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NOTE ON THE SHORT TIME BEHAVIOR OF GROUND MOTION

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Abstract

In this paper we study one aspect of ground motion namely the connection between the high frequency behavior of the spectrum of relative motions and the behavior of the rms relative displacement at small time. We show, for example, that if the spectrum drops fast enough with increasing frequency, then the rms displacement is proportional to time. This conclusion can be used to compare fast vibration tolerances of linear colliders in relation with their repetition rate.

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1 Introduction

Ground motion is one source of loss of luminosity in the TeV scale linear colliders [1]. For example, transverse misalignment of several final quadrupoles, caused by ground motions, results in offset or spot size growth of the e^+ and e^- bunches at the interaction point (IP). Tolerances on the misalignments are so severe that the luminosity can only be kept if an active continuous realignment of the elements of the collider is used. Beam based alignment is one powerful method for doing so. It relies on measuring the beam positions through some elements precisely enough to determine and correct their misalignments. Thus an important parameter for a beam based alignment system is the beam repetition rate f_{rep}: a collider with a higher repetition rate has the advantage that it can be realigned more frequently. Therefore the ground motions that spoil alignment should also be considered in relation with the repetition rate. In this paper we are particularly interested in knowing the time behavior of relative displacement of elements of collider for short time scale [1]. In this paper we will not use any measured data on ground motion spectra but, by making only general assumptions about the large frequency behavior of the spectrum we will derive the functional dependence of displacements versus time for small time scale. The results obtained can be interesting for example to compare vibration tolerances of different linear colliders projects with their repetition frequency properly taken into account [2].

2 Short time displacement from the power spectrum at high frequency

Let us suppose for simplicity that there is only one element in each e^+ and e^- part of linear collider. Let them be separated by the distance L. A relative transverse displacement $x(t)$ of these elements will result in an offset of the beams at the IP. Supposing that at each time τ these elements are perfectly

[1]It is known that for long time scale, relative displacements obey the "ATL law" [3] of diffusional ground motion: thus square of displacement is proportional to time. However in the case of the considered short time intervals ($1/f_{rep}$ can be of the order of 1 ms) the "ATL law" largely overestimates the amplitude of ground motion and cannot be applied.

aligned, one can write [2] for the rms value of the relative displacement after a time t :

$$\sigma_x^2(t) = \langle (x(t+\tau) - x(\tau))^2 \rangle_\tau = 2 \int_0^\infty p(\omega, L)\, 2[1 - \cos(\omega t)]\, d\omega / 2\pi \qquad (1)$$

where $p(\omega, L)$ is the power spectrum of relative displacements $x(t)$ measured in [m^2/Hz] if the displacements are measured in meters. We put a factor of two before the integral because the power spectrum is defined from $-\infty$ to $+\infty$ and is symmetrical relative to $\omega = 0$.

It is known from measurements that this spectrum decreases with increasing of frequency. Also at any finite t the formula (1) should give a finite answer for the displacement: it is easy to show that this implies that the power spectrum should be less singular than $1/\omega^3$ for $\omega \to 0$ and decrease faster than $1/\omega$ for $\omega \to \infty$. We suppose in this paper that these conditions are satisfied [3].

Let us now consider formula (1) for small times $t \to 0$. We asssume that for large frequencies $\omega > \omega_0$ the power spectrum behaves as

$$p(\omega, L) = C/\omega^\alpha. \qquad (2)$$

Let us split the integral in (1) in two parts, I_1 from 0 to ω_0 and I_2 from ω_0 to ∞. For $t \to 0$ the behavior of I_1 is easily shown to be:

$$I_1 \sim \frac{t^2}{\pi} \int_0^{\omega_0} p(\omega)\, \omega^2\, d\omega \;\propto t^2 \qquad (3)$$

By a change of integration variable the second integral I_2 can be written:

$$I_2 = \frac{2C}{\pi} t^{\alpha-1} \int_{\omega_0 t}^\infty \frac{1 - \cos(u)}{u^\alpha}\, du \qquad (4)$$

For $\alpha < 3$ the integral in the above expression has a finite limit when $t \to 0$ so that

$$I_2 = \frac{2C}{\pi} t^{\alpha-1} \int_0^\infty \frac{1 - \cos(u)}{u^\alpha}\, du \;\propto t^{\alpha-1}\ , \alpha < 3 \qquad (5)$$

[2]This formula is obtained using the theorem that autocorrelation $\langle (x(t+\tau)\, x(\tau)) \rangle$ and power spectrum $p(\omega)$ are connected via Fourier transformation [4].

[3]In the limit $t \to \infty$ equation (1) should give an infinite result. Thus the power spectrum should increase faster $1/\omega$ for $\omega \to 0$.

On the contrary, for $\alpha \geq 3$ the integral is diverging when $t \to 0$. It behaves like $\log(\omega_0 t)$ for $\alpha = 3$ and $(\omega_0 t)^{3-\alpha}$ for $\alpha > 3$, in such a way that

$$I_2 \propto \begin{cases} t^2 \log(1/t) & \text{if } \alpha = 3 \\ t^2 & \text{if } \alpha > 3 \end{cases} \tag{6}$$

Collecting the asymptotic behaviors of both I_1 and I_2 leads to the following behavior of the rms relative displacement square, in the short time limit:

$$\sigma_x^2(t) \propto \begin{cases} t^{\alpha-1} & \text{if } 1 < \alpha < 3 \\ t^2 \log(1/t) & \text{if } \alpha = 3 \\ t^2 & \text{if } 3 < \alpha \end{cases} \quad \text{, for } t \to 0. \tag{7}$$

Notice that for $\alpha > 3$ both I_1 and I_2 contribute to the displacement, while for $\alpha \leq 3$ only the high frequency part I_2 dominates.

Measurements of ground motions performed in several quiet places [5] showed that the considered spectrum behaves approximately as $1/\omega^4$. In this case expected value of displacement will be proportional to time. Therefore the figure of merit which allows to compare different linear collider designs should be the value of vibrational tolerances multiplied by the repetition frequency.

3 Conclusion

Behavior of the relative rms displacement of elements of a linear collider at small time can be evaluated from the behavior of the spectrum at high frequencies. For the spectrum that drops fast enough with increasing of frequency the displacement is proportional to time.

References

[1] See for example Proceedings of the International Workshops on Linear Collider LC89 - LC95.

[2] Linear Collider Technical Review. To be published.

[3] Baklakov B.A., Lebedev P.K., Parkhomchuk V.V., Sery A.A., Shiltsev V.D., Sleptsov A.I.,*Investigation of Seismic Vibrations and Relative Displacement of Linear Collider VLEPP Elements*, Proc. of 1991 IEEE Part. Accel. Conf., San Francisco, USA, pp.3273-3275, May 1991.

[4] A. Sery, O. Napoly, *Influence of ground motion on the time evolution of beams in linear colliders*, DAPNIA/SEA 95 04

[5] See for example Juravlev V.M., Sery A.A., Sleptsov A.I., Coosemans W., Ramseier G., Wilson I. ,*Investigation of Power and Spatial Characteristics of Seismic Vibrations in the CERN LEP Tunnel for Linear Collider Studies*, CERN SL/93-53, CLIC-Note 217.