: The wakefields in the flat top region of the normalized energy spread (compare Fig. 4).

Subsequently the value of the lossfactor and the energy spread can be calculated by

$$k_{loss} = \frac{Z_0 c}{2\pi a^2}\mathrm{H}(k_0\sigma_z) \quad (13)$$

and

$$\Delta E = \frac{Z_0 c}{2\pi a^2}\Delta(k_0\sigma_z) \quad (14)$$

respectively.

3 NUMERICAL RESULTS

As an example of the application of the dielectric layer model a tube with a radius of a =2 mm and a roughness depth of δ =20 μm, modeled as Fig.1, is taken. The bunch length $\sigma_z = 250\mu$m is more than 10 times larger than the gaps of the surface roughness.

The longitudinal wakefield resulting from the numerical calculation, is compared to the wakefield, as derived by convolution from the analytical solution 6. The 2 curves show good agreement. The relative permittivity is ε_r = 1.515. The longer the calculated tube is, the more the amplitude of the wakefield approaches the analytic curve.

The wakefield created by a $\sigma_z = 250\mu$m bunch inside a tube with a surface modeled randomly as well in longitudinal and radial direction is used as another example. The tube radius is $a = 5mm$, the mean value of the random distribution is 50μm in radial and longitudinal direction.

The field lines of the electric field (Fig. 7) derived by the numerical simulation in the time domain. show a harmonic oscillating field. The wave length of the field is much higher than the period of the surface roughness. The size of a single roughness cavity does not correspond directly to the wave length. This is in agreement to the fact, that the strict periodicity is not necessary. The random dis-

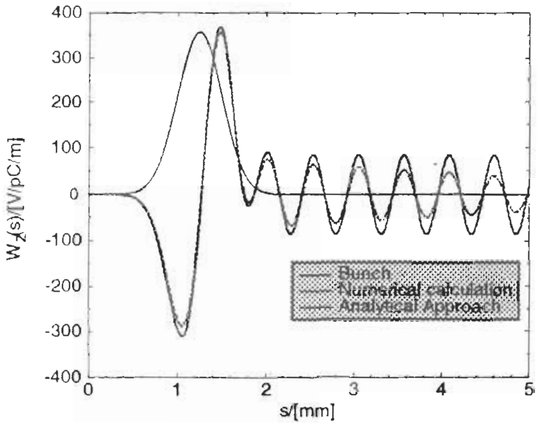

Figure 6: Comparison of the analytical approach to numerical calculations. Wakefield of a bunch with the length of $\sigma_z = 250\mu$m, carrying a charge of 1pC is passing a tube with radius $a = 2$mm. The structure of the surface is modeled as Fig. 1period of the roughness is $p = 40\mu$m, the roughness depth is $\delta = 20\mu$m.

tribution does not lead to any decoherence of the rf-pulse following the bunch.

4 APPLICATION OF THE MODEL OF SURFACE ROUGHNESS

The derived theory is now applied to some components of Linear Colliders and Free Electron Laser. As an example the numbers of three tubes are given. One with a comparatively large diameter and a very smooth surface, one with a rougher surface and a smaller diameter and finally a very narrow undulator chamber, where wakefields are apprehended in particular.

	Tube 1	Tube 2	Undulator
Tube Radius [mm]	10	50	5
Rough. Depth $\delta[\mu m]$	0.5	0.1	0.5
$\sigma_z [\mu m]$	100	10	20
Q [nC]	0.5	1	1
frep [Hz]	200	10	5
Bunch Spacing [ns]	1	100	100
Pulse Length [μs]	0.1	1000	1000

Table 1: Electron beam and geometric parameters of the examples for surface roughness wake fields. Numbers are given for a tube of 1 meter length.

Several assumptions are made to derive the numbers. The roughness depth has a rectangular shape. For the equivalent permittivity ε the value 2 is taken. To calculate the average power different repetition rates are assumed. This number as well as the bunch spacing and the pulse length affects the average power only.

The application of the surface roughness model on some

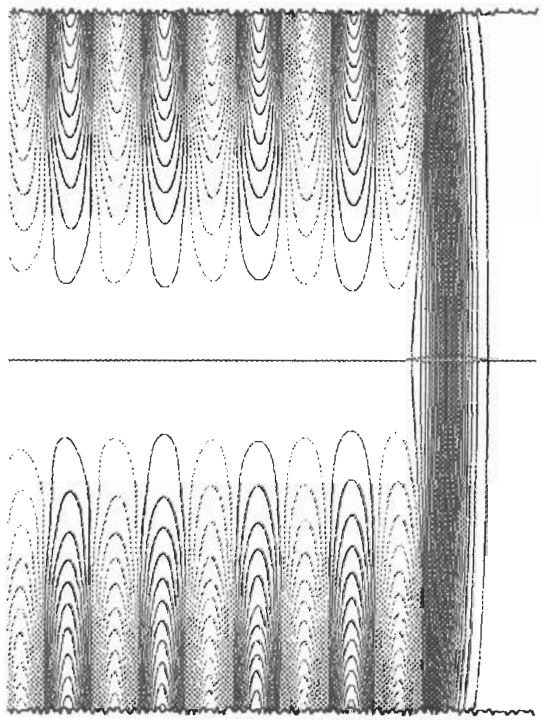

Figure 7: Field lines of the electric wakefield of a bunch with the length of $\sigma_z = 250\mu$m, carrying a charge of 1pC.The radius of the tube is 5 mm. The average period of the roughness is $\delta = 50\mu$m, the average roughness depth is $\delta = 50\mu$m.

elements which might appear in future Linear Colliders and FEL's shows, that enormous peak powers are achieved even with surfaces usually regarded as smooth (Tab. 4). The peak power stays constant as the structure length increases, because the rf-pulse length depends on the structure length too. Thus the power stored in the rf-pulse grows proportional to the length. The increase per meter is estimated by $2\delta/a$. Note that the numbers given in Tab. 4 are expected to be smaller in a three dimensional structure with an arbitrary shaped surface.

Nevertheless the effect on beam dynamics cannot be neglected.

Furthermore the utilization of this wakefields in the

	Tube 1	Tube 2	Undulator
Energyloss [kV/m]	24.3	6.6	381.3
Energyspread [kV/m]	49.8	3.5	216
$k_0\sigma$	1.41	0.28	0.8
Frequency [GHz]	675	1350	1900
Pulselength [fs/m]	333	13.3	670
Peak Power [MW]	72.9	496	570
Average Power [W/m]	0.486	0.66	19.4

Table 2: Wakefields due to the vacuum chamber roughness in several Linear Collider components. Parameters are listed in 4.

beam dynamics calculation in damping rings shows, that the saw tooth instability reported in many cases might be a consequence of the surface roughness[3].

5 RESISTIVE AND SURFACE ROUGHNESS WAKEFIELDS

The collective effect of the surface roughness and the resistive wall wakefield effect is studied. Therefore a periodic rectangular structure is chosen (Fig. 1 I.). The radius $a = 2$mm and the roughness depth $\delta = 100\mu$m stay unchanged for the comparison. For the conductivity σ several different values where assumed. The lowest conductivity $\kappa_1 = 132\ 1/\Omega$m corresponds to a skin depth of $\delta_{skin} = 100\mu$m, the same value as the roughness depth at the bunch frequency. In steps of 3 times the preceding conductivity, σ is increased, i.e. the skin depth is decreased by $\sqrt{3}$. Finally a perfect conducting material is used, as a comparison to the usual surface roughness wakefields.

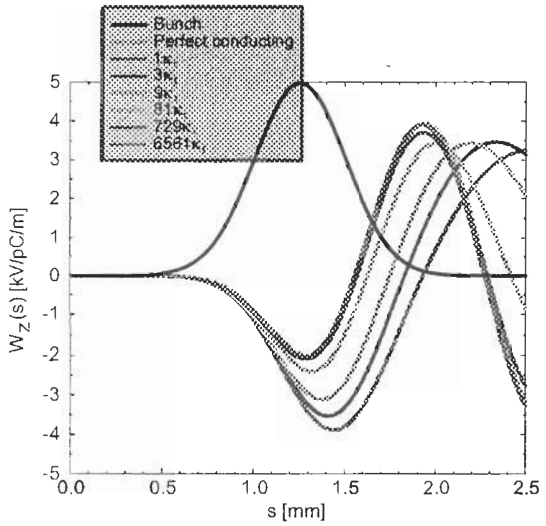

Figure 8: Wakefield of a rough resistive Tube. The tube radius is 2mm the roughness depth is 100μm. The conductivity $\kappa_1 = 1321/\Omega$m corresponds to a skin depth of δ at the bunch frequency.

Fig. 8 shows the wakefields in the bunch region. The lower the conductivity, the higher is the amplitude of the wakefield. The resistive wall and the rough surface effect fortify each other.

Regarding the wakefield in a longer range Fig. 9, the lower conductivity damps the wakefield strongly. The rf-pulse does not reach the length of the perfectly conducting case. The frequency of the pulse is lower. Compared with the description of the resistive wall wakefield in [4], the rough surface wake field is the dominating effect if

$$\delta > \frac{4}{3}\sqrt[3]{a}\left(\frac{c}{2\pi\sigma}\right)^{\frac{2}{3}}, \qquad (15)$$

where σ, the conductivity, is $5.4 \cdot 10^{17}\frac{1}{s}$, for a copper tube, $3.2 \cdot 10^{17}\frac{1}{s}$ for aluminum and $1.3 \cdot 10^{16}\frac{1}{s}$ for stainless

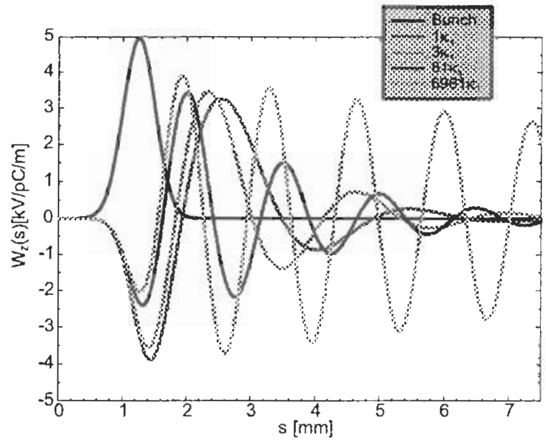

Figure 9: Wakefield of a rough resistive tube. The tube radius is 2mm, the roughness depth is 100μm. The conductivity $\kappa_1 = 1321/\Omega$m corresponds to a skin depth of δ at the bunch frequency.

steel. In case of the aluminum undulator pipe (Tab. 4) the roughness depth $\delta > 64$nm and in case of the transfer line $\delta > 138$nm. In the example above Fig. 8, 9 the transition between the regimes is $\approx 2.5\kappa_1$.

6 CONCLUSION

This approach takes into account the accelerators vacuum chamber disturbed by a rough surface. There is a rough tube mode with the phase velocity equal to the speed of light accompanying the Bunch. The bunch does not experience every single detail of the surface corrugation, but averages over the faults. The wakefields due to this mode are large. Estimations of the influences are given.

7 ACKNOWLEDGMENTS

The authors want to thank Reinhard Brinkmann, Jörg Rossbach and Holger Schlarb for their interest in this work.

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