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Production of MeV γ-rays at the TTF

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Abstract. We discuss two possible laser back-scattering arrangements for the production of MeV γ-rays at the TTF. Fluxes of $\sim 4 \times 10^7 \gamma$-rays per second of energy up to 4.8 MeV can be achieved.

Backscattering of laser photons of frequency ω from an electron beam of energy $E = \gamma m$ yields γ-rays of frequency ω_s [1]

$$\omega_s = n\omega \frac{4\gamma^2}{1 + 2\gamma^2(1 - \cos\theta) + [2n\gamma\omega/m + K^2/2](1 + \cos\theta)} \tag{1}$$

where in the above equation and what follows $\hbar$ and c have been set to 1 (except where they appear explicitly). Here θ is the forward angle of the γ-rays measured from the electron beam direction, and n the harmonic number. K is the "undulator"or "effective mass" parameter which is defined later. The term $[2n\gamma\omega/m](1 + \cos\theta)$ in the denominator of Eq. (1) reflects the energy lost by the electron in the scattering process (referred to as the "recoil term"). For an undulator the factor of 4 in the numerator of Eq. (1) is replaced by a factor of 2 because the virtual photons can transfer momentum but no energy to the electron, and the "recoil term" takes the form $[n\gamma\omega/m](1 + \cos\theta)$ because the electron loses energy only when it emits the high-energy photon.

Since the higher energy scattered photons are contained in the narrow angular range $\theta \simeq 1/\gamma$ (the so-called "forward cone") it is convenient and customary to expand the angular functions in their small argument. Furthermore, as shown below, $K \ll 1$ so

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that it can be neglected with respect to 1, and finally the recoil term is negligible for $E < 1$ GeV even when using visible light; (of course it can be ignored when using undulators since the wavelength is so much longer). With these approximations the scattered γ-ray frequency can be written as

$$\omega_s = \frac{4\gamma^2\omega}{1+\gamma^2\theta^2} \tag{2}$$

For $\gamma \sim 10^3$ and $\omega = 1$ eV, up to 4 MeV γ-rays can be produced.

The total photon yield per incident electron is given by

$$N_T/N_e = (8\pi r_o^2/3)\rho_\omega l \tag{3}$$

where $\sigma_o = (8\pi r_o^2/3) = 6.6 \times 10^{-25}$ cm^2 is the Thomson cross section, and $\rho_\omega = \epsilon_o E_{rms}^2/(2\hbar\omega)$ is the photon density; E_{rms} is the RMS value of the laser field, and l the length of the interaction region. The angular distribution can be expressed for small angles ($\theta \leq 1/\gamma$) as

$$\frac{dN}{d(\gamma^2\theta^2)} = \frac{3N_T}{2}\frac{1+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^4} \tag{4}$$

The spectral width of the γ-ray beam is determined by the angular acceptance, since angle and energy are related by Eq. (2); thus

$$\frac{\Delta\omega_s}{\omega_s} = \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\frac{2\Delta\theta}{\theta} \tag{5}$$

For small angles centered at θ=0

$$\frac{\Delta\omega_s}{\omega_s} \simeq \gamma^2\Delta\theta^2 \tag{6}$$

In contrast, the contribution of the width of the laser line or of the electron beam energy spread are generally smaller: $\Delta\omega_s/\omega_s = \Delta\omega/\omega$ and $\Delta\omega_s/\omega_s \sim 2\Delta\gamma/\gamma$. Electron beam emittance should be included in the scattering angle acceptance $\Delta\theta$.

Before discussing experimental arrangements it is interesting to show the complete equivalence between laser backscattering and undulator radiation. To that effect we can use the laser to set up a standing EM wave in a Fabry-Perot interferometer arrangement. The axis of the interferometer is inclined at a small angle α with respect to the electron beam to provide a clear path for the electrons and γ- rays

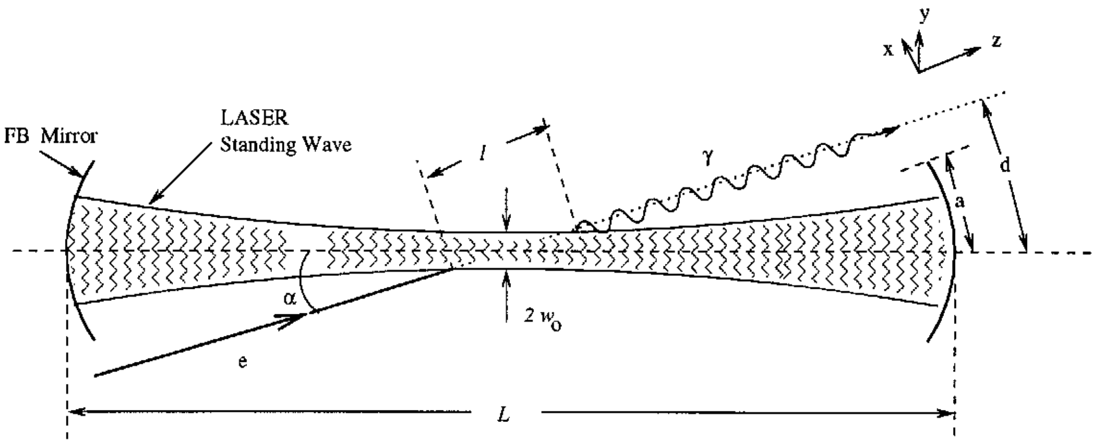

Figure 1: Schematic layout of the Fabry-Perot cavity for an optical undulator.

(see Fig. 1). For counter-propagating circularly polarized laser beams, the resulting standing waves are given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2E_o[\vec{\epsilon}_x \cos\omega t \cos kz + \vec{\epsilon}_y \cos\omega t \sin kz] \tag{7}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = 2\frac{E_o}{c}[\vec{\epsilon}_x \sin\omega t \cos kz + \vec{\epsilon}_y \sin\omega t \sin kz] \tag{8}$$

where the coordinate system is shown in Fig. 1, and the electron moves along the z-axis. These are time-dependent helical undulator fields [2], the **E** and **B** fields being 90° out of phase. Both fields exert a transverse acceleration on the electrons

$$\frac{d\beta_x}{dt} = \frac{e}{cm\gamma}[E_x - \beta_z c B_y], \qquad \frac{d\beta_y}{dt} = \frac{e}{cm\gamma}[E_y + \beta_z c B_x] \tag{9}$$

Introducing the fields from Eq. (7,8) and using $z = \beta_z ct$, $k = \omega/c$ we obtain

$$\frac{d\beta_x}{dt} = \frac{eE_o}{cm\gamma}[(1+\beta_z)\cos[\omega t(1+\beta_z)] + (1-\beta_z)\cos[\omega t(1-\beta_z)]] \tag{10}$$

$$\frac{d\beta_y}{dt} = \frac{eE_o}{cm\gamma}[(1+\beta_z)\sin[\omega t(1+\beta_z)] - (1-\beta_z)\sin[\omega t(1-\beta_z)]] \tag{11}$$

Since $(1-\beta_z) \simeq 1/2\gamma^2$ we can ignore the second term (with effective frequency $\omega_{eff} = \omega/2\gamma^2$) and also replace $(1+\beta_z) \simeq 2$. Clearly the rapidly rotating terms arise from the counter-propagating wave whereas the slowly rotating terms arise from the interaction with the co-propagating wave which gradually overtakes the electrons. Thus

$$\frac{d\beta_x}{dt} = \frac{2eE_o}{cm\gamma}\cos 2\omega t, \qquad \frac{d\beta_y}{dt} = \frac{2eE_o}{cm\gamma}\sin 2\omega t \tag{12}$$

and it also holds $d\beta_z/dt = d\gamma/dt = 0$ because the transverse fields do no work on the electron.

Eq. (12) can be integrated to obtain the electron transverse velocity and trajectory

$$\beta_x = \frac{1}{\gamma}\frac{eE_o}{m\omega c}\sin 2\omega t, \qquad \beta_y = -\frac{1}{\gamma}\frac{eE_o}{m\omega c}\cos 2\omega t \tag{13}$$

$$x = -\frac{1}{2\omega\gamma}\frac{eE_o}{m\omega c}\cos 2\omega t, \qquad y = -\frac{1}{2\omega\gamma}\frac{eE_o}{m\omega c}\sin 2\omega t \tag{14}$$

The undulator parameter is given by

$$K = \frac{eE_o}{m\omega c} = \frac{eB_o}{mc}\frac{\lambda}{2\pi} \tag{15}$$

and the standing wave pattern is equivalent to a magnetic undulator of wavelength $\lambda_u = \lambda/2$ and strength $B = 2B_o$. From Eqs. (13) we see that

$$\beta_x^2 + \beta_y^2 = \beta_\perp^2 = \frac{K^2}{\gamma^2} \tag{16}$$

and therefore

$$1 - \beta_z^2 - \beta_\perp^2 = \frac{1}{\gamma^2} = 1 - \beta_z^2 - \frac{K^2}{\gamma^2} \tag{17}$$

which leads to

$$(1 - \beta_z^2) = \frac{1 + K^2}{\gamma^2} \tag{18}$$

Namely the electron has acquired an effective mass $\bar{m}^2 = m^2(1 + K^2)$ due to its transverse motion. As shown below, in the present applications $K \ll 1$ and can be neglected with comparison to 1. For $K \ll 1$ the transverse motion of the electrons is purely sinusoidal and non- relativistic so that only the fundamental frequency and no harmonics are emitted. Note that $mc^2K^2/2$ is also referred to as the "ponderomotive potential" of the laser field, and for nonrelativistic motion represents the kinetic energy of the oscillatory motion of the electron in the limit $K \ll 1$. Note also that, as already mentioned, the electron recoil term has been neglected.

For a helical undulator the radiated energy per electron, per unit solid angle and unit frequency interval expressed in the small angle approximation is [2]

$$\begin{aligned}\frac{d^2I}{d\Omega d\omega} &= \frac{2e^2\gamma^2}{4\pi\epsilon_o c}\frac{N^2K^2}{(1 + K^2 + \gamma^2\theta^2)^2}\sum_n \left[\frac{\sin[(n - \omega/\omega_s)N\pi]}{(n - \omega/\omega_s)N\pi}\right]^2 \\ &\quad [J_{n+1}^2(z) + J_{(n-1)}^2(z) - \frac{2(1 + K^2)}{K^2}J_n^2(z)]\end{aligned} \tag{19}$$

The argument of the Bessel functions is

$$z = \frac{2K\gamma\theta}{1 + K^2 + \gamma^2\theta^2} \tag{20}$$

and N is the number of undulator periods. If we consider only the $n = 1$ term, drop K^2 with respect to 1, and integrate over $d\omega$, we find

$$\frac{d^2I}{d\Omega} = \frac{e^2\gamma^2 N\omega_s}{4\pi\epsilon_o c}\frac{K^2}{(1+\gamma^2\theta^2)^2}[J_2^2(z) + J_o^2(z) - \frac{2}{K^2}J_1^2(z)] \tag{21}$$

We convert Eq. (21) to radiated photons and expand the Bessel functions in their small argument to obtain

$$\frac{dN_\gamma}{d\Omega} = \alpha\gamma^2 NK^2\frac{1+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^4} \tag{22}$$

The total flux for $\gamma\theta \leq 1$ (in the forward cone) per electron, is

$$N_F = \alpha K^2 N\frac{\pi}{3} \tag{23}$$

Putting in the constants explicitly and rearranging, we obtain

$$\begin{aligned} N_F &= \frac{\pi}{3}\frac{e^2}{4\pi\epsilon_o\hbar c}\frac{e^2E_o^2}{m^2c^2\omega^2}N \\ &= \frac{\pi}{3}(\frac{e^2}{4\pi\epsilon_o mc^2})^2\frac{e^2E_o^2}{\hbar\omega}4\pi(N\frac{c}{2\pi\nu}) \\ &= (\frac{4\pi}{3}r_o^2)\rho_\omega l \end{aligned} \tag{24}$$

This is one half of the total flux obtained from photon scattering considerations, as given by Eq. (3) Here the photon density $\rho_\omega = 2(\epsilon_o E_o^2/2)/\hbar\omega$ because of the presence of the two traveling waves, and $l = N(\lambda/2)$. Furthermore the angular distribution for the laser electron scattering is exactly as given by Eq. (22) [1][3]. We can therefore use either representation in evaluating the γ-ray flux and it's properties.

We consider two laser arrangements: (1) A CW interferometer, which therefore produces a standing wave pattern (undulator) and (2) pulsed operation, where we can directly use the backscattering picture. In comparing the yield from the two arrangements we assume similar average laser power and similar constraints in terms of the clearance for the optics.

CW operation. Recent progress in interferometric mirrors has made possible the construction of a Fabry-Perot cavity 1.75 m long at $\lambda = 1054$ nm and having reached $Q = 5 \times 10^{11}$ [4]. The 10 mW laser was locked onto the cavity by the Pound-Drever technique [5] and remained locked for over 30 minute periods. The energy stored in a cavity is

$$U = \frac{PQ}{\omega} \tag{25}$$

and the energy density at the waist $u = (PQ/\omega)/V$ where $V = \pi w_o^2 L$ is the effective volume of the laser mode; L is the length of the cavity and w_o the waist. The magnetic field in each wave train of the interferometer is

$$B_o = \sqrt{2\mu_o (PQ/2\omega V)} \tag{26}$$

and the undulator parameter [see Eq. (15)]

$$K = \frac{eB_o}{mc}\frac{\lambda}{2\pi} = 94B[T]\lambda[m] \tag{27}$$

If we use $P = 2$ W, $Q = 5 \times 10^{11}$, $L = 2$ m, $w_o = 0.5$ mm, we find $B_o = 2.1 \times 10^{-2}$ T and $K = 2.1 \times 10^{-6}$, justifying the use of the linear approximation.

We can optimize the size of the waist and interaction length to maximize the γ-ray flux. As shown in the sketch in Fig. 1 the crossing angle is α, sin$\alpha = 2d/L$ where d is the required clearance from the mirror center to the beamline; thus the interaction length l and the number of "undulator" periods N are

$$l = (w_o/d)L \qquad\qquad N = l/(\lambda/2) = \frac{2w_o}{d}\frac{L}{\lambda} \tag{28}$$

respectively. The γ-ray flux per electron is given by [see Eq. (23)]

$$\frac{N_\gamma}{N_e} = \frac{\pi\alpha}{3}K^2 N = [\frac{4\pi}{3}(\frac{e^2}{4\pi\epsilon_o mc^2})^2][\frac{PQ}{\omega}\frac{1}{\hbar\omega}]\frac{l}{V} \tag{29}$$

where $l/V = 1/(\pi w_o d)$. However w_o and d are not independent: the waist is given by $w_o \sim \lambda f^*$ and $f^* = f/D = 2f/a$ where a is the mirror radius. For a confocal cavity $L = 2f$, so that $w_o = \lambda L/a$. The clearance distance d should be $2a$, and thus $w_o d = 2\lambda L$; hence we can write $l/V = 1/(2\pi\lambda L)$ leading to

$$\frac{N_\gamma}{N_e} = [\frac{4\pi}{3}(\frac{e^2}{4\pi\epsilon_o mc^2})^2][\frac{PQ}{\omega}\frac{1}{\hbar\omega}]\frac{1}{2\pi\lambda L} \tag{30}$$

For the conditions stated in the previous paragraph one obtains

$$N_\gamma/N_e = 8 \times 10^{-9} \tag{31}$$

and since we expect 7.2×10^4 pulses/s of 1 nC each, namely 4.5×10^{14} electrons/s, the total γ-ray flux in the forward cone is

$$F_\gamma \simeq 3.6 \times 10^6/s \tag{32}$$

Pulsed operation. The energy per pulse will be $U = P/f$ where f is the pulse frequency and the duration of the pulse is Δt with a focal area $A = \pi w_o^2$. Then the photon density is

$$\rho_\omega = \frac{U}{\hbar\omega}\frac{1}{\pi w_o^2 c\Delta t} \tag{33}$$

while the interaction length is the smaller of $[Lw_o/d; c\Delta t/2]$. The waist of the laser beam must be larger than the electron beam size, $\sigma \sim 50\mu$m which matches $c\Delta t/2$ if $\Delta t = 7$ ps. We therefore set $w_o = 2\sigma$ to obtain for the γ-ray flux per electron in the forward cone

$$\frac{N_\gamma}{N_e} = (\frac{4\pi}{3}r_o^2)\frac{P/f}{\hbar\omega}\frac{1}{8\pi\sigma^2} = 8 \times 10^{-8} \tag{34}$$

leading to a total flux

$$F_\gamma = 3.6 \times 10^7/s \tag{35}$$

which is larger by a factor of 10 from the flux expected in the CW case.

Both arrangements are challenging: In the CW case, establishing the high Q-value at a power level of 2 W is demanding. In the pulsed operation mode, maintaining good overlap between the 7 ps laser pulses and the subpicosecond electron bunches is a difficult task; in this latter case the availability of the injection laser may be a special advantage.

Some properties of the γ-rays. The flux spectrum of the γ-rays produced at TTF is shown in Fig. 2. It is obtained by integrating the spectral flux angular density Eq. (19) over the angular range corresponding to the given bandwidth at a given energy. Only the fundamental is visible since the harmonics are suppressed by large factors as mentioned before. As indicated in Eq. (4) the spectral width is determined by the angular acceptance and is given by $\Delta\omega/\omega \sim \gamma^2\theta^2$ so that in general the flux in bandwidth $\Delta\omega$ in the forward direction is given by

$$dF = F_\gamma \frac{\Delta\omega}{\omega} \tag{36}$$

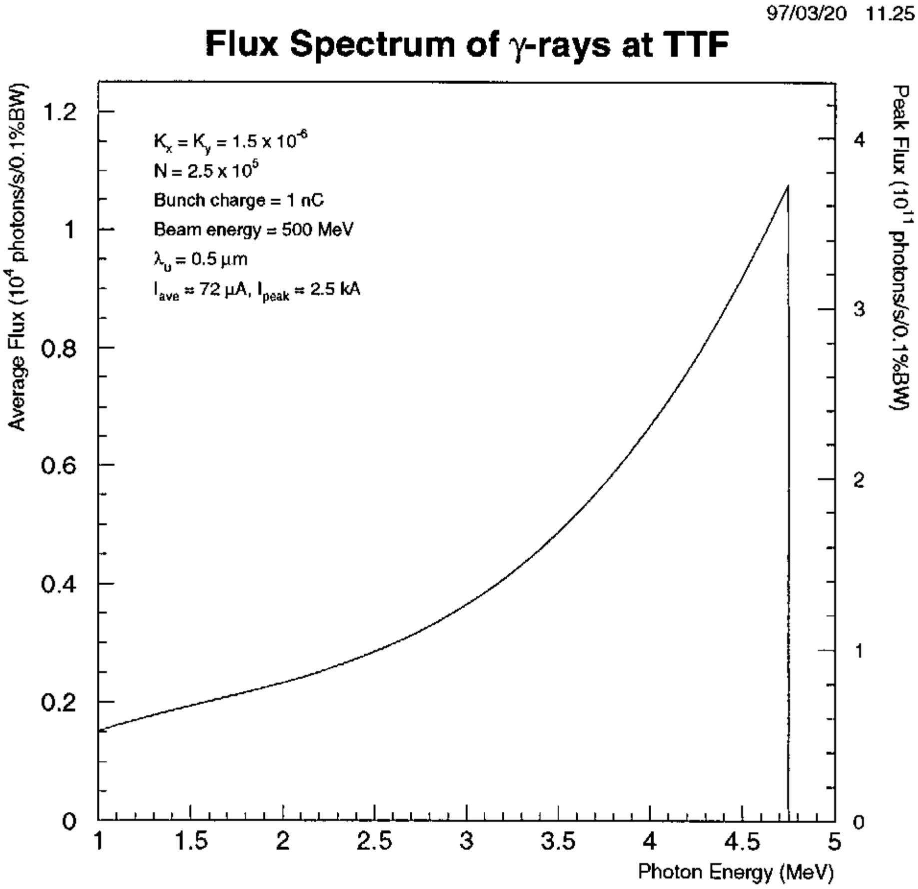

Figure 2: Flux spectrum of γ-rays at produced with an "optical undulator" at the TTF. The time-averaged as well as peak flux values are shown.

The peak intensity is obtained by multiplying the flux/s by $1/f\Delta t$, namely by a factor 3.5×10^7. Although existing monochromators do not operate in this photon energy range, selection of a narrow bandwidth can be achieved by limiting the angular acceptance. For example, the angle at which the photon energy is 1% below the peak of the fundamental is at $\theta = 103\ \mu$rad according to Eq. (2). This is large compared to the electron beam divergence of 26 μrad. The required detector would have an acceptance area of 1 mm $\times$ 1 mm at a distance of 10 m from the source. The time structure of the γ-ray is given by the properties of the electron beam, which is bunched in short bursts of 160 fs duration.

References

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