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5 lectures on Coherent Electromagnetic Radiation of Relativistic Electron Beams

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Lecture 1

Radiation of a Single Relativistic Particle

Certain definitions. Mentioning a particle of a beam we will bear in mind a particle with charge q and total enery γ (in mc^2 units) moving mainly in z-direction with an average longitudinal velocity $c\beta(\gamma)$. It may also oscillate transversally (along $\vec{r}_\perp$) and/or longitudinally with some frequency $\Omega(\gamma)$ (in the lab frame). Physical factors causing the oscillations are not of interest at the moment, i.e., the motion is supposed fixed and mainly z-directed. Tr.1

By an electromagnetic wave (as well as by a wave of density, velocity, etc.) we will understand a perturbation of the corresponding field depending on z and time as $\exp[i(kz-\omega t)]$ with an arbitrary dependence on transverse coordinates $\vec{r}_\perp$. The characteristics $k_0 = \omega/c$ and k are supposed correlated and satisfying a dispersion relation $k_0(k)$ determined by the electrodynamic structure where the propagation takes place. In the simplest case of free waves one can speak, for example, about plane waves $\propto \exp[i(\vec{k}\vec{r} - \omega t)t]$ with definite wave vector $\vec{k}$ propagating at definite angle θ with respect to the z-axis. Then $k = |\vec{k}|\cos\theta$ and the dispersion relation takes the form $k_0 = |\vec{k}|$ as far as the speed of light does not depend on its frequency. However, the wave phase velocity along z is $\beta_p = k_0/k = 1/\cos\theta$ and exceeds unity. Wave field vectors are normal to $\vec{k}$ and may have z-components.

In a regular waveguide of an arbitrary cross section electromagnetic waves belong either to TM or to TE class and obey a hyperbolic dispersion relation

$$k_0^2 = k^2 + k_{co}^2$$

where the product ck_{co} is a cut-off frequency. Their phase velocity exceeds unity but can be decreased $\sqrt{\varepsilon}$ times if propagation takes place in a medium with a dielectric constant ε.

Another method used in practice to retard electromagnetic waves consists of emploiment of periodic structures – iris-loaded and spiral waveguides, gratings, etc. Dispersion relation of these systems reveals frequency bands of transparency separated by bands of total internal reflection. Generally, electromagnetic fields have all six components.

The electromagnetic field of an arbitrary moving particle as a function of the vector Tr.2
$\vec{R}$ directed from the charge to the observation point can be found in every textbook on electrodynamics. The second term behaving as $1/R$ for $R \to \infty$ is identified usually as a radiation field as far as it provides a finite flux of outgoing electromagnetic power. However, this definition is not adequate in many practical cases. First of all it assumes free space propagation and does not take into account possible metallic and/or dielectric surfaces in the vicinity of the charge. For example, if a source of radiation is placed inside an ideal waveguide the radiation field (if it exists!) is concentrated inside the channel and does not decrease with distance at all. Secondly, all values in rhs of the formula are to be calculated at a previous moment of time which is a solution of a rather complicated equation. At last, ignoring the first time-dependent term one looses information about the field in the vicinity of the source which can be very essential in a case of many interacting particles.

For these reasons it is more convenient to define the radiation field as a set of eigenmodes which can propagate in the system in the absence of a source. A question arises

DEFINITIONS

TR. 1

PARTICLE:

(MOVING OSC-R)

POSITIVE CHARGE

ENERGY γ

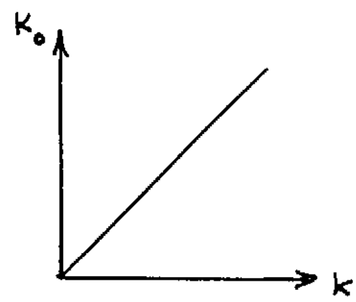
FREQ. $\Omega(\gamma)$ LAB. FRAME

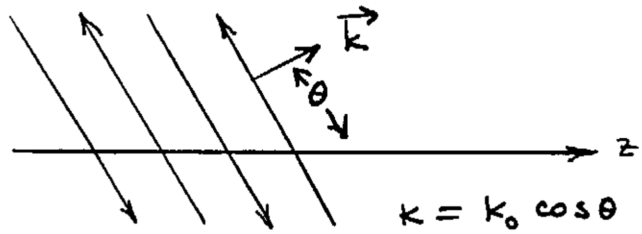
AVERAGE LONG. VELOCITY $\beta(\gamma)$

WAVE:

SOMETHING BEHAVING AS $\exp i(kz-\omega t)$

o PLANE VACUUM WAVE

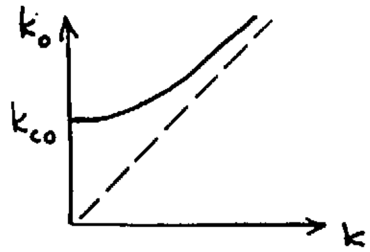


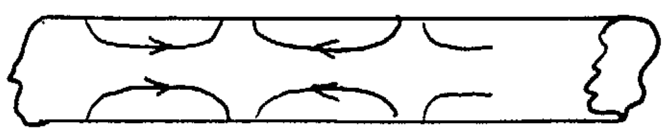


$k = k_0 \cos\theta$

$$\beta_p = \frac{k_0}{k} = \frac{1}{\cos\theta} > 1$$

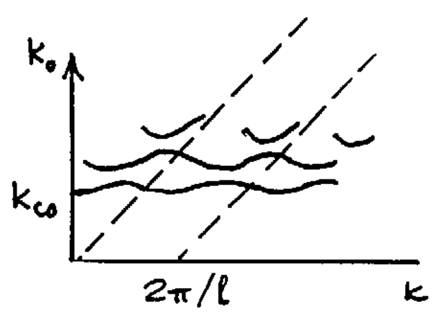
o REGULAR WAVEGUIDE

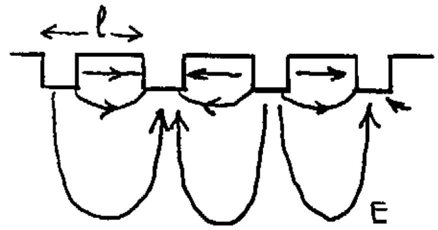




$$\epsilon\beta_p^2 = 1 + \frac{k_{co}^2}{k^2}$$

PERIODIC STRUCTURE





Tr. 2

DEFINITIONS

RADIATION FIELD

USUAL REPRESENTATION:

$$\vec{E} = \frac{q}{(R - \vec{R}\vec{v})^3}\left\{(1-v^2)(\vec{R} - R\vec{v}) + \left[\vec{R} \times \left[(\vec{R} - R\vec{v}) \times \dot{\vec{v}}\right]\right]\right\}$$

$$t' + R(t') = t$$

WILL NOT BE USED:

- GOOD ONLY IN VACUUM (NO SURROUNDINGS)
- — " — FOR SIMPLE TRAJECTORIES
- DOES NOT DEFINE RAD. FIELD IN BEAM VICINITY

INSTEAD:

- DIRECT SOLUTION OF MAXWELL EQS WITH BOUNDARY COND-S
- RADIATION FIELD AS A SUPERPOSITION OF "FLYING" EIGENMODES ($\vec{E} = ik_0 \,\mathrm{rot}\, \vec{B}$; $\vec{B} = -ik_0 \,\mathrm{rot}\, \vec{E}$)
- EXTERNAL FIELD DEFINED AT $z \to -\infty$ AND/OR $t \to -\infty$
- EXCITATION AND ABSORBTION OF A MODE ARE TWO SIDES OF A COMMON PROCESS AND ARE DETERMINED BY PARTICLE PHASING IN THE WAVE.

then how to distinguish the "proper" radiation of the source from external radiation fields. Hence, additional boundary and initial conditions are to be taken into account. However, this problem is of importance mainly for calculations of radiation energy losses of a single particle. For what follows more essential is the interaction of particles with collective radiation fields which can be considered, in a way, as external ones. Moreover, a general consideration of the particle-wave interaction with phase relations taken into account reveals a certain symmetry of absorbtion and emission processes which plays an essential role in explaining of so called stimulated emission of radiation.

Tr.3

Wave-particle synchronism. If the energy of the wave quantum $\hbar\omega$ is considerably less than the particle energy γmc^2 and the amount of transfered energy is large, then the number of quanta emitted during the interaction period must be large. Consequently, the interaction should endure many periods of the wave, the average work of the field over a particle not being zero. The latter means that a phase position of the particle is almost constant. It is easy to see that the same considerations refer also to the field of an oscillator whose center moves along a straight line if such a motion occurs for a sufficiently long time. In this case the relative phase $\varphi = kz - k_0ct \pm \Omega t$ should be kept approximately constant (within the limits of $\pm\pi$) along the interaction distance.

Cherenkov radiation. For a free particle the synchronism condition is identical to the condition of Cherenkov radiation for which the equality between the wave phase velocity and a particle velocity takes place. For plane waves it can happen for $1 < \sqrt{\varepsilon}\beta$. The wave propagates then at the angle $\theta = \arccos(\beta\sqrt{\varepsilon})^{-1}$. In a case of a dispersive medium ($\varepsilon = \varepsilon(\omega)$) the radiation exists at different angles and have different colours. However, one should stress that from the viewpoint of the questions addressed below it is not the most typical case. As a rule, the beam moves in a vacuum channel surrounded by metallic and/or dielectric walls building up a certain dispersion of waves, i.e., a specific dependence $k_0(k)$. If there are slow waves among them, whose phase velocity in the direction of a particle motion coincides with particle velocity for a sufficiently long time then just those waves will be emitted. Then one should regard the synchronism relation as an equation for determining their frequency and the type of the radiated wave will depend on the type of the system.

Sometimes it is also useful to regard the action of an external accelerating field on a particle from the same viewpoint. The electrodynamic structure of a linear accelerator (an iris-loaded waveguide, a set of cavities etc.) is designed in such a way as to enable the accelerating wave to propagate with a fixed phase velocity being below or equal the velocity of light. Then the process of systematic acceleration appears as a reverse process to the Cherenkov radiation emission. The radiation field is subtracted coherently (i.e., phase taken into account) from the external field and appears as a load of the accelerating system with the accelerated current.

One can conclude that in regular vacuum waveguides Cherenkov radiation is impossible while any retarding system with slow eigenmodes provides radiation at a particular frequency marked by a circle in the figure. Tr.4

Doppler-shifted radiation. For $k_0 = \beta k \pm \Omega/c$ the synchronism condition gives the Doppler's formula, i.e., the frequency of the wave radiated (or absorbed) by an oscillator

Tr. 3

WAVE-PARTICLE SYNCHRONISM

CONDITIONS OF INTEREST:

- $\Delta\gamma < \gamma$ (FIXED MOTION)
- $\hbar\omega << mc^2\Delta\gamma$ (CLASS. SYSTEM)
- LARGE NUMBER OF EMITTED/ABSORBED PHOTONS
  $t >> 1/\omega(1-\beta)$

$\downarrow$

- LONG-TIME PARTICLE-WAVE ENERGY EXCHANGE

$\downarrow$

- CONSTANT (SLOWLY VAR.) PHASE $\varphi = kz(t) - k_0 t \pm \Omega t$

$$\boxed{k_0 - k\beta \mp \Omega = 0}$$ (WITHIN $\pm\pi/L$)

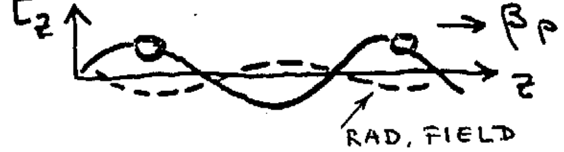
FOR A FREE PARTICLE $k_0 = k\beta$ OR $\beta = \beta_p(k)$

- PLANE WAVES $k \to k_0 \cos\theta / \beta_p$

$$\cos\theta(k_0) = \frac{\beta_p(k)}{\beta}$$ CHERENKOV ANGLE

- THE ONLY COND-N OF CHERENKOV EMISS/ABS:
  SLOW EIGENMODES WITH LONG. EL. FIELD.

SLOW-WAVE SYSTEMS:

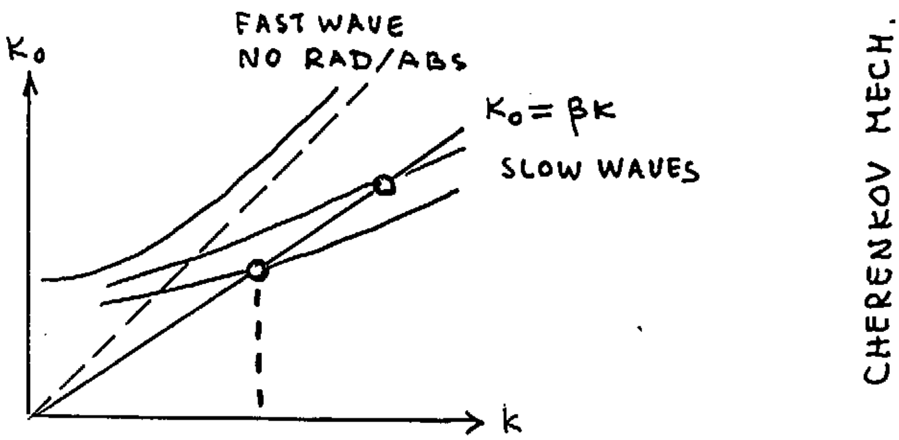


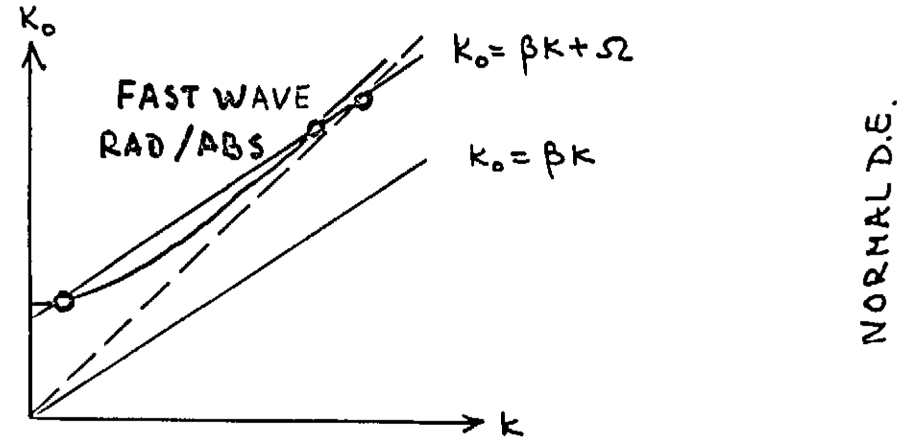
LINAC; RAD. FIELD SUBTRACTED FROM EXT. ONE (BEAM LOADING)

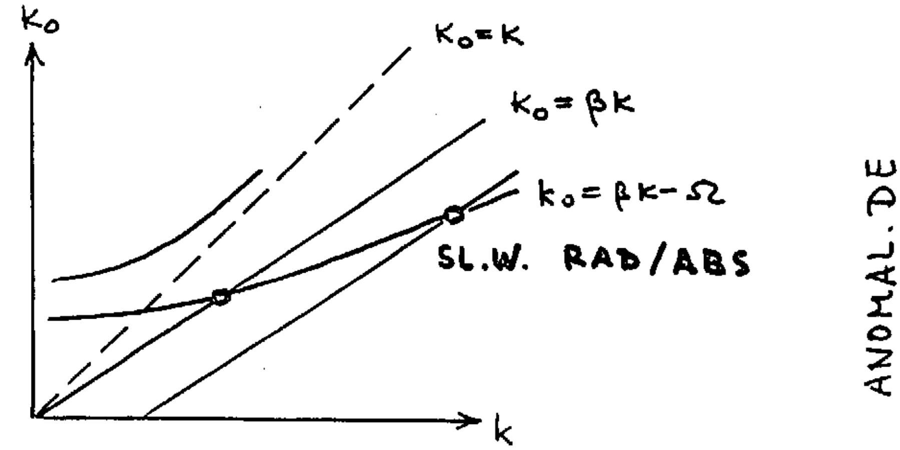
TWT; RAD. FIELD ADDED (AMPLIFIER)

PHASING TO BE PROVIDED!

TR. 4


FAST WAVE
NO RAD/ABS
K_0
$K_0 = \beta K$
SLOW WAVES
k
CHERENKOV MECH.


K_0
FAST WAVE
RAD/ABS
$K_0 = \beta K + \Omega$
$K_0 = \beta K$
k
NORMAL D.E.


K_0
$K_0 = K$
$K_0 = \beta K$
$K_0 = \beta K - \Omega$
SL.W. RAD/ABS
k
ANOMAL. DE

posessing the frequency Ω in the laboratory frame (not to be confused with the frequency in the rest frame!). For plus sign (normal Doppler effect) a particle interacts with the fast wave that overcomes it by one wavelength during one period of the oscillator and for minus sign (anomalous Doppler effect) a particle interacts with the slow one lagging behind it by one wavelength during one period. Notice that an oscillator moving along a straight line may interact with free vacuum plane waves due to normal Doppler effect if its oscillations are transverse, that is directed along the electric field. The synchronism condition for a flying oscillator can be met even in a fast-wave (regular) waveguide. Both absorbtion and emission are possible depending, of course, on oscillator phasing.

Radiation of an oscillator moving with a relativistic velocity $\beta \approx 1$ has several interest- Tr.5
ing and important features. The oscillator optical activity reveals itself at the frequency

$$ck_0 = \frac{\Omega}{1 - \beta\cos\theta}.$$

For small angles and $\beta \to 1$

$$ck_0 = \frac{2\Omega\gamma^2}{1 + \theta^2\gamma^2} \qquad \text{where} \qquad \gamma^2 = (1-\beta^2)^{-1}.$$

So the oscillator radiates at a very high frequency $\approx 2\Omega\gamma^2$ within a small solid angle $\approx \gamma^{-2}$ along the z-direction and at a comparatively low frequency $\approx \Omega$ outside of the angle. As far as numbers of forward and back directed photons in the rest frame are equal there are equal numbers of photons in the lab frame inside and outside of the cone. One can conclude that the radiation power is sharply directed forward where the hard photons are emitted to, and that the total power is about γ^2 times larger than for an oscillator of the same frequency and of the same amplitude being in rest.

By the way: a low frequency (below a cut-off) oscillator can radiate in a waveguide if moving with a sufficient velocity.

Another unexpected and even shocking feature of particle radiation under conditions Tr.6
of anomalous Doppler effect follows from simple considerations based on conservation laws. If Γ is an internal energy of the oscillator and $\vec{P}$ is its total momentum one can write down the usual relativistic kinematic relation

$$\gamma^2 = \Gamma^2 + P^2; \qquad \vec{\beta} = \vec{P}/\gamma\,.$$

Taking into account that a single emitted photon carries out $\hbar k_0/mc$ of the total energy and $\hbar\vec{k}/mc$ of the total momentum one gets for a change in the internal energy

$$\Delta\Gamma = \frac{\hbar\gamma k}{mc}\,(\beta\cos\theta - \beta_p)$$

Thus, a slow photon emitted within the Cherenkov angle increases (!) the internal energy of the oscillator (of course for account of its directed motion). So, slow waves (if capable to propagate in the system) can be emitted by a zero amplitude oscillator which gains amplitude as a result of the emission. One may expect that such slow waves will be self-exited, i.e., will grow up spontaneously starting from noise fluctuations.

TR. 5

FLYING OSCILLATOR:

PLANE WAVES $k = k_0 \cos\theta / \beta_p$

$$\boxed{k_0 = \left[\frac{\pm \Omega / c}{1 - \frac{\beta}{\beta_p} \cos\theta} \right]}$$

NORMAL $(\beta_p > \beta)$

DOPPLER EF

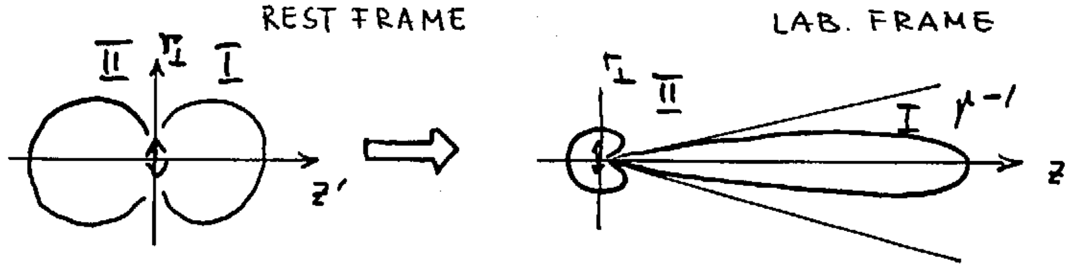
ANOMALOUS $(\beta_p < \beta)$

WAVE SLIPS FORWARD (BACK) – ONE WAVELENGTH / PERIOD

NORMAL DE: $\beta \to 1$ $\theta \to 0$

$$k_0 \approx \frac{2\Omega\gamma^2/c}{1 + \theta^2\gamma^2}$$

- SMALL ANGLES $(<\gamma^{-1}) \to$ HF $(2\Omega\gamma^2)$ RAD/ABS
- LARGE $(>\gamma^{-1}) \to$ LF $(\sim\Omega)$ RAD/ABS
- RAD-N CONC NTRATED IN A SOLID ANGLE OF γ^{-2}



EQUAL NUMBER OF PHOTONS IN I (HARD IN LAB FRAM
AND II (SOFT IN LAB FRAME

- TOTAL POWER γ^2 TIMES INCREASED

- RADIATION IN A WAVEGUIDE IS POSSIBLE

EVEN FOR $\Omega < k_{co} c$

TR. 6

FLYING OSCILLATOR. CONSERVATION LAWS

"REST" ENERGY Γ (mc^2 UNITS)

MOMENTUM $\vec{P}$ (mc UNITS)

TOTAL ENERGY $\gamma^2 = \Gamma^2 + P^2$

VELOCITY $\vec{\beta} = \vec{P}/\gamma$

SINGLE PHOTON EMITTED:

$$\Delta\gamma = -\hbar\omega/mc^2 \; ; \quad \Delta\vec{P} = -\hbar\vec{k}/mc$$

$$\omega = k\beta_p \cdot c$$

$$\gamma\Delta\gamma = \Gamma\Delta\Gamma + \vec{P}\Delta\vec{P}$$

$$\boxed{\Delta\Gamma = \frac{\hbar\gamma k}{mc}\left(\frac{\vec{\beta}\vec{k}}{k} - \beta_p\right) = \frac{\hbar\gamma k}{mc}\left(\beta\cos\theta - \beta_p\right)}$$

- FAST PHOTON EMISSION DECREASES $\gamma, \vec{P}, \Gamma$
- SLOW PHOTON EMITTED WITHIN THE CHEREN. ANGLE DECREASES $\gamma, \vec{P}$ AND <u>INCREASES</u> Γ !
- SLOW WAVES (IF EXIST) MAY BE EMITTED BY A ZERO-AMPLITUDE OSC-R, BUT CAN NOT BE ABSORBED
- SLOW WAVE EMISSION <u>COULD BE SELF-AMPLIFIED</u>

Undulator radiation. From a viewpoint of the emission process it does not matter whether the oscillations are due to elastic forces and initial conditions or are forced by some external field. A particle passing across a static magnetic field alternating with a spatial period l will oscillate transversally and will emit specific "undulator" radiation. This type of emission is a basic process for free electron lasers and is worth of a brief discussion. Tr.7

If the oscillation amplitude is small enough the motion is nonrelativistic in the rest frame and the radiation intensity can be readily estimated using the well known expression for a dipole radiation. Being recalculated in the lab frame this expression shows the abovementioned relativistic γ^2 amplification of intensity for fixed transverse force. According to Doppler's formula the basic frequency at the angle θ exceeds the frequency of oscillations:

$$k_0 = \frac{2\pi\beta/l}{1-\beta\cos\theta + K^2/\gamma^2}$$

where the additional term in the denominator comes from the decrease in the average longitudinal velocity due to a finite value of the transverse particle velocity. It can be easily shown that the undulator coefficient K depends only on the product of the transverse magnetic field and the undulator period. Increase in K increases radiation intensity but shifts down the basic frequency, so its optimal value is around unity. However, for $K \approx 1$ the spectre of the undulator radiation is already enriched with harmonics of the basic frequency due to the abovementioned directiveness of radiation. For small oscillations the natural radiation cone of γ^{-1} angle always overlaps an observer looking along z. So the latter sees more or less harmonic radiation field with frequency γ^2 times exceeding the undulator frequency. For larger K the cone quickly sweeps over the observer at each period, so only sharp spikes of the field can be registered which are enriched with higher harmonics. Note that for the same reason the spectre of synchrotron radiation contains high harmonics of the revolution frequency up to the γ^3-th one.

Tr.8

Radiation reaction. Up to now we have been considering electromagnetic radiation under assumption that the electron motion is fixed. Obviously, this is not quite correct. Radiation carries out certain energy, momentum, angular momentum etc. and hence changes the corresponding characteristics of the particle motion. (For example, it violates the synchronism condition.) Sometimes the assumption of the fixed motion is justified, but generally it is not consistent with the essential wave-particle energy exchange declared above.

At this point we meet a non-trivial physical problem due to a principal difference between conservation laws in point-like particle dynamics and electrodynamics. The latter can not be "localized" in space and time meaning that a change of energy concentrated in a closed volume is equal to the Poynting's vector flux averaged over time (plus the change in mechanical energy of particles within the volume). The same is valid for the momentum conservation law.

To keep a standard form of a point-like particle equation of motion taking reaction of radiation into account one should introduce a supplementary force supporting the conservation laws at least in average. As far as this force does not include degrees of freedom of the electromagnetic field one can not expect this complemented equation to have a canonical (Hamiltonian) form. For this reason the radiation reaction force is called sometimes a "radiation friction".

TR. 7

UNDULATOR RADIATION

DIPOLE:

REST FRAME $\quad I = \frac{2q^2}{3c^3} |\dot{\vec{v}}|^2$

GENERAL $\quad \frac{dp_i}{ds} = \frac{2q^2}{3c} \left(\frac{du_k}{ds}\right)^2 u_i$

LAB. FRAME

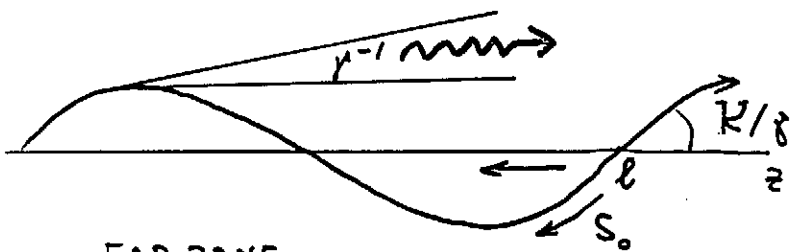
$$I = \frac{2q^2}{3c^3}\gamma^2 \left[\left(\frac{d}{dt}\gamma\vec{v}\right) - c^2\left(\frac{d\gamma}{dt}\right)^2 \right] = \frac{2q^2}{3m^2c^3} \left[\gamma^2 F_\perp^2 + F_\parallel^2 \right]$$

RELATIV. INCREASE

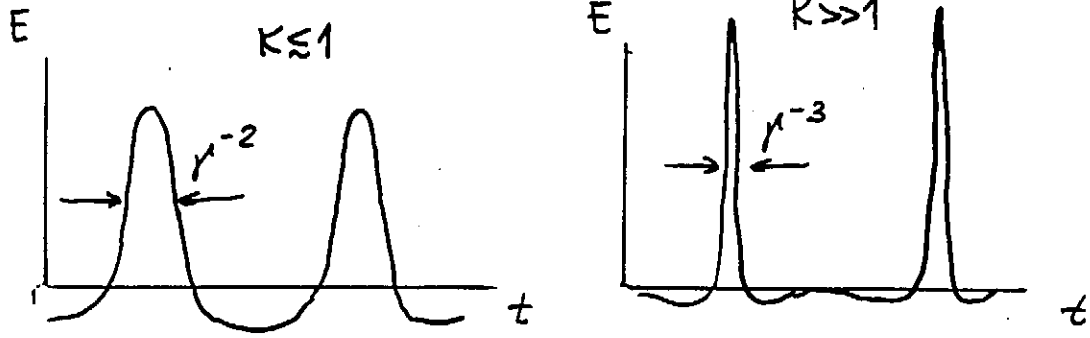
LARGE AMPLITUDES:

HARMONICS OF $\quad \omega^* = \frac{2\pi v/\ell}{1-\beta\cos\theta + K^2/2\gamma^2}$

$$K^2 = 2\gamma^2\left(\frac{s_0}{\ell} - 1\right)$$



FAR ZONE



TR. 8

RADIATION REACTION

CONSERVATION LAWS

LOCALIZED FOR A POINT-LIKE PART.
NON-LOCAL FOR THE E.M FIELD
} NO PROMPT REACTION TO BE EXPECTED!

TO KEEP A STANDARD FORM OF EQS OF MOTION
RAD. REACTION (NON-HAMILTONIAN) FORCE IS NEEDED

$$\langle \vec{f}\,\vec{v}\rangle + \left\langle \frac{2q^2}{3c^3}\left(\frac{d\vec{v}}{dt}\right)^2\right\rangle = 0 \quad ; \quad \text{OR:}$$

$$\vec{f} = \frac{2q^2}{3c^3}\frac{d^2\vec{v}}{dt^2} \qquad \text{— REST FRAME}$$

$$f_i = \frac{2q^2}{3c}\left[\frac{d^2u_i}{ds^2} - u_i\left(\frac{du_k}{ds}\right)^2\right] \quad \text{— GENERAL}$$

$$\vec{f} = -\frac{P}{c}\vec{\beta} + \frac{2q^2}{3c^2\gamma}\frac{d}{dt}\gamma^3\frac{d\vec{\beta}}{dt} \qquad \text{— LAB. FRAME}$$

$$P = \frac{2q^2}{3c}\left[\gamma^4\left(\frac{d\vec{\beta}}{dt}\right)^2 - \gamma\frac{d^2\gamma}{dt^2}\right] \quad \text{— TOTAL POWER}$$

$$\frac{d\vec{\beta}}{dt} \approx \vec{F}_\perp / mc\gamma$$

RADIATION "FRICTION" OR RECOIL FORCE

$$\boxed{\text{"SELF-ACCELERATION"} \quad \sim \exp\frac{3ct}{2r_0} \quad !?}$$

In the coordinate frame where the instantaneous particle velocity is small radiation intensity is proportional to the acceleration squared. Hence, the radiation reaction force $\vec{f}$ must be proportional to the time derivative of the acceleration. To obtain a corresponding relation in the lab frame one should write down a relativistic covariant expression for the force. Then the three space components give the force while the fourth component describes an averaged work performed over the partice, i.e., the total radiation power integrated over the radiation spectre.

Due to the high power of the factor γ in the expression for $\vec{f}$ one can keep the first term only. Then the radiation reaction force literally behaves as a friction force being antiparallel to the particle velocity. In this approximation it can be treated as a recoil per unit time related to a momentum carried out by radiation. The expression for energy losses is also simplified as far as the acceleration of a relativistic particle is due mainly to the change in the direction of its velocity rather than in its absolute value.

The concept of the radiation friction is valid for sufficiently slow time variations of dynamical variables, i.e., when at least in one coordinate frame it is small as compared with external forces. A general criterion for this can be formulated as a strong unequality

$$r_0 \ll \lambda$$

where $r_0 = q^2/mc^2$ is a classic radius of the particle and λ is a characteristic wavelength of the external field or a radius of curvature of the particle trajectory. For an electron $r_0 = 2.8 \times 10^{-13}$ cm, so the condition is safely satisfied for most parameters of practical interest.

One can conclude that the radiation reaction force is, in a way, a fictitious one meaning that it is related not only to the mechanical particle motion but to changes in the surrounding electromagnetic field as well. It is worth to note in this connection that introducing the force on the conservation laws basis we ignored a possibility of excitation of internal degrees of freedom. This is justified, of course, for an elementary particle as far as in a relativistic theory it can be only a point-like one. However, for a small but finite size bunch the arguments above should be reconsidered.

Of course, the radiation energy losses can be provided only with a proper electric field generated by the particle acceleration . This field can be calculated directly under assumption that a change in the particle acceleration during light propagation through the particle is small as compared with the acceleration itself. Coming then to the limit of a particle "radius" $a \to 0$ one can consider the result as a strict one if time exceeds Tr.9
a/c. However, this approach is still not satisfactory. For a free particle it leads to a solution when its velocity and acceleration spontaneously increase exponentially with a characteristic time $\tau = 2q^2/3mc^2$. Actually the result is meaningless because for the reasons above the equation of motion with the fictitious radiation friction can not be solved with any fixed initial conditions.

It is interesting to note that in the abovementioned case of a small dense bunch this difficulty does not exist. If the number of particles in the bunch is high enough to make its "classic radius" comparable with the physical size the radiation reaction force will be comparable with the mechanical inertia. The behaviour of such an object would be quite unusual.

The assumed smallness of the radiation reaction as compared with external forces does not imply that it can not change essentially the motion of the electron and characteristics of radiation. For example, the work performed by the radiation friction slowly

TR. 9

SELF FIELD ACTION

TO GET THE FORCE: SOLVE MXW EQ AND PUT $\vec{r} \to 0$

$$\vec{f}(t) = \frac{2q^2}{3c^3}\ddot{\vec{v}} + \frac{4q^2}{3\pi c^2}\dot{\vec{v}}_0 \frac{\sin\frac{ct}{a}}{ct} + O(a) ; \quad a \to 0$$

O.K. for $ct \gg a$

MAKES MEANINGLESS ANY SOLUTION WITH MECHANICAL „INITIAL" CONDITIONS

APPLICABILITY: $a \ll r_0 \ll \lambda$

SMALL AND SMOOTH ACCELERATION

QED FOR $\lambda < \Lambda = 137\, r_0$

UP TO NOW THE ONLY OBJECT WITH $a \ll r_0$ IS AN ELECTRON

WHAT IF $a \simeq r_0$ AND $\Lambda \ll r_0$?

$$r_b = \frac{(Nq)^2}{Nmc^2} = N r_0 \ ; \quad \Lambda_b = \frac{\hbar}{Nmc} = \frac{\Lambda}{N}$$

?

$$N_{cR} \simeq \frac{\lambda}{r_0} \ ; \quad N \gg \sqrt{137}$$

but monotonously changes the particle energy and leads to a drift in the condition of synchronism. The latter determines spectral and angular characteristics of radiation. As a result, for example, spectral lines typical for periodic charge motion will have some "natural" broadening. (However, in what follows the spectral line width is determined by other factors.)

Sometimes the radiation reaction plays a principal role in the particle dynamics as well even if energy losses are compensated by an external electric field and the particle energy is Tr.10 supported constant. Such conditions are typical for electron and positron accelerators and storage rings. One of their most important characteristics is a beam brightness defined as a phase space density of particles. A pecularity of this characteristic follows from a seemingly academic fact that in absence of radiation the particle equations of motion are canonical. The Hamiltonian $\mathcal{H}(\vec{r}, \vec{P}, t)$ depends on time, on coordinates and on canonially conjugated components of a generalized momentum $\vec{P}$. This is known to be sufficient to support the brightness nonincreasing (constant in the best case) under any manipulation with the beam. This statement has a fundamental character and is known as Liouville's theorem. It states that a six-dimensional phase-space region occupied by particles can be transformed in almost arbitrary way but its volume (so called beam emittance) is an exact integral of motion.

The non-hamiltonian character of the radiation friction mentioned above means the Liouville's theorem violation in presence of radiation and open a way for beam "cooling", i.e., for increase in its brightness.

To prove this statement let us consider a phase-space particle density $\Psi\left(\vec{r}, \vec{P}, t\right)$ satisfying a discontinuity equation (conservation of total number of particles). Being supplemented with the non-canonical radiation friction it gives an expression for the total derivative $d\Psi/dt$ along a phase trajectory. In absence of radiation it would be equal to zero (Liouville's theorem). But now it has a non-zero right hand. Using the identity

$$\gamma^2 = 1 + \left(\vec{P} - \frac{q}{c}\vec{A}\right)^2 / m^2 c^4$$

one can put

$$\vec{\nabla}_{\vec{P}} \frac{P}{\gamma}\left(\vec{P} - \frac{q}{c}\vec{A}\right) \approx 3\frac{P}{\gamma} + \gamma\left(\frac{\partial}{\partial\gamma}\frac{P}{\gamma}\right)_{\vec{r}=\mathrm{const}} = \frac{1}{\gamma^2}\left(\frac{\partial P\gamma^2}{\partial\gamma}\right)_{\vec{r}=\mathrm{const}}$$

Taking into account that $P \propto \gamma^2$ we get

$$\frac{d\Psi}{dt} = \frac{4P}{\gamma mc^2}\Psi \qquad \text{and} \qquad \Psi \propto \exp\int^t \frac{4P}{\gamma mc^2}\,dt$$

Hence, the beam brightness exponentially grows with time, i.e. the total beam emittance damps. The characteristic damping or cooling time $\tau_c \approx \gamma mc^2/P$ is equal by order of magnitude to the time a particle needs to loose its energy because of emission of radiation.

TR. 10

VIOLATION OF LIOUVILLE'S THEOREM
(RADIATION COOLING)

CONT. EQN FOR A PHASE-SPACE DENSITY $\Psi(\vec{r},\vec{p},t)$:

$$\frac{\partial}{\partial t}\Psi + \nabla_{\vec{r}}\left(\Psi\frac{d\vec{r}}{dt}\right) + \nabla_{\vec{p}}\left(\Psi\frac{d\vec{P}}{dt}\right) = 0 \qquad \left(\frac{\partial\rho}{\partial t} + \mathrm{div}\,\vec{j} = 0\right)$$

FOR $\frac{d\vec{r}}{dt} = \nabla_{\vec{p}} H$;

FRICTION DUE TO EN. LOSSES
($\mathcal{P}$ – RADIATION POWER)

$$\frac{d\vec{P}}{dt} = -\nabla_r H + \left(-\frac{\mathcal{P}}{\gamma mc}\vec{\beta}\right)$$

$$\frac{mc^2}{\Psi}\frac{d\Psi}{dt} = \nabla_p \frac{\mathcal{P}}{\gamma}\left(\vec{P} - \frac{q}{c}A\right) \approx 3\frac{\mathcal{P}}{\gamma} + \gamma\frac{\partial}{\partial\gamma}\frac{\mathcal{P}}{\gamma} = \underline{\frac{1}{\gamma^2}\frac{\partial}{\partial\gamma}\mathcal{P}\gamma^2}$$

$$\boxed{\Psi \sim \exp\int^t \frac{1}{\gamma^2}\frac{\partial}{\partial\gamma}\mathcal{P}\gamma^2 \,\frac{dt}{mc^2}}$$

VALID FOR ANY TYPE OF COOLING

RADIATION COOLING $\qquad \mathcal{P} \sim \gamma^2$

$$\Psi \sim \exp\int \frac{4\mathcal{P}}{\gamma mc^2}\,dt \qquad ; \quad \gamma \simeq \mathrm{const}$$

$\left.\begin{array}{l}\text{DENSITY INCREASES}\\ \text{EMITTANCE DECREASES}\end{array}\right\}$ EXPONENTIALLY WITH TIME

Lecture 2

Spontaneous and Stimulated Emission of Radiation

Coherency. At the previous lecture we did not pay much attention to absolute values of energy losses of a single radiating particle. This is a "proper" electromagnetic field which Tr.11 is determined by the particle motion while the radiation power or energy loss crucially depends on a total field including an external field and/or the radiation field of other particles.

The difference can be enormous. A single electron passing through the undulator at ATA accelerator (Livermore Lab.) looses only about 5 meV. This value, of course is not of interest for practical purposes even being multiplied by a large number of simultaneously radiating particles. At the same device an 1 kA beam produces around 1 GWt of power which corresponds to 1 MeV/particle of energy losses. This nine orders of magnitude difference shows that the collective radiation power is not just a sum of individual particles radiation losses.

The key point here is a coherency of the individual radiation field $\vec{E}_r$ with the already existing field $\vec{E}_0$ of the same frequency and the same wavenumber. If both fields behave as $\exp[i(kz - \omega t)]$ and have a fixed phase shift ψ then their sum being squared and averaged over time produces the total Poynting vector which can be presented as a sum of the external field vector, of a negligible "proper" one, and of a cross term depending on the phase shift. From a viewpoint of single particle dynamics the latter can be explained as a trivial particle acceleration or deceleration. However, interpreting it as absorbtion or amplification of the incident power flux by large number of phased radiating particles one can see that it readily exceeds the first term. Of course, for random phases it vanishes because 50% of particles absorb field energy while another 50% provide it. But even small mutial phasing may change the incoming flux essentially if the number of particles is large enough!

The same arguments are valid if applied to the collective radiation field. Of course, Tr.12 different electromagnetic modes can not be mutually coherent as far as they have different frequencies and wavenumbers. But a Poynting vector of a fixed mode can be presented as a single particle one multiplied by a so called coherency factor which is a double sum of exponential phase factors of individual particles (double summation comes from field squaring). Depending on mutual particle phasing this factor can vary within enormously wide limits – from unity up to N^2 where N is a number of phase correlated particles. Of course, this coherent amplification can take place for select modes only because phasing can not be provided for all radiation modes at the same time.

Perhaps the simplest illustration of the select mode coherent amplification can be obtained considering a regular one-dimensional chain of particles under condition of Cherenkov radiation. The angles which correspond to π phase shift of the adjacent particles radiation fields will be shadowed while those of 2π phase shift will reveal large intensity amplification. Hence, the z-directed angular distribution of a single particle radiation becomes a many petal one with sharp minima and maxima determined by particle separation and by the wavelength. Note that these angles are different for different frequencies, so that the angular distribution averaged over spectre would reveal much weaker coherency than the spectral density.

Of course, this example of "nailed" particles is an extreme one. The opposite limit

COHERENCY

TR. 11

ATA UNDULATOR AMP R (LIVERMORE)

SINGLE-PART. RADIATION = 5 meV / part

1 kA BEAM RADIATION = 1 MeV / part

9 ORDERS OF MAGN. DIFFERENCE !

SINGLE PARTICLE + EXT. WAVE COHERENCY

$$\vec{E} = \left(\vec{E}_0 + \vec{E}_r e^{i\psi}\right) \cdot \exp i(kz - \omega t)$$

$$\vec{B} = \ldots.$$

$$\vec{P} = \vec{P}_0 + \vec{P}_r + \frac{c}{8\pi} \left\{ [\vec{E}_0 \times \vec{B}_r] + [\vec{E}_r \times \vec{B}_0] \right\} \cos\psi$$

POYNTING VECTOR

$\vec{P}_r \ll \frac{c}{8\pi}\{\ldots\}$

TRIVIAL EXPL-N: PARTICLE AC/DE CELERATION

FOR UNIFORM PHASE DISTRIBUTION

50% OF PARTICLES RADIATE AND 50% CONSUME ENERGY

EVEN SMALL PROPER PHASING MAY BE CRUCIAL !

IF EXISTING WAVE IS STRONG ENOUGH

TR. 12

INTRABEAM COHERENCY

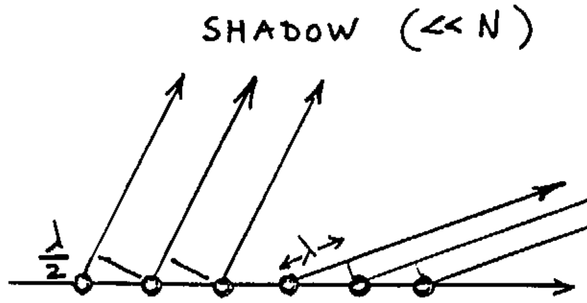
FOR A PARTICULAR EIGENMODE

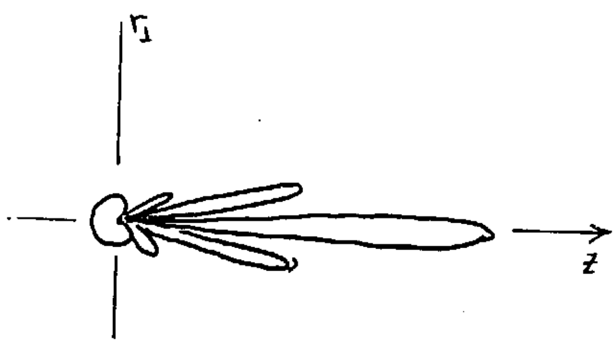
$$\vec{P} = \frac{c}{4\pi}\left[\vec{E}_r \times \vec{B}_r\right] \cdot C$$

COHERENCY FACTOR

$$C = \sum_{n,m=1}^{N} \exp i(\psi_n - \psi_m) \begin{cases} \sim 1 & \text{IF } \psi_n = \pi n \\ = N^2 & \text{IF } \psi_n = 2\pi n \end{cases}$$

LINEAR REGULAR DISTRIBUTION, DIFFERENT MODES SAME FREQUENCY





MANY PETAL DI.

$I_{max} < N^2$

$I_{MIN} \ll N$

is presented by a random distribution when the coherency factor is equal to N being averaged over all realizations. Note that if the distribution can be treated as random for all modes then all spectral and angular characteristics of radiation identically repeat the individual particle ones. This type of completely non-coherent radiation is called a spontaneous one (the term genesis will be explained below). Tr. 13

A conditional boundary between coherent and non-coherent modes is determined by particles position correlation. Suppose the particle phases obey a normal distribution law with an average phase shift of adjacent particles μ and phase dispersion δ (both proportional to a frequency). Then δ^{-1} plays the role of a correlation length and the averaged coherency factor can be presented as N plus a term exponentially small for small δ^{-2} (the first term appears because a particle is always correlated with itself). Hence, the radiation will be coherent only for wavelengths larger than the correlation length and reveals in this region spectral maxima with $C \approx N^2$ and minima with $C \approx N$. Note that for $N \to \infty$ these variations vs frequency are very compressed, so that being averaged over a certain spectral interval the intensity never reaches the N^2 limit and radiation remaines partly incoherent. One may conclude that complete coherency (meaning N^2 factor) takes place only for wavelengths exceeding a size of a particle bunch and for select modes with wavelengths larger than the correlation length. In the last case the coherency is observable only in high spectral and angular resolution experiments. Tr. 14

This selectiveness of coherency is more expressed for 2-D and 3-D particles distributions. Even for absolutely correlated positions of particles in a lattice the N^2 factor may appear only for particular directions, i.e., along main crystallographic axes. It is interesting to note that the emitted quantum momentum is then an integer of $2\pi\hbar/l$ where l is the lattice constant. For a random distribution a certain degree of coherency is kept if a number of particles in a λ^3 volume is large enough. For large dense lattices ($L = Nl \gg \lambda$) the coherency factor is then $NL/\lambda \ll N^2$, by order of magnitude. A case of rarified lattices with $N < (L/\lambda)^3$ (less than one particle per λ^3 volume) is hard to estimate. Some computations show that the integral (averaged over angles) coherency disappears while differential peaks for particular modes remain for regular distributions. Tr. 15

Two essential remarks should be made in this connection. The first one looks like a paradox: acccording the arguments above a point-like bunch must radiate in all directions and at all frequencies N^2 times more intensively than a single particle does. However, one should bear in mind that it is valid only for $\lambda \gg L$ (for example, for external low frequency wave scattering). Charged particles always interact and pressing them together will produce new high frequency and short wavelength degrees of freedom (plasma oscillations). They have to be taken into account in calculations of proper fields. To make the long story short a small dense bunch in principle can not be considered as a point-like particle, so one should be very careful with the coherency factor in the limit of $L \to 0; N =$ const. The physics of this has much common with the radiation reaction problem mentioned at the previous lecture. Tr.15

The second remark is more practical. All the arguments above look like an academic exersize in calculating waves inerference unless a certain mechanism of particles phasing is suggested. We shall show that such a mechanism really exists. The self-organization of particles and the resulting appearance of coherent effects is an essence of so called induced or stimulated emission of radiation

TR. 13

RANDOM DISTRIBUTION

$$\langle C \rangle = \sum_{n=m=1}^{N} \exp i(\psi_n - \psi_m) \equiv N$$ → NO COHERENCY

IF DISTRIBUTION IS RANDOM FOR ALL MODES SPECTRAL AND ANGULAR DENSITY OF RADIATION REPEATS SINGLE-PART. ONE BEING MULT-D BY N

THIS IS NON-COHERENT or SPONTANEOUS RADIATION

CORRELATED 1-D DISTRIBUTION

WITH PROBABILITY

$$\prod_{s=1}^{N} \frac{1}{\delta\sqrt{\pi}} \exp\left[-\frac{(\psi_s - s\mu)^2}{\delta^2}\right]$$

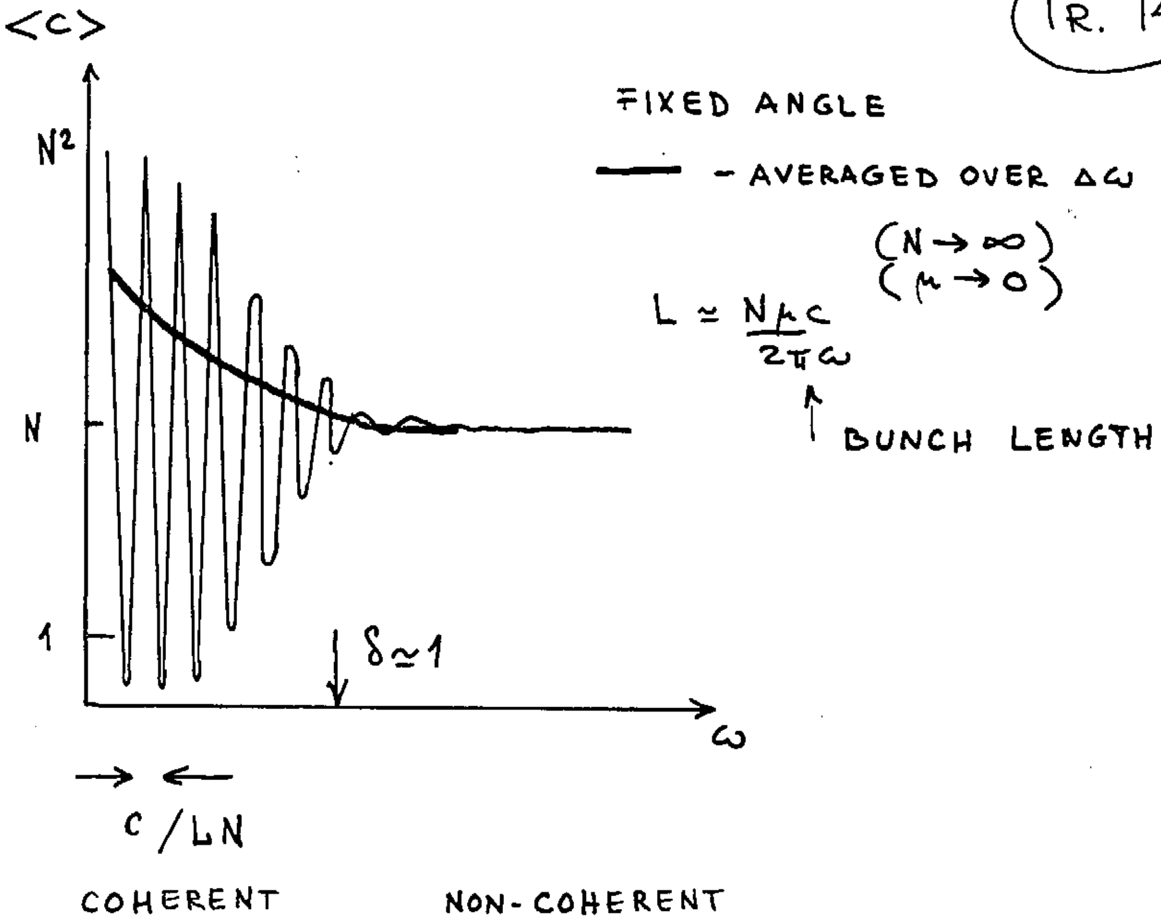
μ — AVERAGE PHASE SHIFT
δ — PHASE DISPERSION
} $\propto$ MODE FREQUENCY

INVERSE CORRELATION LENGTH)

$$\langle C \rangle = N + \left[\frac{\sin^2 \frac{\mu N}{2}}{\sin^2 \frac{\mu}{2}} - N\right] e^{-\frac{\delta^2}{2}}$$

PARTIAL COHERENCY FOR LARGE WAVELENGTHS

TR. 14



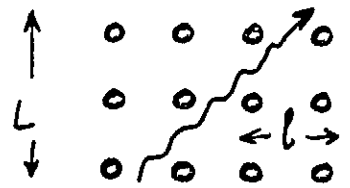
WITH WAVELENGTH SHORTENING COHERENCY LOST:

- INTEGRAL CHARACTERISTICS — FOR $\lambda \ll L$ (BUNCH LENGTH)
  (TOTAL INTENSITY, LOW SP. AND/OR ANGULAR RESOLUTION, etc)
- DIFFERENTIAL CHARACTERISTICS: (SINGLE MODE)
  (SPECTRAL AND/OR ANGULAR DENSITY etc) FOR $\lambda \gg L_{cor}$ (CORRELATION LENGTH)
- BUT MIGHT BE KEPT EVEN FOR $\lambda > \frac{L}{N}$
  (LESS THAN ONE PARTICLE / WAVELENGTH)

Tr. 15

3-D DISTRIBUTIONS

INTEGRAL COHERENT EFFECTS ARE WEAKER EVEN FOR REGULAR LATTICES WITH IDEAL PHASING. ONLY PARTICULAR FREQS ALONG CRYST. AXIS GIVE COHERENCY



RECOIL MOMENTUM HAS TO BE $= \frac{2\pi\hbar}{l} \times$ INTEGER

FOR $N \gg \underbrace{\left(\frac{L}{\lambda}\right)}_{} \gg 1$; $\frac{L}{\lambda} \simeq N^{1/3}$ AND RANDOM DISTRIBUTION:

LARGE NUMBER OF PARTICLES / λ^3 VOLUME SUPPORTS A CERTAIN DEGREE OF COHERENCY

$$C \simeq N \cdot \frac{L}{\lambda} \ll N^2$$

FOR $N \simeq L/\lambda$ (ONE PART / λ^3 VOLUME)

INTEGRAL COHERENCY DISAPPEARS (COMPUTED) WHILE DIFFERENTIAL PEAKS REMAIN FOR REGULAR DISTRIBUTIONS (PARTICULAR MODES)

ACHTUNG: A LOGIC TRAP N PARTICLES PACKED IN A λ^3 BUNCH RADIATE AS A SINGLE PART. $\times N^2$ IS THIS CORRECT ?

Spontaneous and stimulated emission in a phenomenological approach. It is widely known that the terms "spontaneous" and "stimulated" emission were introduced by Einstein in connection with a thermodynamic equilibrium of radiation and elementary radiators (Planck formula). However, we forget sometimes that it had happened before the quantum mechanics of an atom appeared. So, this argumentation is worth of brief repeating using the update terminology, of course. Tr.16

The probability of a stimulated emission transition from an energy level E_n to E_m was (and had to be) assumed to be proportional to the radiation spectral density and to be equal to the probability of the inversed process, i.e., to that of absorbtion. In the equilibrium total probabilities of emission and absorbtion must be equal, but the energy population of radiators obeying Boltzmann law contradicts the equality because it contains more absorbing particles then radiating ones. So additional probability independent of radiation spectral density should be introduced. This was called a "spontaneous" emission probability.

In this context the "spontaneous" emission meant just unprovoked one. As far as in the large wave length limit the result must coincide with Rayleigh-Jeans formula it is easy to show that the spontaneous probability is equal to the stimulated one calculated for a radiation field represented just by one photon. Obviously, the spontaneous radiation is non-coherent; that is why the term was kept for non-correlated particles radiation even in a classic theory.

Today one can not call this terminology adequate. With appearance of quantum mechanics a non-stimulated transition between steady states sounded strangely (W. Pauli) and only in quantum electrodynamics it was shown to be related with vacuum non-zero electromagnetic fluctuations. So the emission, in a sense, is stimulated too but keeps its name. In a classic theory the emission is always caused by particle acceleration or Cherenkov mechanism, so "spontaneous" means just "random phase". This is stimulated emission or its physical analogy that should be explained in a classic theory. Following the same logic we consider it as an inversed absorbtion process. Tr.17

First of all, the conception of absorbtion, stimulated emission, and spontaneous radiation as elementary processes can be extended to non-stationary situations. Then a kind of a kinetic equation for number of photons with a wave vector $\vec{k}$ can be written down. Note that we kept the definition of the spontaneous emission as induced by a single photon and took into account the difference in population for emission and absorbtion processes. In a steady case this equation readily yields Planck's formula. However it predicts also Tr.18 an exponential multiplication of identical photons (at least for $n_k \gg 1$) if the energy population $f(E)$ is inversed ($f(E) > f(E - \hbar\omega)$) and the second term prevails over the last one. This is, of course, the lasing effect well known nowadays.

Stimulated emission in classic electrodynamics. Let us come now to a classic limit Tr.19 assuming that the energy distribution is wide as compared with $\hbar\omega$. Then $f(E - \hbar\omega) \simeq f(E) - (\partial f/\partial E)\hbar\omega$. Multiplying the kinetic equation by $\hbar\omega$ and performing integration over E one gets a power balance for the spectral density $W_k = \hbar\omega n_k$, where the Planck constant disappears. It is easy to see that the obtained relation still predicts the lasing effect which is proportional to a partial derivative of the spontaneous radiation spectral intensity over particle energy. On one hand this is a sequence of a small asymmetry of absorbtion and induced emission; on the other hand it predicts lasing if the spontaneous radiation spectral line is sharp enough. Note that this is exactly what is provided by the

SPONTANEOUS AND INDUCED / STIMULATED RADIATION

TR. 16

A BIT OF HISTORY

- PROBABILITY OF $E_m \to E_n$ TRANSITION (INDUCED BY E.M. FI

$$W^i_{mn} = B_{mn}\,\rho(\omega_{mn}) = W^i_{nm} = W^i$$

SPECTRAL DENSITY

- ABS-N AND RADIATION (INDUCED) PROB-S WERE POSTULATED EQUAL

- FOR THERMODYN. EQUILIBRIUM TOTAL PROBABILITIES

$$W_{mn} N_m = W_{nm} N_n$$

POPULATION

- FOR BOLTZMANN POPULATION $\left(\frac{N_n}{N_m} = \exp \frac{E_n - E_m}{T} \right)$

$$\frac{W_{mn}}{W_{nm}} = \exp\left(\frac{\hbar\omega_{mn}}{T}\right) > 1 \quad \text{WITH } W_{nm} \equiv W^i$$

- "SPONTANEOUS" PROBAB. INDEPENDENT OF ρ (EINSTEIN)

$$W^{sp}_{mn} = W_{mn} - W^i \quad \text{AND} \quad B_{mn} = \frac{\pi^2 c^3}{\hbar\omega^3_{mn}} W^{sp}_{mn}$$

- THEN $\rho \sim \dfrac{\omega^3}{\exp(\hbar\omega/T) - 1}$ (PLANCK FORMULA)

- Q. WHAT CAUSES "SPONTANEOUS" RADIATION?

  A. NOTHING (30 YEARS LATER: VACUUM FLUCT-NS)

TR. 17

VOLUNTARY EXTENSION TO NON-EQ. SITUATIONS

Kinetic relation for photons

$$\frac{\partial n_\mathrm{k}}{\partial t} = \int dE \, \{ \underbrace{f(E) w_\mathrm{k} \downarrow}_{\text{sp. rad.}} + \underbrace{n_\mathrm{k} f(E) w_\mathrm{k} \downarrow}_{\text{ind. rad.}} - \underbrace{n_\mathrm{k} f(E - \hbar\omega) w_\mathrm{k} \uparrow}_{\text{abs.}} \}$$

$w_\mathrm{k} \downarrow\uparrow$ – probability of transition with radiation (absorbtion) of a photon $\vec{\mathrm{k}}$

$f(E)$ – energy distribution of radiators

TR. 18

$$\frac{\partial}{\partial t} = 0 \qquad \text{and} \qquad f(E) \propto \exp(-E/\kappa T)$$

yield Planck formula

$$n_k = \frac{1}{\exp(\hbar\omega/\kappa T) - 1};$$

$$\frac{\partial}{\partial t} \neq 0 \qquad \text{and} \qquad f(E) > f(E - \hbar\omega)$$

yield lasing

induced radiation is a process reversed with respect to resonant absorbtion and reveals itself as lasing in presence of free energy (inverse population)

TR 19

CLASSICAL LIMIT

"SPONT" RADIATION IS OBVIOUS → PART. ACC-N

" INDUCED" RADIATION ?

$$f(E-\hbar\omega) \approx f(E) - \frac{\partial f}{\partial E}\hbar\omega \qquad \text{for} \qquad \hbar\omega \to 0$$

power balance

$$\boxed{\frac{\partial W_k}{\partial t} = P_k - W_k \frac{\partial P_k}{\partial E}} \qquad W_k \sim e^{-\int^t \frac{\partial P_k}{\partial E} dt}$$

assuming

$$w_k \uparrow = w_k \downarrow = w_k;$$

$$f(E_{min}) = 0;$$

$W_k = \hbar\omega n_k$ – spectral density of em energy;

$P_k = \int dE f(E) w_k \hbar\omega$ – total power of spontaneous radiation

So:

- Planck constant is not involved
- induced radiation presented
- lasing is possible if $\frac{\partial P_k}{\partial E} < 0$
- sharp line of spontaneous radiation should be a function of energy or, for a given frequency, a resonant value of energy exists

↑

CONDITIONS OF WAVE-PARTICLE SYNCHRONISM !

wave-particle synchronism condition. Tr.20

Moreover, the synchronism condition gives the profile of the spectral line intensity. Being calculated in a standard way it is a function of a frequency and of a particle energy via a single parameter – the kinematic phase shift μ of a particle along the system, or detuning. Differentiating over μ one finds two bands of energy where the beam optical activity is revealed at a fixed frequency. The energy values slightly lower than the synchronous one correspond to absorbtion and those larger than the synchronous one correspond to the stimulated emission, i.e., to lasing. In terms of waves behaviour for a fixed beam energy one can say that slow quasi-resonant waves (positive μ) are subject of induced emission and fast ones will be absorbed by the beam.

This conclusion shows an importance of phase motion of particles wih respect to the Tr.~~19~~ 21 wave. For accelerator people this obviously reminds autophasing or a phase stability mechanism in linear accelerators, so the description is ready. To solve the equations of phase motion we shall assume, at first, that the possible induced effects are small, so that the effective field amplitude is almost constant. In most cases of interest the parameter g/k is small, so a successive approximation method is adequate at least at an initial stage of the phasing process. The first approximation for the phase ψ shows a kinematic phase shift due to initial detuning and an additional induced phase shift due to modulation of the particle energy. So the change in the energy (i.e., the work performed by the wave) contains a term linear with g and a quadratic one. The first one vanishes being averaged over all initial phases. That means that 50% of particles absorb the wave energy and 50% provide it for initially uniform phase distribution. But the quadratic term breaks the symmetry and does not vanish being averaged. Let me remind that the effect is due to the induced phasing of the beam. Its proportionality to the wave intensity rather than to the field shows unambigiously its relation to the stimulated emission.

Now we may define a gain parameter as a relative work performed by the beam over Tr.~~20~~ 22 the wave. It appears to be proportional to the beam current I and to the interaction length cubed and is a function of the initial detuning. Note that this function is the exact derivative of the spectral line profile with respect to the detuning as has been predicted using phenomenologic approach. The coincidence (including an omitted coefficient) is remarkable because the latter was based on energy considerations only and did not include any phasing mechanism (a good example of the correspondence principle in quantum mechanics!). As a matter of fact, the phase has no physical meaning when a number of photons serves as a characteristic of the field.

Of course, our calculations are valid for an initial stage of phasing only and one can ~~Tr.21~~ predict a forthcoming saturation. The same is true, however, for the phenomenologic description as well where an inevitable exhaustion of an initially inversed energy population was neglected. Anyway, as a result of the induced effects certain modes of spontaneous radiation will be amplified (preferably those at the maximal slope of the spectral line profile). A small gain was assumed but a positive feedback (better if selective) will convert it to the temporal exponential increase of the particular radiation mode. Note that at this point we successfully invented an FEL oscillator, unfortunately being late for about 30 years. For relief we can stress that the same principles govern the operation of rather ancient travelling wave tubes.

The calculations above might be illustrated by phase-space diagrams showing the ini- Tr.~~24~~ 23 tially monoenergetic and phase uniform beam phasing in a deceleration (radiation) region Tr.~~25~~ 24 and then spreading again with saturation coming. It happens for positive detunings; the

TR. 20

SPONT. RADIATION OF A FIXED MODE VS PART. ENERGY

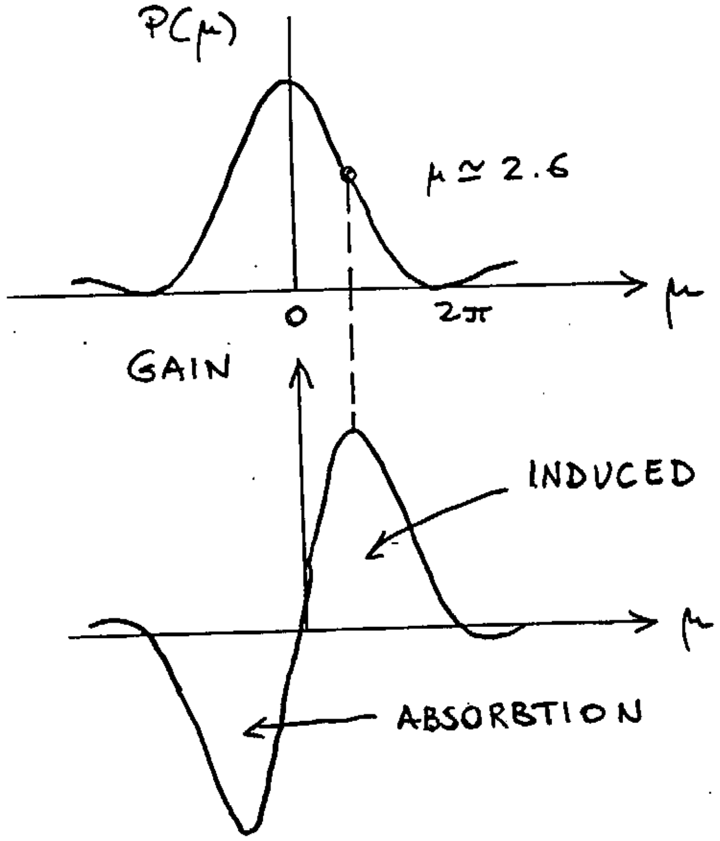
$$E_{\vec{k}} \sim K \int_0^L \exp\left[\frac{iz}{\beta}\left(k_0 \pm \Omega/c - k\beta\right)\right] dz = ikL \; \frac{e^{-i\mu}-1}{\mu}$$

$$\mu = \left(1 - \frac{k_0 \mp \Omega/c}{\beta k}\right) Lk \; -$$ PHASE SHIFT AT THE TOTAL INTERACTION LENGTH (DETUNING)

$$P(\mu) = P(0) \left(\frac{\sin \mu/2}{\mu/2}\right)^2$$

SYNCHR. PARTICLE

BEAM OPTICAL ACTIVITY VS ENERGY (VS μ)



TR. 21

AUTOPHASING IN A QUASI-SYNCHRONOUS WAVE $(\gamma \approx \gamma_s)$

$$\frac{d\psi}{dz} = k\alpha(\gamma - \gamma_s) \quad ; \quad \frac{d}{dz}(\gamma - \gamma_s) = g\cos\psi$$

$\alpha = -\beta_p\left(\frac{\partial}{\partial\gamma}\frac{k_0 \mp \Omega/c}{k_0\beta}\right)$ – SENSITIVITY OF PHASE SLIPPING TO ENERGY DEVIATION

$g = \frac{qE_0}{mc^2}\left(\frac{\beta_\perp}{\beta_\parallel}\right)$ – MAXIMAL ENERGY GRADIENT $\ll k$

$\approx$ const (LOW GAIN APPROXIMATION)

$$\psi = \psi_i + \mu + \frac{g}{\alpha k(\gamma_i - \gamma_s)^2}\left[\cos\psi_i(1-\cos\mu) - \sin\psi_i(\sin\mu - \mu)\right]$$

$\uparrow$KINEMATIC + $\uparrow$INDUCED PHASE SLIPPING $\qquad \mu = k\alpha L(\gamma_i - \gamma_s)$

$$\gamma = \gamma_i + \frac{g}{\alpha k(\gamma_i - \gamma_s)}\left[\sin\psi_i(\cos\mu - 1) + \cos\psi_i \sin\mu\right]$$

$$- \frac{g^2}{\alpha^2 k^2(\gamma_i - \gamma_s)^3}\left[\underline{1 - \cos\mu + \frac{\mu}{2}\sin\mu} - \frac{1}{2}\sin(2\psi_i + \mu)(\sin\mu - \mu)\right]$$

$\uparrow$

NONZERO AFTER AVERAGING OVER ψ_i

GAIN

$$G = \frac{\Delta g^2}{g^2} \propto \frac{L^3 \alpha I}{\mu^3}\left[1-\cos\mu+\frac{\mu}{2}\sin\mu\right]$$

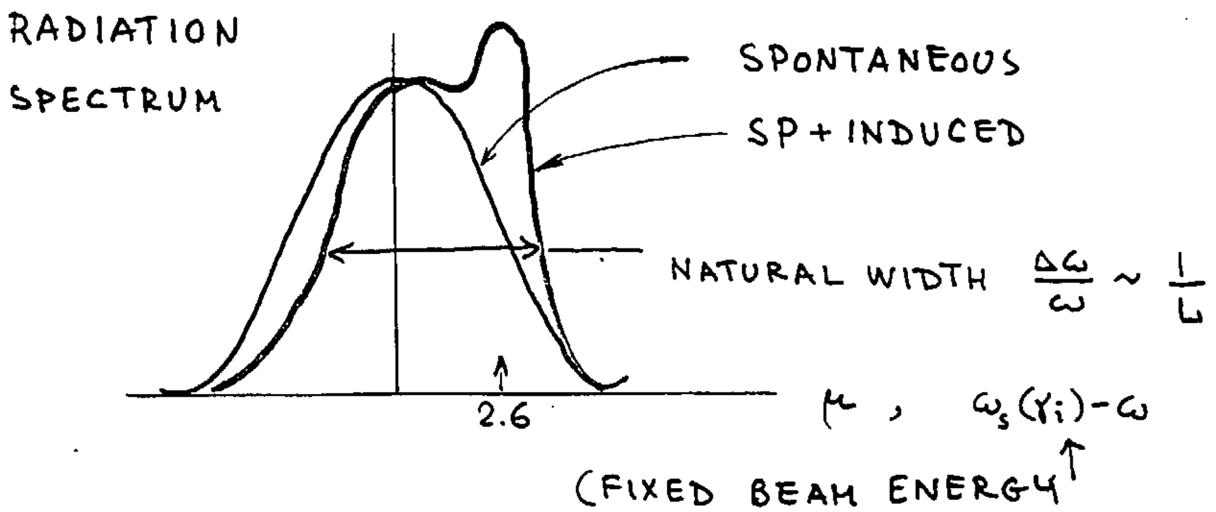
$$\propto \frac{L^3 I \alpha}{4}\,\frac{d}{d\mu}\left(\frac{\sin\mu/2}{\mu/2}\right)^2$$

EXACTLY THE SAME AS IN THE "CLASSICAL LIMIT" !

SATURATION LENGTH $= \max\left(\frac{\gamma_i-\gamma_s}{g}\ ;\ (\alpha K g)^{-1/2}\right)$

SUPPOSING $L <$ SATURATION LENGTH

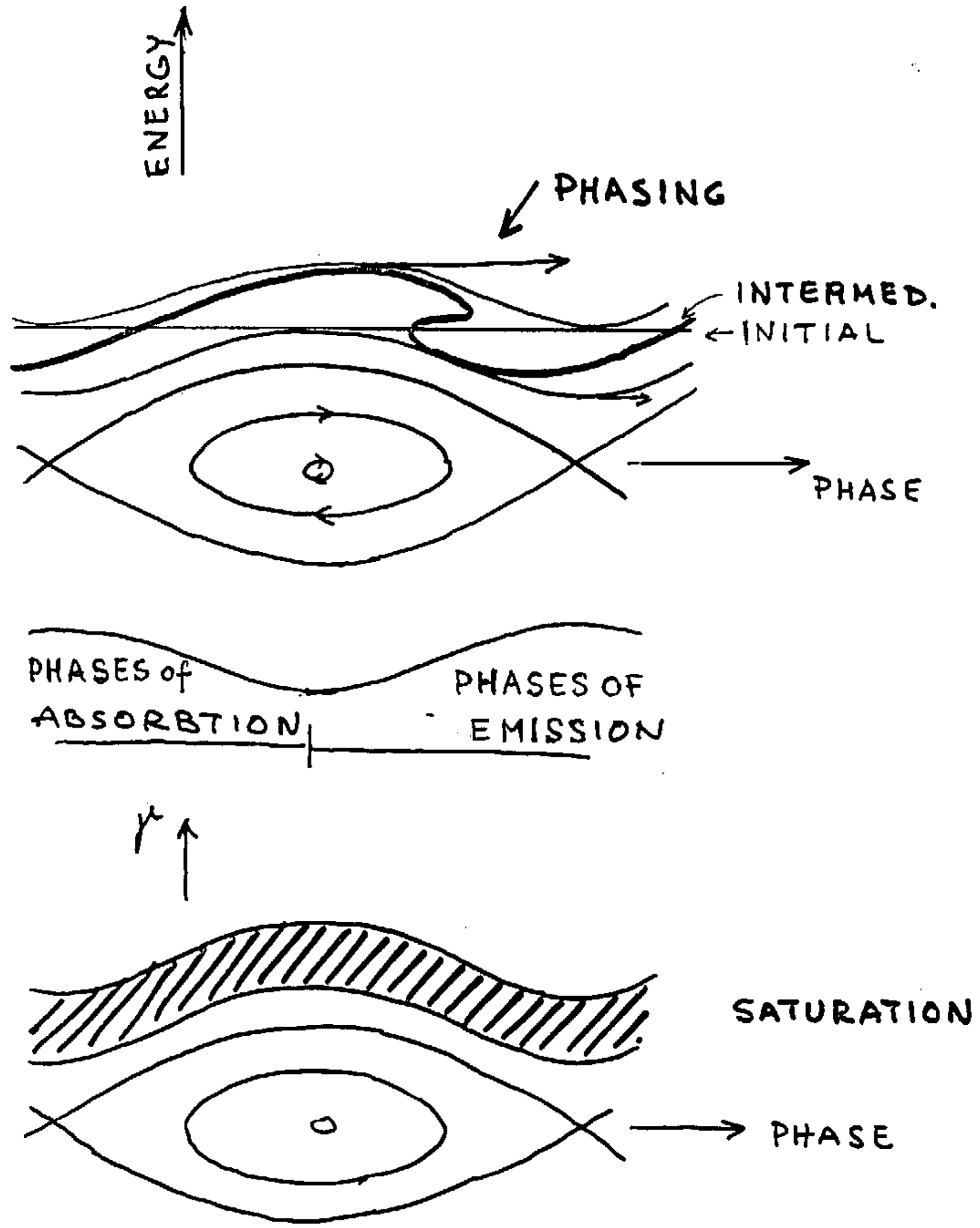
THE MODE WITH $\mu \simeq 2.6$ WILL BE MOSTLY AMPLIFIED



FOR SMALL GAIN A POSITIVE (BETTER IF SELECTIVE) FEEDBACK IS NEEDED TO GET SELFEXCITATION

$\boxed{\text{THIS IS FEL OSCILLATOR}}$

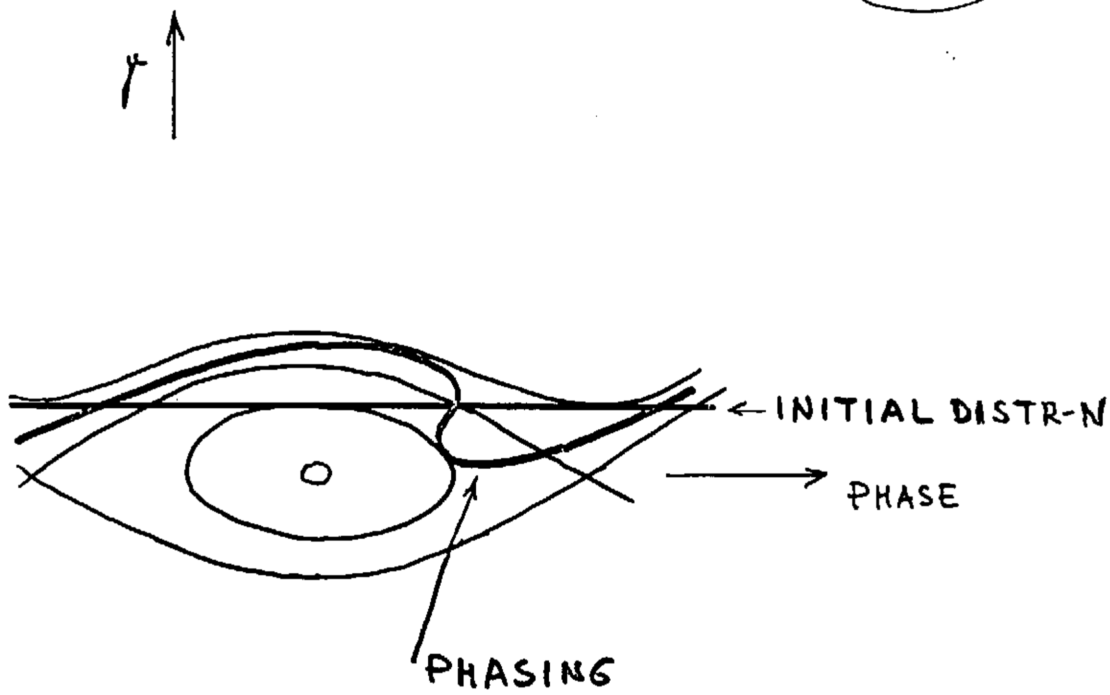
TR. 23

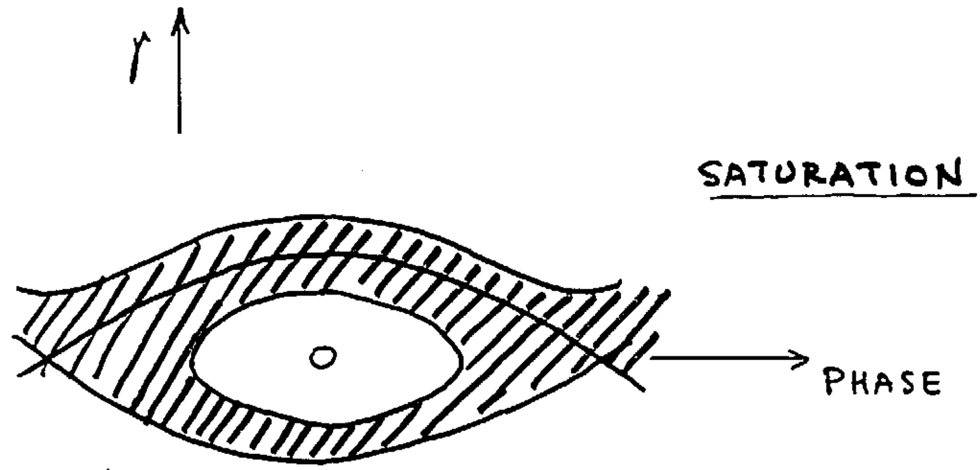


PHASE SPACE DISTRIBUTION OF INITIALLY MONOENERGETIC AND NON-PHASED BEAM DUE TO INTERACTION WITH A QUASI-SYNCHRONOUS WAVE

FOR NEGATIVE DETUNING ROTATE THE PICTURE 180°

Tr. 24





THE SAME FOR SMALLER DETUNING

NO PHASING FOR ZERO DETUNING } !
NO COHERENT RADIATION

negative ones lead to the wave absorbtion. Tr.25

Several words are to be said about high-current beams when the gain is not small. The main equations of phasing remain the same but have to be supplemented with an equation for a complex amplitude of the growing mode. Note that change in phase occurs now not only for particle dynamics reasons but due to z-dependence of the amplitude as well. One can find then again the induced bunching and, after averaging, to write down an integral equation for the amplitude variations along the system.This equation contains an initial detuning and a characteristic radiation length ζ_r as parameters. The latter is proportional to $I^{1/3}$ and depends on so called serial impedance R which is inversely proportional to the effective cross-section of the mode. The equation can be solved analytically showing the previous result if the beam current is small and the interaction distance is much smaller than the radiation length. For large gains (i.e. for large currents and/or large Tr.26
interaction lengths) the equation predicts an exponential spatial growth of the amplitude of modes with μ close to zero. Hence, this system without a feedback can be considered as an amplifier with a frequency band determined by the inversed radiation length. An input signal will be amplified exponentially with distance if its frequency lies in this band. The role of the input signal can be played by spontaneous radiation as well. As far as its spectral line is narrower than the radiation width in a large gain regime it is approximately kept constant along the system. This situation is called SASE – Stimulated Amplification of Spontaneous Emission. In the next figure one can see how fast are amplitude growth Tr.27
and beam phasing in this regime.

The main conclusions are listed in the next transparency and do not need additional Tr.28
comments but the last one. Up to now we neglected particle interaction by means of "near zone" (Coulomb) fields. They will obviously play an important role for large currents influencing, if not preventing, the particles phasing. One can foresee that the individual particle motion description hardly can be adequate then. These questions are worth of a separate discussion.

TR. 25

HIGH GAIN REGIME (LARGE CURRENTS)

IF FIELD CHANGES ESSENTIALLY WITH $\zeta = kz$

THE SYNCHR. MODE COMPLEX AMPLITUDE DEPENDS ON ζ

$$C(\zeta) = g(\zeta)\, e^{i\phi(z)}$$

$i\phi(z) \leftarrow$ TO BE INCLUDED IN PHASE SLIPPING (CHANGE IN β_p)

$\frac{dC}{d\zeta}$ IS DETERMINED BY PHASE BUNCHING

$$\frac{d\psi_i}{d\psi} = \int_0^{\zeta} \frac{g(\zeta')}{k}(\zeta-\zeta')\sin\left[\psi-(\gamma_i-\gamma_s)(\zeta-\zeta')\alpha+\phi'-\phi\right]d\zeta'$$

$\downarrow$ INTEGRAL EQN

$$\frac{dC}{d\zeta} = -\frac{i}{\zeta_r^3}\int_0^{\zeta} C(\zeta')(\zeta-\zeta')\cdot \exp\left[-i\alpha(\gamma_i-\gamma_s)(\zeta-\zeta')\right]d\zeta'$$

$$\zeta_r = \left(\frac{2I_0\ \alpha k^2\beta}{\pi I R \beta_p \beta_\perp^2 c}\right)^{1/3} \quad ; \quad I_0 = \frac{mc^3}{e} \simeq 17\,\mathrm{kA}$$

$$R = \frac{(\text{ACTING FIELD AMPLITUDE})^2}{\text{MODE POWER FLUX}}$$

TR. 26

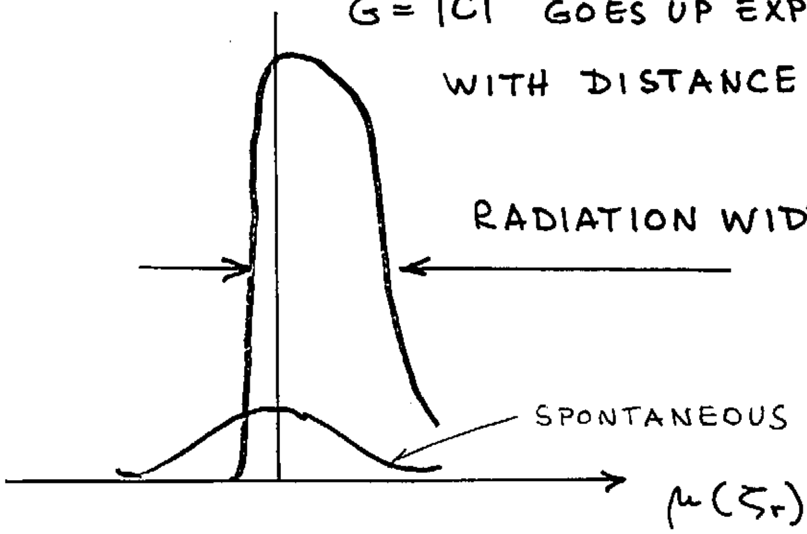
HIGH GAIN ASYMPTOTICS

FOR $C_i = 1$; $C_i' = C_i'' = 0$; $\gamma_i = \gamma_s$

$$C \simeq \frac{1}{3}\left[\exp\left(\frac{i\zeta}{\zeta_r}\right) + 2\exp\left(-\frac{i\zeta}{2\zeta_r}\right)\cosh\left(\frac{\sqrt{3}\,\zeta}{2\zeta_r}\right)\right]$$

$G = |C|^2$ GOES UP EXPONENTIALLY WITH DISTANCE AND $I^{1/3}$

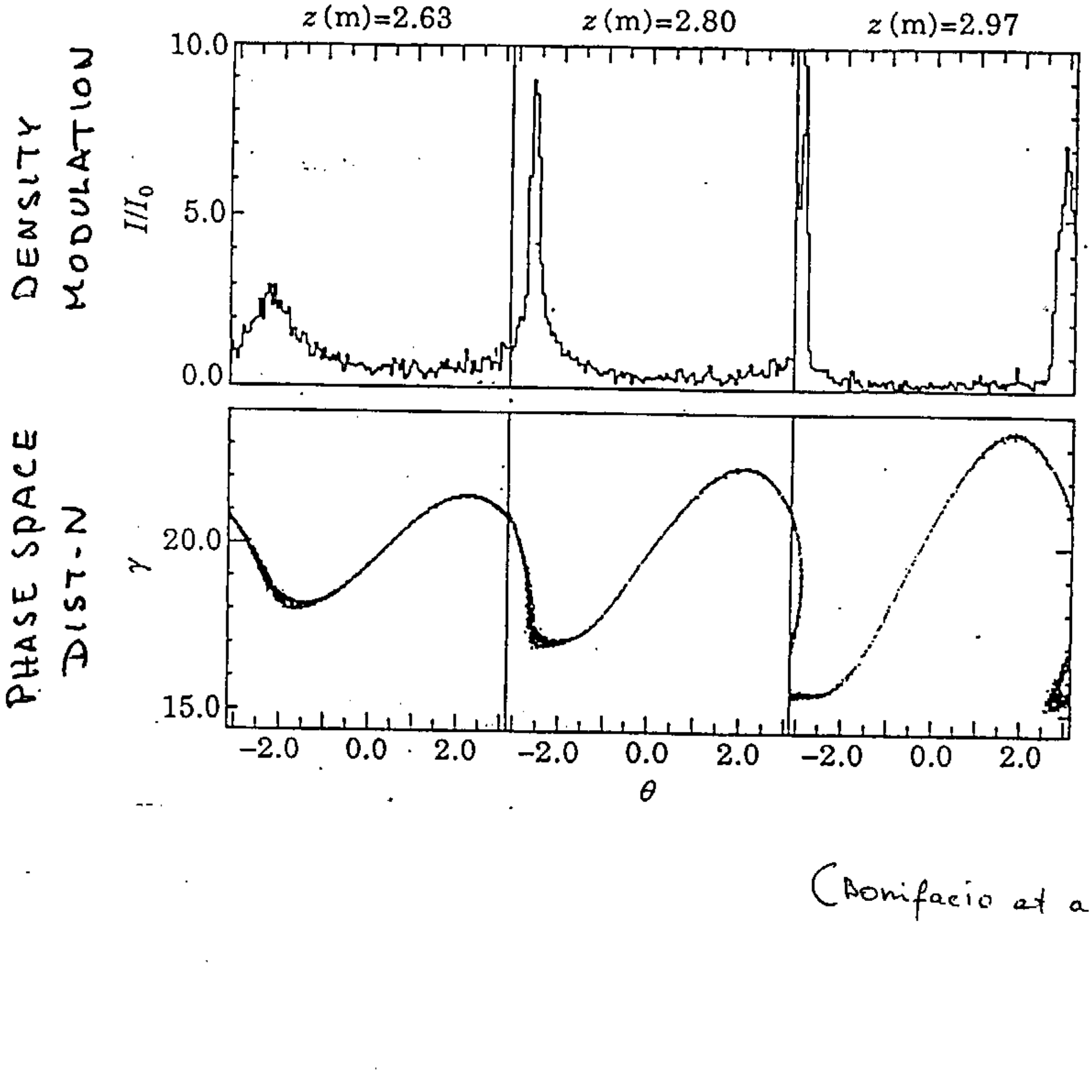
RADIATION WIDTH $\sim \zeta_r^{-1}$



THE BEAM BEHAVES LIKE ACTIVE (AMPLIF.) OPTICAL MEDIUM. IN ABSENCE OF AN INPUT SIGNAL A CERTAIN PART OF SPONTANEOUS SPECTRUM WILL BE AMPLIFIED (SASE). THE RESULTING RADIATION IS PARTLY COHERENT BECAUSE THE AMPLIFICATION IS SELECTIVE (MOSTLY ω_s)

TR 27

HIGH GAIN



(Bonifacio et al)

TR. 28

CONCLUSIONS

- TO RESTORE A THERMODYN. EQUIL. THE BEAM BEHAVES LIKE AN OPTICALLY ACTIVE MEDIUM REVEALING BANDS OF ABSORBTION AND ADDITIONAL <u>INDUCED</u> EMISSION
- IND. EM-ON MECHANISM IS BASED ON PARTICLES PHASING AND ON APPERANCE OF COHERENCY
- THE RATE OF RADIATION GROWTH DEPENDS ON THE SPONT. RAD.-N SPECTRUM PROFILE (SHOULD BE NARROW)
- THE MOST SUCCESSFUL MODE IS SLIGHTLY "SLOWER" THAN THE CENTRAL ONE OF THE SPONT. SPECTRUM AND CORRESPONDS TO ~2.6 PHASE SHIFT ALONG THE INTERACTION LENGTH L (SMALL GAIN REGIME)
- SMALL GAIN GOES UP AS $L^3 I$ (FOR FIXED PHASE SHIFT) REVEALIN INITIAL BEAM "LETARGY"
- SATURATION PREDICTED FOR $L \approx L_{SAT}$
- LARGE GAIN ASYPTOTICALLY GROWS UP AS $\exp(C \cdot z I^{1/3})$ UPTO L_{SAT}. LARGE AMPLIFICATION OF AN INPUT SIGNAL FOLLOWS (INCLUDING SASE)
- HOWEVER SHORT RANGE PARTICLES INTERACTION INCREASES WITH I

Lecture 3

Collective Radiation of Interacting Particles (Dense Beams)

General considerations. Up to now we have been considering particles interacting by means of radiation fields only. This interaction was shown to produce beam bunching in proper phases being a basic mechanism of the stimulated emission. This is correct for rarified beams only because for dense ones an interaction by means of short-range forces may take place and interfere the process. Tr.29

The problem can not be reduced to simple mutual electrostatic repelling. The relativistic particles will excite proper magnetic fields as well, not speaking about boundary conditions which can distort essentially the electrostatic fields. For example, the near zone field of a particle under condition of Cherenkov radiation looks very complicated even in the rest frame and hardly can be identified as a good old Coulomb force.

We better start with the fact that particles in a dense beam do not respond to an external field independently. One can not kick just one particle not influencing others. A dense beam has its own collective degrees of freedom very much like plasmas where collective Langmuire oscillations are known to be possible. The appearance of these collective degrees of freedom as space charge and space current oscillations is quite different from the individual particle motion. Waves on a water surface have nothing common with the motion of individual moleculas. Hence, a different approach should be used when particles are interacting. We should consider a beam rather as a flow of compressible charged fluid with its own collective beam modes. This is interaction of these modes with electromagnetic radiation ones which determines the collective emission.

Of course, fields of the collective modes do not satisfy the equation

$$\operatorname{rot}\vec{B} = -ik_0\vec{E};$$

because for them

$$\operatorname{div}\vec{E} = 4\pi\rho \neq 0$$

Hence, they can not propagate without space charge variations, always exist in the vicinity of the beam and influence the bunching process.

Dispersion relations. The common field being a superposition of electromagnetic waves and beam ones still has an exponential z and time dependence $\sim \exp[i(kz-\omega t)]$. This is the dispersion relation $k_0(k)$ or $k(k_0)$ which determines a behaviour of the systems. If for some k an eigenfrequency is complex, then the total field exponentially grows up with time (including the radiation field part). If for a real value of k_0 wavenumbers k have an imaginary part the total field grows in space along the beam starting from a point where an external signal of frequency k_0c is introduced. The picture is well know for everybody who has suffered from collective beam instabilities in accelerators. As a matter of fact, the induced radiation is always an appearance of a beam instability even for non-interacting particles; remember the bunching process in a proper radiation wave discussed above.

INTERACTING PARTICLES (DENSE BEAMS)

TR. 2

- PREVIOUS CONSIDERATIONS WERE BASED ON SINGLE PARTICLE + COMMON RADIATION FIELD CONCEPT NEGLECTING PART+PART INTERACTION VIA SHORT-RANGE "COULOMB" FORCES

- PARTICLES MOTION IS NOT INDEPENDENT → DIFFERENT INERTIA PROPERTIES AND COLLECTIVE DEGREES OF FREEDOM ARE TO BE INCLUDED → DIFFERENT APPROACH

- BACK TO DEFINITIONS: ARE MODES WITH $\mathrm{rot}\,\vec{E} = -ik_0\vec{B}$; $\mathrm{rot}\,\vec{B} = -ik_0\vec{E}$ THE ONLY ONES WHICH CAN PROPAGATE?

- $\mathrm{div}\,\vec{E} = 4\pi\rho \neq 0$ (SPACE CHARGE AND/OR CURRENT) WAVES MUST BE INVOLVED AS WELL IN PRESENCE OF PART

- ALTHOUGH NOT ABLE TO LEAVE THE BEAM THEY ALWAYS CONTAMINATE THE E.M. MODES IN BEAM VICINITY AND INFLUENCE BUNCHING (INDUCED EMISSION)

- <u>MEANS</u>: TO FIND DISP. RELATION FOR ALL EIGENMODES $\sim \exp i(kz - k_0 ct)$ INCLUDING „POTENTIAL" ONES

  <u>GOAL</u>: POSSIBILITIES FOR

  FIELD AMPLIFICATION ($\mathrm{Im}\, k < 0$)

  SELF-EXCITATION ($\mathrm{Im}\, k_0 > 0$)

  generally: COLLECTIVE RADIAT. INSTABILITIES (INDUCED RAD-N EFFECTS)

To find a required dispersion relation in a general case and to solve it is a rather complicated task. But considering radiation effects as perturbation, meaning that all instabilities lead to slow variations of amplitudes and phases, one can use again the quasi-synchronous interaction concept. Really, two slightly coupled waves may exchange their energy only if their frequencies and wavenumbers are close enough. Hence, in the dispersion diagram the beam waves and electromagnetic waves are represented by their own, or partial, dispersion curves $k_0(k)$ everywhere but in the vicinity of intersection points. Only there a weak interaction can split and rejoin the partial branches delivering unstable (complex) roots of the total dispersion relation. In this region the exact or normal eigenmodes are neither pure electromagnetic nor beam ones being hybrids of both.

This splitting can be of two kinds shown in the figure. In the first case all k_0 remain real, there is no temporal increase and only beating of the amplitudes takes place. In the second case a small "window" of instability exists meaning that an initial perturbation grows up exponentially; i.e. induced emission of radiation with well defined wavenumbers and frequencies (hence coherent!) appears. It is worth to note here that in the absence of dissipation all roots of the dispersion relation are complex conjugated. Therefore, both resonant absorbtion and stimulated emission are described in this manner, but have different fates. Those components of the initial perturbation which correspond to damping will die and be forgotten while those inducing emission will grow up.

Collective mode description requires, of course, an adequate model of the beam. In the simplest case it can be considered as a compressible charged fluid obeying a discontinuity equation and an equation of motion (Euler equation). The hydrodynamic velocity as a function of time and of coordinates is usually assumed then coinciding with the velocity of a particle located at the same point at the same moment of time. In other words the internal ("thermal") motion of particles is neglected, so this model is known as "cold hydrodynamical" one. Radiation energy then can be provided only for the account of the directed longitudinal motion. However, sometimes this internal invisible motion is of importance as a source of energy transferred to the radiation field. Then a phase-space description becomes necessary provided by a kinetic equation (usually a self-consistent Vlasov equation). Tr.~~29~~ 3c

Beam conductivity. To illustrate this approach let us consider the simplest model of an one-dimensional electron flow along z axis in a longitudinal uniform magnetic field B_0. Let a plane wave propagate the same direction. To have a freedom to vary its phase velocity a dielectric constant ε is assumed.

Dividing transverse and longitudinal components of the total electric field one gets immediately from Maxwell equations two separate linear equations relating the fields to corresponding components of the space current density $\vec{j}$. Note that the transverse components of the field have rights for existence even in the absence of the current if $k_0^2 = k^2/\varepsilon$, i.e., if the dispersion relation for electromagnetic waves is satisfied. The longitudinal component has no poles and, hence, can not propagate if $j_{||} = 0$. Tr.31

Now we need another relation between $\vec{j}$ and $\vec{E}$ which is to be provided by the equation of motion. For small fields exciting small currents inside the beam this relation should be linear. The tensor σ may be called a beam conductivity as far as the relation looks like the Ohm law. Poles of σ in the (k_0, k) plane (if any) indicate a possibility of currents propagating in a very rarified beam, i.e., represent a dispersion relation for beam waves in the low density limiting case. As was pointed above neither electromagnetic nor beam

TR. 30

COLLECTIVE DESCRIPTION

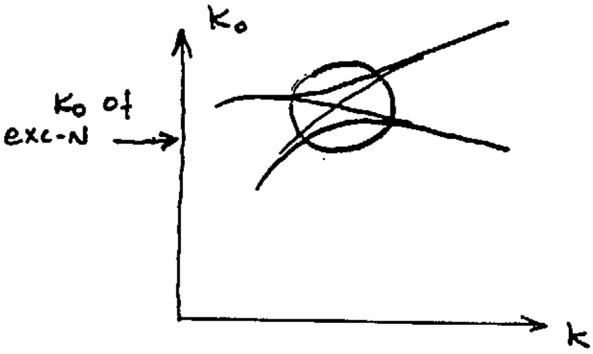
- NO INDIVIDUAL PARTICLES ANYMORE ; INSTEAD :
  - CHARGED LIQUID → HYDRODYNAMIC DESCRIPTION
  - CHARGED PHASE-SPACE DISTR-N – KINETIC APPROACH
- FORGET ABOUT SPONT. RADIATION
- NORMAL EIGENMODES PROBLEM TREATED AS INTERACTION BETWEEN "BEAM MODES" AND "E.M" MODES (PARTIAL MODES)

WAVE – WAVE SYNCHRONISM

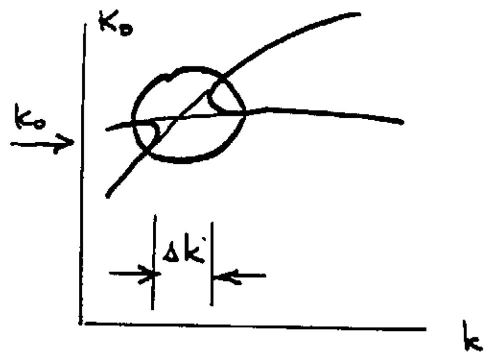
- PARTIAL MODES INTERACT VIA INDUCED PARTICLE MOTION IF THE LATTER IS QUASI-SYNCHR. FOR BOTH, I.E.

$$K_0^{(1)} \approx K_0^{(2)} \quad ; \quad k^{(1)} \approx k^{(2)}$$

- "CROSSING POINTS" ON A DISPERSION DIAGRAM ARE MOSTLY SUSPICIOUS : THE PARTIAL MODES ARE COHERENT THERE
- NEAR "CROSSING POINTS" THE NORMAL EIGENWAVES ARE HYBRID WAVES OF TWO TYPES (IF NOT DEGENERATED)



BEATING
(k REAL FOR BOTH)



EXPON. GROWTH AND/OR ABSORBTION
(Re k ≷ 0) ← for both

TR. 31

SELF-CONSISTENT EIGENMODES PROBLEM (1-D)

- MAXWELL EQNS FOR $\vec{E} = \vec{e}\,E_{\parallel} + \vec{E}_{\perp}$; $\vec{k} = k\vec{e}$

$$\vec{E}_{\perp} = \frac{4\pi i k_0}{c\,(k_0^2 - k^2)} \cdot \vec{j}_{\perp} \;; \quad E_{\parallel} = -\frac{4\pi i}{c k_0}\, j_{\parallel}$$

- TO CLOSE THE SYSTEM ONE NEEDS A LINEAR RELATION

$$\vec{j} = \overleftrightarrow{\sigma}(k_0 k)\,\vec{E}$$

LINEAR OPERATOR OF BEAM COMPLEX CONDUCTIVITY TO BE OBTAINED FROM THE EQN OF MOTION. IF KNOWN IT GIVES 3 HOMOGEN. ALGEBRAIC (?) EQNS WITH A COMPATIBILITY CONDITION (DISPERSION RELATION)

$$k_0 = k_0(k) \qquad \text{OR} \qquad k = k(k_0)$$

THEN LOOK FOR COMPLEX ROOTS

- IN ABSENCE OF DISSIPATION σ IS PURELY IMAGINARY FOR REAL k_0, k (CONSTANT AMPLITUDE ≡ NO WORK)

THUS: THE COMPLEX ROOTS APPEAR AND DISAPPEAR AS C.C. PAIRS

ABSORBTION AND INDUCED EMISSION ARE ALWAYS TOGETHER !

waves can independently propagate if the corresponding poles coincide in the (k_0, k) plane where a collective instability (or stimulated emission) can be expected. Tr.32

Assuming the equilibrium particles velocity βc and density ρ_0 independent of coordinates let us consider small wave-like perturbations of these values $\delta\vec{v}$ and ρ. They have to satisfy, firstly, a discontinuity equation, so that

$$\rho = \frac{k\rho_0\,\delta v_{||}}{c(k_0 - k\beta)}.$$

Secondly, the hydrodynamic equation of motion must be satisfied which equalizes the total time derivative of the momentum density with the Lorentz force density. Note that expressing $\vec{p}$ in terms of $\delta\vec{v}$ one should take into account an anisotropy of this linear relation due to a relativistic character of the directed motion.

Then, after some arithmetics, we get the sought expressions for the longitudinal and transverse conductivities which again are splitted. Both are proportional to the beam equilibrium density which plays a role of a coupling constant for the wave-beam interaction. Note that the transverse part is gyrotropic indicating a Hall effect in the magnetized beam.

So, in the low density limit the beam waves are of two different kinds:

- transverse ones conjugated with transverse currents (cyclotron waves with $k_0 = \beta k \pm k_c$);
- longitudinal ones conjugated with space charge density variations (space-charge waves with $k_0 \approx \beta k$).

Tr.33

Cyclotron waves. The cyclotron waves are waves of the transverse current density and of the transverse electric field not producing space charge density modulations and a longitudinal electric field. (Of course, if the beam is restricted transversally some surface charge density and corresponding $E_{||}$ appear). Being substituted into Maxwell equations the expressions obtained above give the dispersion relation which splits in two cubic equations corresponding to left/right circular polarization. Solving these equations and omitting certain pecularities connected with the mentioned double degeneracy one comes to an important conclusion. At the point of intersection of the fast cyclotron wave with the electromagnetic one the system remains conditionally stable. The same is true in a case of propagation along a waveguide with a finite cut-off frequency when two points of intersection may exist as shown in the fugure. This hydrodynamic stability could be expected because no transverse equilibrium motion is assumed in our beam model. For a normal Doppler effect (the waves are fast!) all transverse oscillations must damp due to emission. So if they do not exist initially there is no source of energy to supply a growing wave. For a beam posessing some "internal" energy the situation is different, so we better put at the moment a question sign at these suspicious points.

Slow waves which can exist for $\beta^2\varepsilon > 1$, i.e., in the Cherenkov radiation domain show remarkably different behaviour. There is a band of instability where an eigenwave grows up exponentially. In the middle of the band the frequency is

$$k_0 c \approx \frac{k_c c}{\beta\sqrt{\varepsilon} - 1} \pm i\frac{k_p c}{2\sqrt{\varepsilon}}\sqrt{\beta\sqrt{\varepsilon} - 1}\,.$$

The increment of the instability is proportional to the Langmuire frequency and, hence, grows with the beam current as $I^{1/2}$.

TR. 32

HYDRODYNAMICS

- ALL PARTICLES HAVE THE SAME VELOCITY $\vec{v}(\vec{r},t)$

  SP. CHARGE DENSITY $\rho = \rho_0 + \rho \cdot \exp i(kz - k_0 ct)$

  CURRENT DENSITY $\vec{j} = \rho_0 \beta c \vec{e} + \vec{j} \cdot \exp \ldots$

  EXT. FIELD (if any) $\vec{B_0} = B_0 \vec{e}$

- EQNS OF MOTION (relativistic hydrod. momentum $\vec{p}(\vec{r},t)$)

$$\underbrace{\frac{\partial}{\partial t}\vec{p} + (\vec{v}\vec{\nabla})\vec{p} = q\vec{E} + \frac{q}{c}[\vec{v}\vec{B}] \;;\quad \frac{\partial\rho}{\partial t} + \mathrm{div}(\vec{v}\rho) = 0}$$

$$\left[\begin{array}{l} \dfrac{4\pi}{c} j_{\parallel} = \dfrac{i k_p^2 k_0}{\gamma^2 (k_0 - \beta k)^2} E_{\parallel} \\[2ex] \dfrac{4\pi}{c}\vec{j}_{\perp} = -\dfrac{k_p^2 (k_0 - \beta k)}{k_0 [k_c^2 - (k_0 - \beta k)^2]} \left(i\vec{E}_{\perp}(k_0 - \beta k) + k_c [\vec{e}\vec{E}_{\perp}] \right) \end{array}\right.$$

SOLUTION FOR PLANE WAVES

$$k_p^2 = \frac{4\pi \rho_0 q}{m\gamma c^2} \sim (\text{LANGMUIRE FREQ})^2$$

$$k_c^2 = \left(\frac{q B_0}{m\gamma c}\right)^2 = \Omega^2/c^2 (\text{CYCLOTR. FREQ})^2$$

NOTE: RESONANT DENOMINATORS ARE ZERO FOR WAVE – SINGLE PART. SYNCHRONISM CONDITIONS

- THIS GIVES BEAM CONDUCTIVITY TENSOR

  ↓

  ANISOTROPIC AND GYROTROPIC

  (HALL EFFECT)

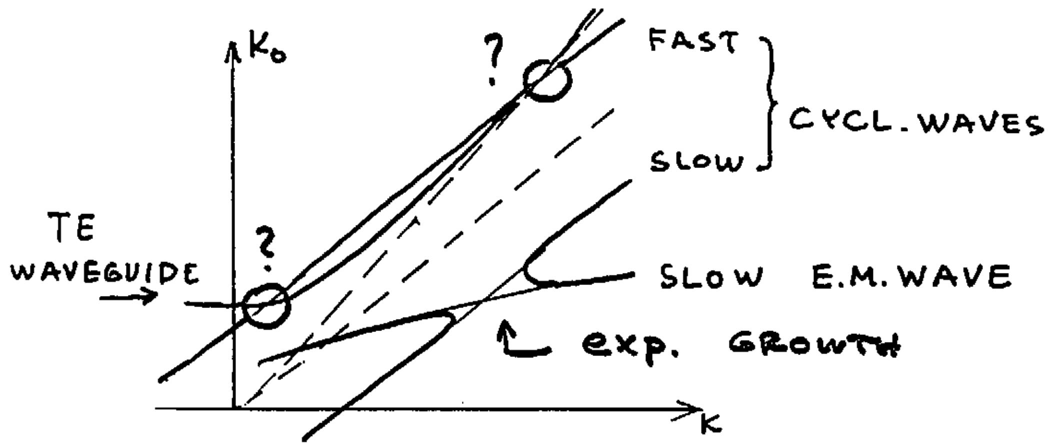
TR. 33

CYCLOTRON WAVES (TRANSVERSAL)

DISPERSION EQN

$$(k_0 - \beta k \pm k_c)(k_0^2 - k^2) = k_p^2 \qquad (6 \text{ ROOTS})$$

- WAVES OF TRANSVERSE CURRENT
- NO SPACE CHARGE DENSITY EXCEPT BEAM BOUNDARY
- INTERACT WITH TRANSVERSE E.M. WAVES (IMPORTANT FOR SHORT-WAVELENGTH CASE OF THE FAST WAVE)
- DOUBLE DEGENERATION $\wedge$ ($\pm$ CIRCULAR POLARIZATION)



IN A SLOW-WAVE STRUCTURE SCW IS UNSTABLE, I.E, EXPONENTIALLY GROWING RADIATION TAKES PLACE

FAST WAVE IS STABLE HYDRODYNAMICALLY (NO INTERNAL TRANSVERSE MOTION INCLUDED)

Space-charge waves. In the model under consideration the space-charge waves are longitudinal charge density waves associated with a longitudinal electric field only. Of course this is valid for an unrestricted uniform flow only. Transverse magnetic fields and longitudinal electric field will appear at a beam boundary but in the first approximation can be ignored close to the beam axis. The dispersion relation for space-charge waves is rather simple and follows immediately from the separated longitudinal components of the wave equation and the equation of motion Tr.34

$$k_0 = \beta k \pm k_p^*$$

where $k_p^* c$ is a "heavy" plasma frequency containing the additional factor γ^{-1} coming from the relativistic character of the longitudinal motion. The existence of fast and slow space-charge waves is quite obvious in the rest frame of the beam. Here they are just two plasma waves propagating along z in opposite directions with fixed plasma frequency independent of k. The forward directed plasma wave appears as the fast space-charge wave in the lab system while the backward directed one gives the slow wave.

Regretfully, in our model no interaction is possible between space charge waves and plane electromagnetic ones as far as the latter have no longitudinal electric component in the 1-D case. However, the calculations can be easily repeated for a beam filling uniformly a regular waveguide where TM electromagnetic modes with a longitudinal electric field may propagate with a phase velocity controlled by ε. The calculations are too cumbersome to be reproduced here but the result is simple and transparent: the fast space-charge wave remains stable and the slow one is a subject of instability. The only complication, as compared with the cyclotron waves, follows from the smallness of frequency splitting between the fast and slow waves which has the same order of magnitude as the coupling constant ck_p has. As a result, the interaction between three waves takes place both space-charge waves being taken into acount. The corresponding instability increment is then proportional to $I^{1/3}$.

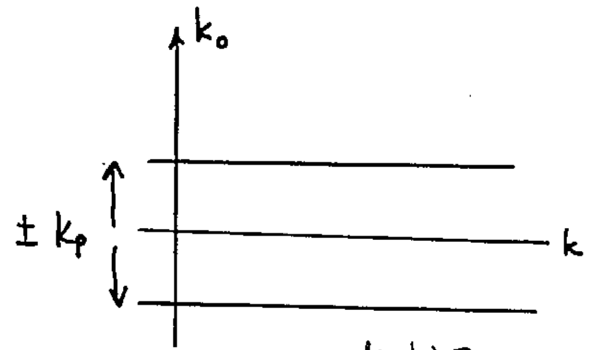
Of course, our "local" dispersion relations valid in the vicinity of the points of intersection only are determined by general topology of partial dispersion curves and should not depend qualitatively on a particular electrodynamic system. This is why we expressed the increment in terms of local characteristics of the electromagnetic wave, i.e. in terms of its phase and group velocities. This is to be stressed because the instability of the slow space-charge wave plays a specific role being a basic physical mechanism for operation of a travelling wave tube – a grandmother of all RF amplifiers with distributed parameters. In practice, of course, the required retardation of the electromagnetic wave is provided with periodic structures rather then with the dielectric constant as above.

Negative energy waves. Let us now discuss briefly a remarkable feature of the slow beam waves (both cyclotron and space-charge ones) – their intrinsic capability for self-excitation. The mechanism is almost clear for a cyclotron wave which is a macroscopic result of coherent rotations of particles around the external magnetic field lines. The slow wave interaction with an electromagnetic wave can take place only under condition of the anomalous Doppler effect when an oscillator (or rotator) gains transverse energy as a result of emission. Gaining energy it radiates more and the radiation field is self amplified. Tr.35

LONGITUDINAL SPACE-CHARGE WAVES

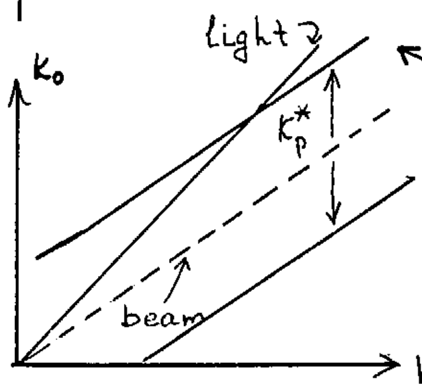
$$\boxed{k_0 = \beta k \pm k_p^*}$$

$k_p^* = \left(\dfrac{4\pi \rho_0 q}{mc^2 \gamma^3} \right)^{1/2}$ – ("HEAVY") PLASMA FREQUENCY



- REST FRAME:

TWO PLASMA WAVES PROPAGATING IN OPPOSITE DIRECTIONS WITH $k_0 = k_p$ (INDEPENDENT OF k)

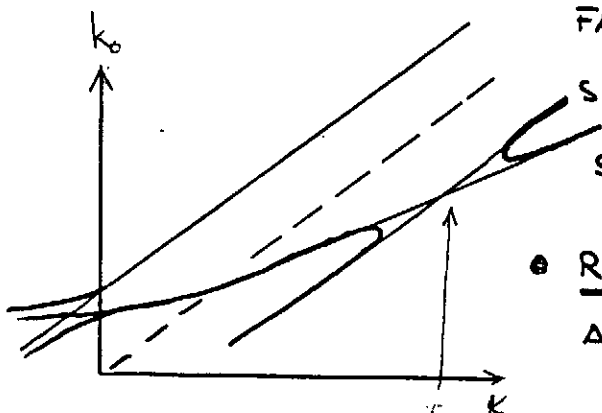


- LAB. FRAME

FAST } SPACE-CHARGE LONG. WAVES
SLOW }

NO INTERACTION WITH VACUUM E.M. WAVE (LACK OF LONG. FIELD).

- IN A RETARDING SYSTEM:



FAST } SP.-CH. WAVES
SLOW }

SLOW E.M. WAVE WITH $E_{\parallel}$

- RAD. INSTABILITY IN A SLOW-WAVE SYSTEM

$$\Im m\, k = \pm \frac{\sqrt{3}}{2} \left(\frac{k_p^{*2} k (\beta_p - \beta_g)}{2\beta_p^2 \beta_g} \right)^{1/3} \sim I^{1/3}$$

β_g – group velocity at the cross. point

TR. 35

NEGATIVE ENERGY WAVES

- SLOW BEAM WAVES ARE UNSTABLE (BOTH CW AND S-CH.W.) IF THEY HAVE AN E.M. SYNCHRONOUS PARTNER CARRYING ENERGY OUT (RADIATING)
- ALMOST EVIDENT FOR CYCLOTRON WAVES: RADIATING OSC GAINS AMPLITUDE AT THE ACCOUNT OF LONG-L MOTION UNDER CONDITION OF ANOMALOUS DOPPLER EFF.
- FOR LONG. WAVES: VELOCITY IS IN ANTIPHASE WITH DENSITY

$$\tilde{\rho} = \frac{k \rho_0 \tilde{v}_{\parallel}}{c(k_0 - \beta k)} \qquad \text{(FROM DISCON-Y EQN)}$$

MECHANICAL ENERGY OF A SLOW WAVE IS

$$\Delta W \propto \Delta(\gamma(v) \cdot \rho)$$

$$\langle \Delta W \rangle = \gamma^3 \rho_0 \frac{|v_{\parallel}|^2}{c^2} \left\{ \gamma^2 (1 + 2\beta^2) + \frac{k\beta}{k_0 - k\beta} \right\}$$

NEGATIVE IF $k_0 \lesssim k\beta$

- NEGATIVE ENERGY WAVES ARE SELF-EXCITED IF CAPABLE TO SPEND ENERGY, E.G.,
  - RADIATE VIA E.M. PARTNERS
  - „RADIATE" BY MEANS OF OHMIC LOSSES IN WALLS
- THIS IS A RESISTIVE INSTABILITY !
- "RESISTIVE WALL" OSCILLATOR AND/OR AMPLIFIER IS PREDICTABLE

For longitudinal space-charge waves the mechanism is less transparent. From the discontinuity equation it folows that the density oscillations are always in phase or in antiphase with the velocity oscillations depending on the unequality

$$k_0 > k\beta \quad \text{or} \quad k_0 < k\beta.$$

One can calculate the mechanical energy of the oscillations as a difference between the beam energies in the excited state and in the equilibrium. Close to the resonance it appears to be negative if $k_0 < k\beta$! The reason for this is that the beam is rarified at points of maximal velocity being compressed at the points of minimal velocity.

For this reason the slow beam waves are called sometimes negative energy ones meaning that the energy of the system decreases when the wave is excited. The situation is obviously instable: a negative energy wave just expects a positive energy partner to transfer its energy and to be self-excited. This transfer really takes place when interacting with an electromagnetic mode: both grow consuming energy from the longitudinal particles motion. More than that: if the beam propagates in a non-ideal electrodynamic structure where ohmic losses exist the self-excitation of the negative energy waves is possible regardless of the synchronism condition. Unfortunately, this is not a discovery – the fenomenon is a basis of the well known resistive wake-field instabilities in accelerators. Up to my knowledge, even a travelling wave resistive amplifier was tested upon the time. Its characteristics were far from ideal ones, but physically it could look as simple as just a beam propagating along a resistive conductor.

Kinetic effects. Let us come back now to fast cyclotron waves predicted to be stable hydrodynamically. One can expect that this stability exists only in the absence of internal rotational motion as a source of energy to be lost for radiation. This suspicion is enforced by the single particle theory result predicting induced radiation in the system of preliminary excited transverse oscillators. Tr.36

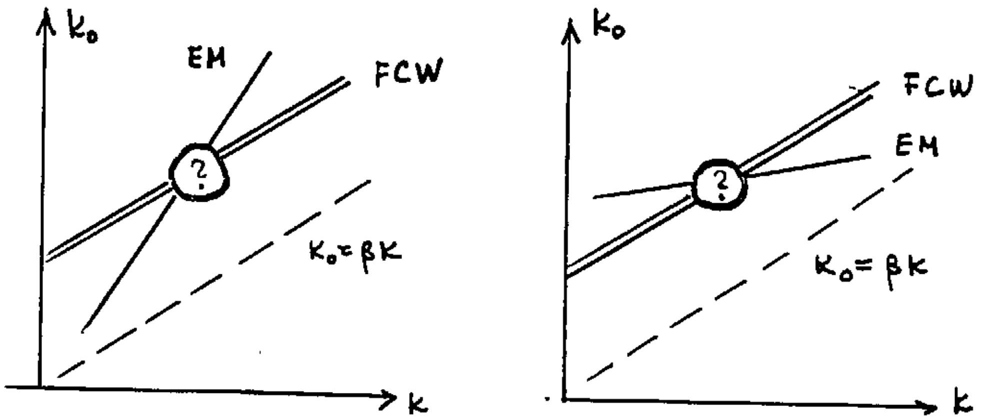
As was mentioned above the cyclotron waves are double degenerated in the absence of internal motion. If the latter is presented the corresponding branch is splitted because the beam consisting of rotating particles can not be indifferent to a direction of circular polarization. As a result, "conditionally stable" intersection points are really 3-wave intersections and provide two complex roots of the dispersion relation. A qualitative character of the normal modes dispersion is shown in the figure.

To get a less formal explanation of the instability one can have a look at the microscopic rotational motion. Having an angular momentum each particle rotates around the magnetic field force line with the Larmor frequency which is inversely proportional to its total energy. If the rotations are non-coherent, i.e. have random phases, they do not reveal a macroscopic motion being averaged. However, any phasing (preliminary and/or induced) excites rotating coherent currents interacting with a circularly polarized transverse electric field rotating in the proper direction. Tr. 37

Just one step remains to explain the instability. The required bunching is provided again by the autophasing mechanism applied now to the rotational motion. If the rotational velocity is in phase with the transverse electric field a particle gains energy and, therefore, looses rotational frequency. A particle in antiphase gains the frequency. So, they are tending towards accelerating phase (i.e., to absorbtion) or decelerating phase (i.e., to induced emission) depending on the rotational frequency of the transverse electric

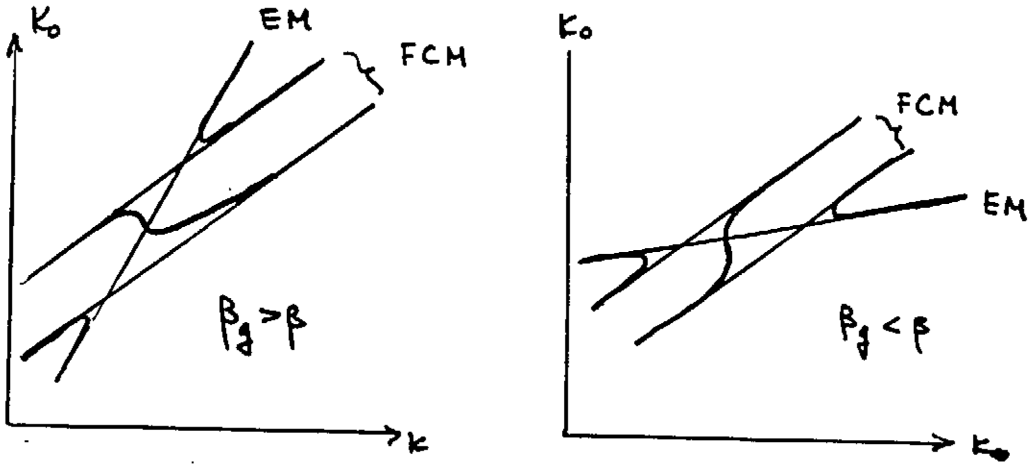
TR. 36

FAST CYCLOTR. WAVE SELF-AMPLIFICATION (KINETICS)



- NO INDUCED RADIATION IN HYDRODYNAMICS BECAUSE THE PARTICLES HAVE NOTHING TO RADIATE UNDER NORMAL DOPPLER EFFECT COND-NS (NO ROTATION). WHAT ABOUT SASE ?

MAGNIFIED ? :



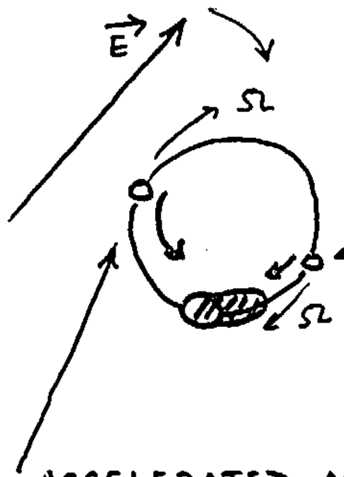
- IN PRESENCE OF INTERN. ROTATIONAL ENERGY DEGENERATION DISAPPEARS, COMPLEX ROOTS EXIST, AND SASE TAKES PLACE IN ACCORDANCE WITH SINGLE PART. THEORY.

TR. 37

ROTATION SELF PHASING

$$\Omega = \frac{q B_0}{m c \gamma}$$

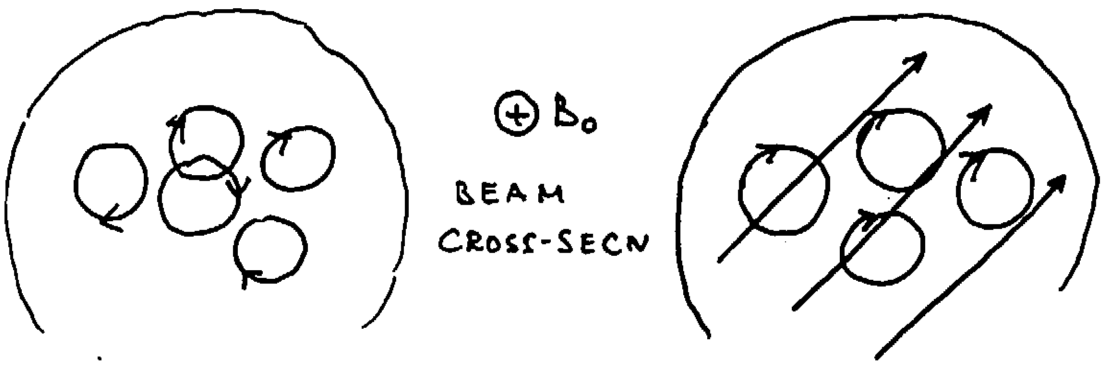
$\Omega \downarrow$ WITH $\gamma \uparrow$



DECELERATED AND PHASE SHIFTED FORWARD

ACCELERATED AND PHASE SHIFTED BACK

INDUCED MACROSCOPIC CURRENT



NON-COHERENT ROTATION

NO NET CURRENT

PHASING DUE TO DEPENDENCE $\Omega(\gamma) \rightarrow$ TRANSVERSE ROTATING CURRENT field. We came to the same conclusion as for directed beams: some quasy-resonant waves will be absorbed and some self-amplified. The only difference is that the cyclotron angular velocity is inversely proportional to energy while a linear velocity of a particle increases with γ. In reality both dependencies should be considered bearing in mind the particles longitudinal motion as well, but this would not change the main result – the appearance of stimulated emission driven by the rotational internal motion.

This mechanism of the stimulated emission of cyclotron radiation is a basis of so called cyclotron resonance masers (CRM) including the well known gyrotrons.

Lecture 4

Schemes of Realization

General requirements. The physical principles discussed above provide a general approach to realization of wave-beam interaction to produce high power and high frequency electromagnetic radiation. First of all, an electromagnetic system is to be chosen where an Tr.38
individual electron can radiate a quasi-monochromatic electromagnetic wave in a desired spectral band where the condition of synchronism is fullfilled. Note that to get radiation in a short-wave region relativistic electrons must be used unless very fine delicate structures are employed with caracteristic dimensions of order of $\lambda\gamma$ which are obviously nonpractical under high power and high current conditions.

If the beam-wave interaction occures under conditions of the normal Doppler effect some precautions should be made to excite transverse motion in the beam. This can be done either at injection, as in the case of a cyclotron resonance masers, or with additional transverse forces using, for example, an undulator system. In the case of the anomalous Doppler effect the beam can be initially cold, the transverse oscillations being self-excited. Note that in the first case higher frequencies of radiation are available.

The beam current determines the e-fold radiation length for stimulated emission. If the available current is small the radiation length can exceed any reasonable distance L of interaction, so that the system may operate as an oscillator only. Then the single pass gain parameter proportional to $L^3 I$ is essential. To convert the system to an oscillator an effective feedback must be provided by an external cavity or by a negative group velocity component of the wave. In the first case the cavity is to be longer than L and will be excited at a very high standing mode. The second way can be used only for comparatively long waves which might be controlled with external electrodynamic surroundings. The short wavelength waves are almost free and have a positive group velocity close to c.

As a matter of fact, one does not need a large gain to get a self-excited system. The gain must just overcome inevitable losses at the cavity mirrors and diffraction losses including the output power. These losses determine a certain starting beam current. With currents exceeding this value the electromagnetic field increases exponentially in time with an increment depending on the current and on the cavity Q-factor. Then it comes to a steady-state level determined by saturation when a particle makes about 1/4 of a synchrotron oscillation in the excited wave along the system. Note that enlengthening the system helps to decrease the starting current but decreases the steady state power for fixed current. Besides, the transient time increases with the length of the cavity requiring a larger duration of the current pulse (10 meters of the cavity length require at least tens of microseconds to be excited).

Prospects of the high gain regime (if available) are much more attractive. The system serves then as an amplifier and does not need an optical feedback. The last is essential for high power levels when radiation damage of mirrors might occure and/or for very large frequencies (UV and X-rays) where effective reflection is hardly possible. The output power level is limited only by saturation which comes at pretty high fields. In a way, the saturation process with its induced spread in electrons energy is equivalent to beam "heating". But, unlike solid state of gas lasers where overheating of the working medium may lead to diastrous effects, the electron beam is pumped out with the velocity of light. Of course, the overall efficiency of a high power amplifier becomes a very essential

HOW TO INVENT A NEW FEL

01. FIND A SHARP SPECTR. LINE OF SPONTANEOUS RADIATION
02. USE DOPPL. EFF. FOR SHORT WAVES ($\gamma \gg 1$)
03. EVALUATE BEAM MODES QUALITATIVE BEHAVIOUR
04. SELECT AN E.D. STRUCTURE TO EXCITE (FREE SPACE FOR SHORT W.)
05. LOOK AT "CROSSING" POINTS FOR IND. RADIATION CHARACTERISTICS
06 IF $\beta_p > \beta$ PROVIDE INTERNAL ENERGY OF OSC-NS TO BE SPENT FOR RADIATION. REMEMBER: YOU ARE LOOSING DOPPL. EFFECT
07 IF $\beta_p < \beta$ — DON'T BOTHER: YOU EMPLOY LONG. ENERGY
08. CHECK YOUR NET GAIN $\sim IL^3$: (DO YOUR BEST TO SAVE EFF-CY)
    IF < 1 GOTO 01
    IF $\gtrsim 1$ GOTO 09
    IF $\gg 1$ GOTO 10
09. USE FEEDBACK (OPTICAL CAVITY); YOU STILL CAN AFFORD A FEL OSCILLATOR. BE SHURE THE CURRENT PULSE IS LONG ENOUGH TO EXCITE THE CAVITY
10. YOU ARE MAKING AN AMPLIFIER. TAKE $L \gg L_r \sim I^{-1/3}$
11. DON'T BE GREEDY: ESTIMATE SATURATION LENGTH. INCREASE IN CURRENT WIDENS FREQ. BAND AND LEADS TO ADDITIONAL INSTABILITIES AND PARASITIC MODES
12 CHECK WHETHER YOUR FEL HAS ALREADY BEEN SUGGESTED (THIS POINT CAN BE OMITTED)
13. SEND YOUR PAPER TO THE NEXT FEL CONFERENCE AND APPLY FOR FUNDING.

parameter.

FEL family. From a viewpoint of general principles of the induced emission all electron beam systems might be called "free electron" lasers or masers meaning that the radiating electrons are not bound in atoms or crystall lattices. However, the name is usually kept for short wavelength relativistic beams devices where Doppler effect is employed and the radiation fields are very close to those of well directed plane waves. Tr.39

Nevertheless, it is worth to stress common features of all beam systems with distributed parameters. According to different types of the synchronism condition they may belong to two classes: those based on the Cherenkov mechanism or employing Doppler shifted radiation from a moving oscillator. As was discussed above the Cherenkov mechanism requires slow waves, i.e., retarding electrodynamic structures are to be used with characteristic dimensions of order of magnitude of the wavelength. Comparatively low frequency waves may be generated and/or amplified in this way. Variations of the structures are really countless. The most common ones are iris-loaded or spiral waveguides or a chain of cavities (klystrons). For short waves generation dielectric coatings or metallic gratings (Smith-Purcell effect) can be used.

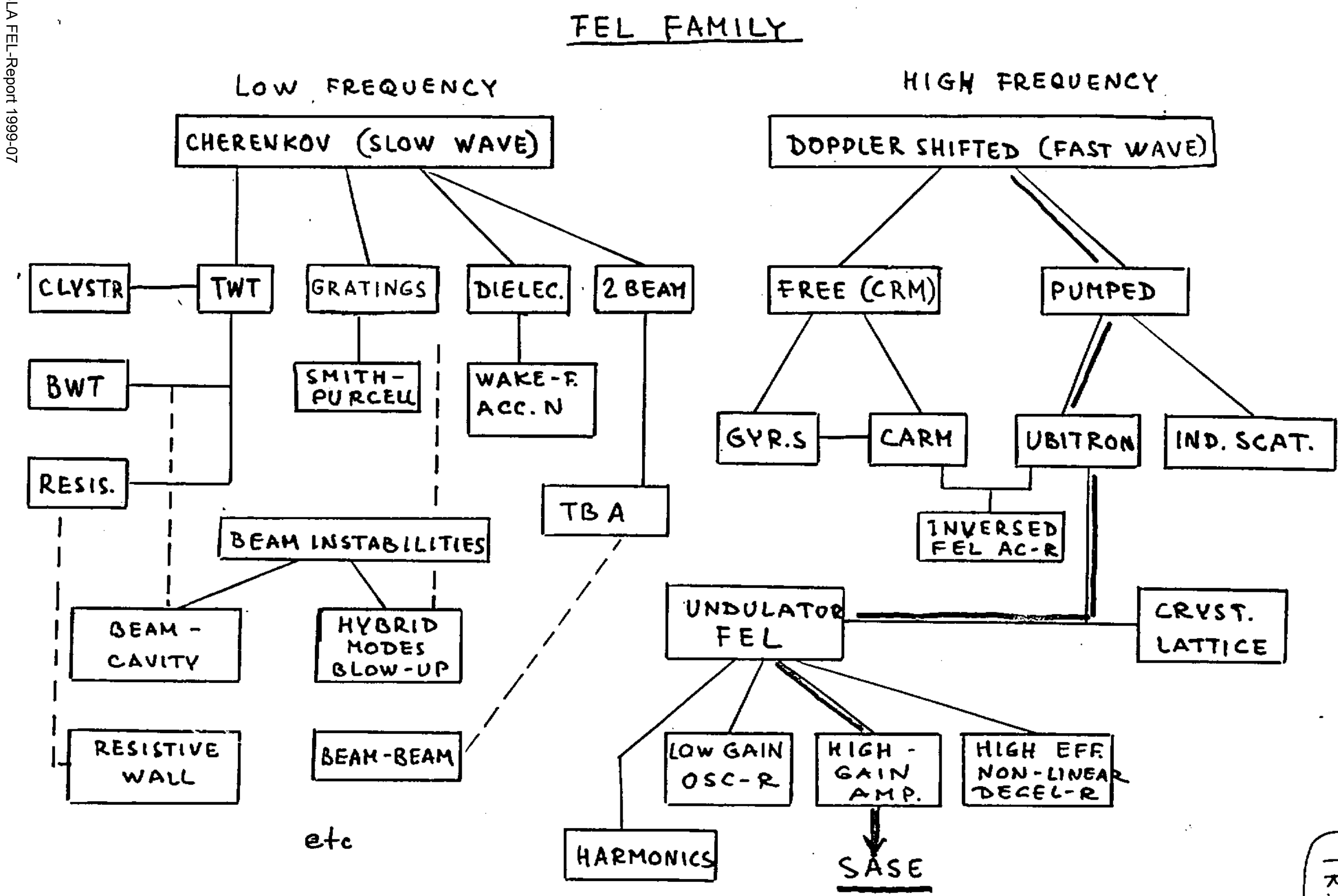
By the way, the Cherenkov radiation fields lagging behind the radiating particle are easily recognizable as wake fields. So modern schemes of wake field acceleration are really very close relatives to the Cherenkov type devices. One may include even two-beam acceleration in this family where one beam serves as a retarding medium for another beam Cherenkov radiation.

In this connection it is worth to remind that the stimulated emission mechanism is actually a mechanism of a beam instability. It easily reveals in situations where one needs a steady beam propagation, i.e., in accelerators. Here collective instabilities are to be avoided, but the physics is common. Certain relations between collective instabilities and mechanisms of induced emission are marked in the figure with dashed lines.

The second class is more suitable for high frequency radiation when the electromagnetic waves are free or almost free. There is no retarding system and fast beam waves are used. Transverse particle oscillations must be excited to be a source of the radiation energy. This can be done either at injection in a longitudinal magnetic field (cyclotron resonance masers) or by pumping them in the interaction region. The latter is actually employed in undulator systems. The stimulated emission of undulator radiation was suggesred by Phillips in early 60-s for nonrelativistic beams (ubitron). Being extended to relativistic beams with a strong Doppler shift this principle became a basis for modern free electron lasers. Their features will be considered separately. At this moment I would like just to mark in the figure the logic way from the general principles to Stimulated Amplification of Spontaneous Emission with the double line and to return to the Cherenkov type devices.

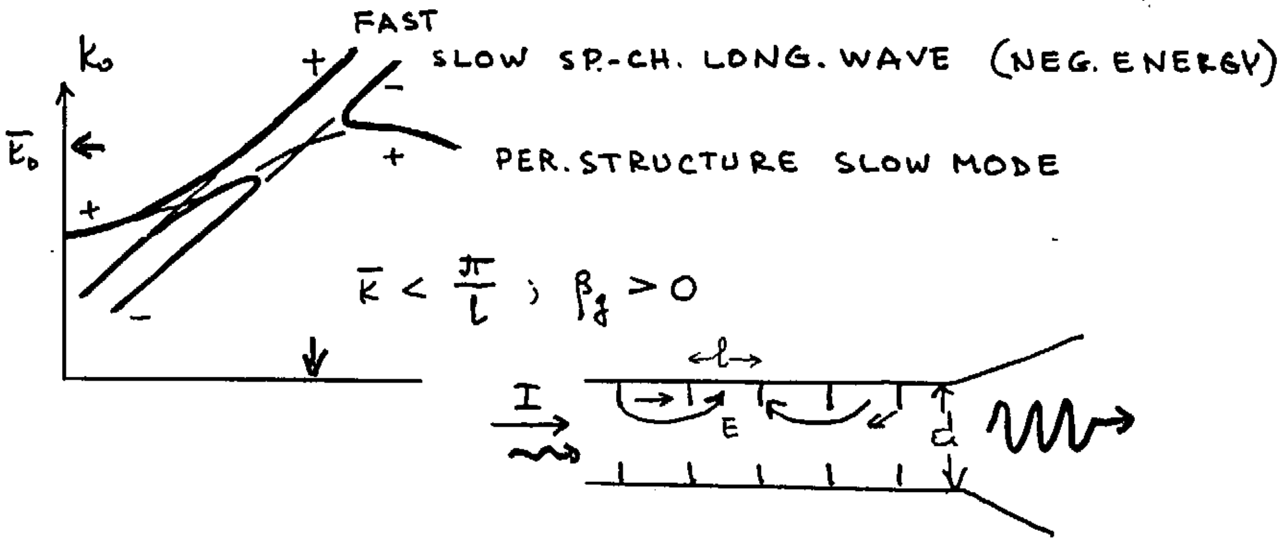
Travelling wave tubes and others. In a travelling wave tube a longitudinal instability of the space charge slow wave is employed as shown in the figure. To get a retarded electromagnetic wave with a longitudinal electric field component a periodically structured waveguide is commonly used and operation takes place in the first transparency band well separated from the others. Hence, the operational frequency is close to the cut-off frequency of the waveguide. Tr.~~38~~ 40

TR.39


FEL FAMILY
LOW FREQUENCY
HIGH FREQUENCY
CHERENKOV (SLOW WAVE)
DOPPLER SHIFTED (FAST WAVE)
CLYSTR
TWT
GRATINGS
DIELEC.
2 BEAM
FREE (CRM)
PUMPED
BWT
SMITH-PURCELL
WAKE-F. ACC.N
GYR.S
CARM
UBITRON
IND. SCAT.
RESIS.
TBA
INVERSED FEL AC-R
BEAM INSTABILITIES
UNDULATOR FEL
CRYST. LATTICE
BEAM-CAVITY
HYBRID MODES BLOW-UP
RESISTIVE WALL
BEAM-BEAM
LOW GAIN OSC-R
HIGH-GAIN AMP.
HIGH EFF. NON-LINEAR DECEL-R
HARMONICS
SASE
etc

TR 40

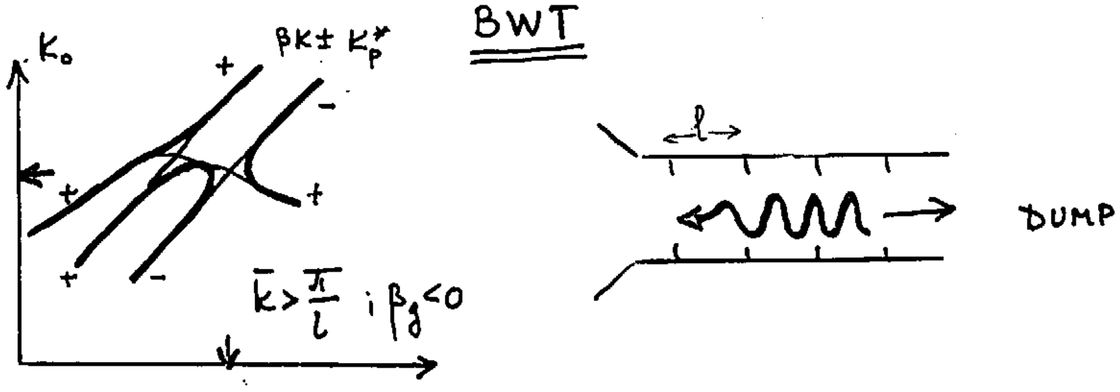
TWT



$\bar{k} < \frac{\pi}{l}$; $\beta_g > 0$

- HIGH GAIN, HIGH EFF-CY FOR $\lambda \gtrsim 1$ cm
- LOW FREQ-CY (LIMITED BY PERIOD AVAILABLE)
- POWER $\propto d^2 \propto l^2$ FOR FIXED CURRENT DENS-Y
- $j \propto (\gamma^{2/3}-1)^{3/2}$; $G \propto k_p^{2/3} \propto \gamma^{-2/3}$ (γ does not help)

BWT



$\bar{k} > \frac{\pi}{l}$; $\beta_g < 0$

- DISTRIBUTED FEEDBACK → OSCILLATOR
- SIGNAL LEVEL, EFFIC-CY etc DETERMINED BY POWER BALANCE AND/OR BY SATURATION
- LIMITED TUNABILITY

A travelling wave tube provides rather high gain and high efficiency for centimeter long waves and serves usually as an amplifier. Its intrinsic disadvantage is common for all Cherenkov type devices. To increase the operational frequency one has to decrease proportionally not only the period of the structure but all transverse dimensions as well. The cross section drops as λ^2 so does the total current for fixed current density determined by the Child-Langmuire law. Increasing voltage increases the available current density but one can not go this way too far. The second (again common) disadvantage is a limited TWT tunability.

As a high-gain amplifier a travelling wave tube can be easily converted to an oscillator with a kind of feedback. Usually it is done by selecting the operational point of the dispersion diagram to make the group velocity of the wave negative. Then the power flux is directed oppositely to the beam providing the desired distributed feedback. For currents exceeding a certain starting value this backward wave tube operates as an oscillator.

As a variation of the general scheme one should mention a grating used instead of a periodic waveguide. Originally the idea was published in a short note by Smith and Purcell who suggested an elegant interpretation of the effect. Really, a particle passing across the conducting grating will be accompanied by an electrostatic image which jumps transversally when passing through a surface step. The jups of the image may be considered as a source of radiation. One can hardly apply these arguments to relativistic particles but the Smith-Purcell effect remains in this case as well. It can be treated as Cherenkov emission of a slow surface wave which propagates along the surface being concentrated in its vicinity. Reflection from a concave mirror placed upon the grating and the stimulated emission mechanism do the rest converting the system in an orotron – a nice compact oscillator of millimeter long waves. Tr.~~39~~ 41

To finish with the Cherenkov mechanism one should mention, perhaps, two-beam schemes which do not need a retarding electromagnetic structure. This is interaction of two pairs of space-charge waves (negative and positive energy ones) which produces the radiation. The scheme is elegant physically but is useless for high frequencies well above the plasma frequency of the beams. In revenge, this type of interaction is a basis of numerous beam-beam instabilities well known in accelerators.

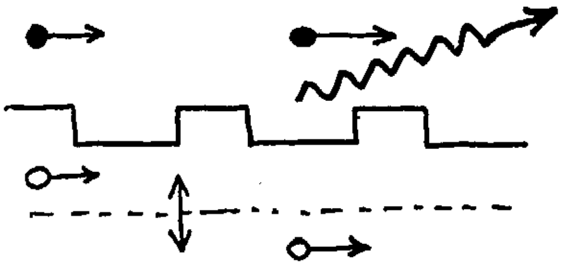
To stress once more the relationship between instabilities and induced emission effects I take a liberty to say that any linear accelerator may serve as a Cherenkov radiator being fed with a high energy beam. As a matter of fact it works in this way, but the radiation field of the main mode is just subtracted from the larger accelerating field as far as the beam is properly bunched. However it may radiate, and it does radiate, also in the next transparency band where its bunching does not play any role unless k_H is an integer of k_A. As a rule, the next band belongs to a non-symmetric mode with transverse magnetic field at the axis. The radiation then distorts the beam transversally: this is a blow up convective instability which appears in long linear accelerators even at low currents.

Cyclotron resonance masers. Coming to the second class we consider at first cyclotron resonance masers where the transverse oscillations with gyrofrequency Ω are induced at injection in a longitudinal magnetic field. For free waves the beam optical activity can be expected at Tr. 42

$$k_0 c = \frac{\Omega}{1+\beta}$$

TR. 41

GRATINGS (SMITH-PURCELL EFF)



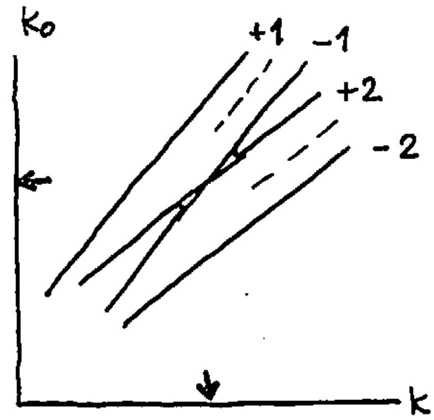
NONRELATIV. SINGLE-PARTICLE REPRESENTATION:

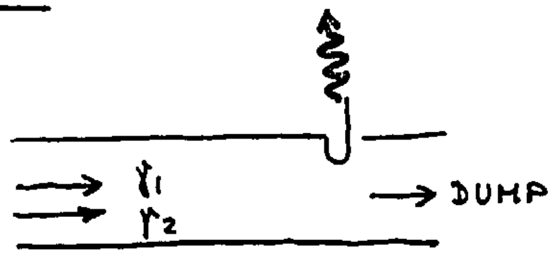
OSCILLATING CHARGE IMAGE

OROTRON

- SURFACE SLOW WAVE CHERENKOV RADIATION
- FREQUENCY - ANGULAR DEPENDENCE
- BEAM - GRATING DISTANCE < $\lambda\gamma$. LARGE CURRENT — ?

2 BEAMS SCHEME

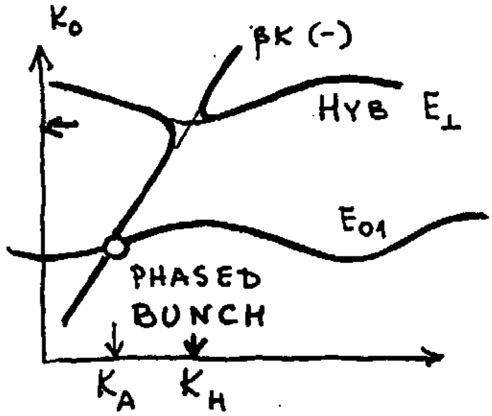




NO SLOW-WAVE STRUCTURE

- LOW GAIN FOR UHF
- WAVE TRANSFORMER TO EXTRACT E.M. WAVE

BLOW-UP IN LINACS



TRANSVERSE INSTABILITY

NOT INFLUENCED BY BUNCHING

UNLESS $K_H = N \times K_A$

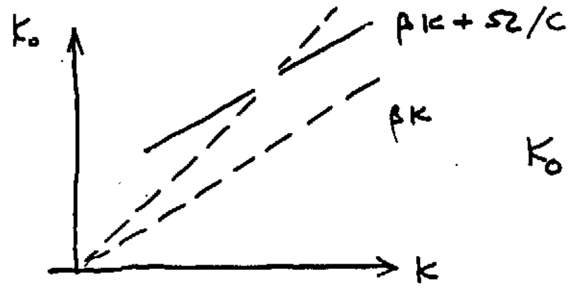
hybrid mode

accelerating mode

TR. 42

UHF (DOPPLER SHIFTED) FELs. GENERAL FEATURES.

- SHORT WAVELENGTH $\longrightarrow$ NO SLOW-WAVE STRUCT.; $\beta_{ph} \geq 1$
  TRANSVERSE WAVE FOR $\beta_{ph} = 1$
- TRANSV. OSCILLATIONS AND NORMAL DOPPL. EFF.



$$k_0 = \frac{\Omega/c}{1-\beta} \simeq \frac{2\gamma^2 \Omega/c}{1+\mathcal{K}^2}$$

for $\beta_p = 1$

- INTERNAL ENERGY OF TRANSV. OSC-NS TO BE SPENT FOR RAD-N
- TRANSV. OSC-NS NON COHERENT AT INJECTION, THEN $\longrightarrow$
  $\rightarrow$ SELF ORGANIZED (PHASED) DUE TO INDUCED RAD-N
- IF EXCITED AT INJECTION ONLY $\longrightarrow$ DAMP
- REALIZATION: INJECTION IN LONGIT. MAGNETIC FIELD

CYCLOTRON RESONANCE MASERS

- EVEN FOR SMALL $\mathcal{K}$ (SMALL "TRANSV." ENERGY; HENCE SMALL EFFICIENCY) $k_0 = 2\gamma^2 \cdot \frac{q B_0}{mc^2 \gamma} \propto \gamma$ INSTEAD OF γ^2 (FOR VISIBLE LIGHT AND $B_0 = 10T$ $\gamma \simeq 10^3$)
- MOST APPROPRIATE FOR $\lambda \gtrsim 1mm$ (MASERS)
  WITH GOOD EFFICIENCY IF $\mathcal{K}$ LARGE

$\swarrow$ $\searrow$

GYROTRONS CARM

where the fast cycloton wave interacts with the backwards directed electromagnetic wave and at

$$k_0 c = \frac{\Omega}{1-\beta}$$

where the forward directed wave is used and strong Doppler effect takes place. Note, however, that the Larmor frequency is itself inversely proportional to γ, so that the net frequency multiplication factor for fixed magnetic field goes as γ rather than as γ^2 for undulators. For example, one would need the field about 10 T and $\gamma \approx 10^3$ to reach the visible light frequency.

If a regular waveguide is used for wave propagation a finite cut-off frequency appears. Now there are two intersection points:

$$k_0 = \frac{\Omega/c \pm \beta\sqrt{\Omega^2/c^2 - k_{co}^2\,(1-\beta^2)}}{1-\beta^2}; \quad k = \frac{k_0 - k_{co}}{\beta}.$$

A choice between these two possibilities depends on the available energy and distance. For a table top device γ is always of order of unity and the difference in the operational frequency is not dramatic. One can sacrify the Doppler effect and select the low frequency point of intersection getting $\Omega = k_{kc}$ and, consequently, $k_0 = k_{kc} \quad k = 0$. Then the operational frequency becomes independent of the longitudinal particle velocity in contrast with the Cherenkov type interaction when high precision of this parameter is required. This scheme was suggested in early 60-s by Gaponov and was called a gyrotron. Tr.4~~0~~3

The choice of $k = 0$ means that the electric field is uniform along the beam at any moment of time and the waveguide operates at a standing mode like a cavity does. The output end of the waveguide performed as a selective Bragg mirror being corrugated with a proper period. This provides certain modes selection and the output power control. Note that the internal power circulating inside the cavity is Q times larger than the output one and determines the electric field level inside. This is essential for getting the maximal efficiency which is inversely proportional to the number of particle turns but needs a sufficiently large electric field to be realized (i.e., to equalize the saturation length and the cavity length). Optimization of gyrotron parameters from various viewpoints including such technical problems as output windows, a beam collector, an external solenoid, etc is too complicated to discuss it now. Nowadays many articles and monographs are devoted to this subject.

The second point of intersection is more interesting for the high frequency operation but requires larger γs. By the way, this is the only point of interest if the wave is almost plane, i.e., if $k_0 \gg k_{co}$. Besides, it gives an unique opportunity to support the synchronism condition under very large variations of the particle energy. Really, k_0 can be represented in the form Tr.44

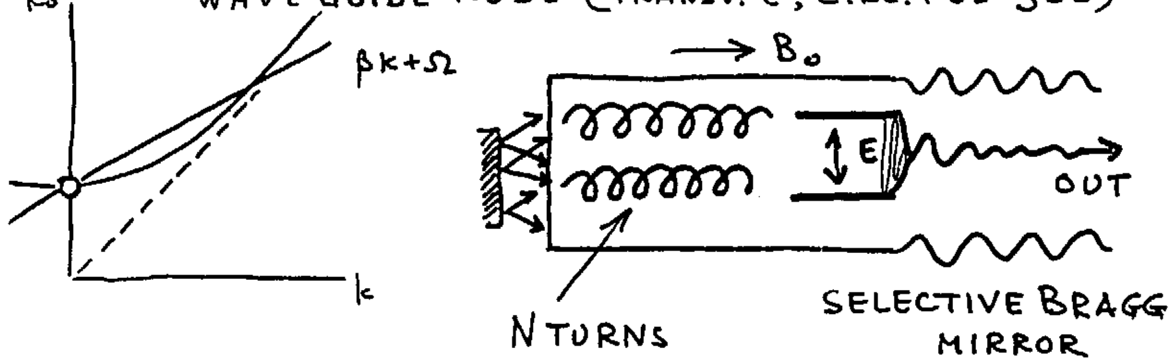
$$k_0 = \frac{qB_0}{mc^2 I} = 6 \times 10^{11} I^{-1} B_0\,[\mathrm{T}]\,\mathrm{cm}^{-1}$$

where the multiplication coefficient $I = \gamma(1-\beta)$ is an exact integral of motion. The latter follows from the fact that an emitted photon carries out $\Delta\gamma = \hbar k_0/mc$ and $\Delta(\gamma\beta) = \hbar k/mc$ which are exactly equal for free electromagnetic waves. As a result the three dimensional cyclotron resonance in a plane wave is kept inspite of changes in the particle energy. This effect was called an "autoresonance" and the corresponding device got the name CARM (Cyclotron AutoResonance Maser).

Tr. 43

GYROTRON OSCILLATOR

- $\Omega(\gamma)$ DEPENDENCE SUFFICIENT FOR PHASING
- TO ELIMINATE SENSITIVITY TO LONG. VELOCITY SPREAD PUT $k=0$ IN $k_0 - \beta k \simeq \Omega$
- STANDING MODE (HIGH Q CAVITY CAN BE USED FOR FEEDBACK)
- WAVE GUIDE MODE (TRANSV. E, CIRC. POL-SED)



- OPERATES AT CUT-OFF FREQ. = LARMOR FREQ.
- SELECTIVE (SINGLE MODE) AND CONTROLLED Q-FACTOR
- INT. POWER = Q × OUTPUT POWER
- NECESSARY GAIN $\propto I \cdot N^3 \propto \frac{1}{Q}$. "STARTING" CURRENT

  MAX. EFFICIENCY (SAT-N) $\propto N^{-1}$

  OUTPUT POWER $\propto I \cdot E_{int} \cdot N$

  INPUT POWER $\propto I \times V$

  MATCHING
- OPTIMAL PITCH ANGLE, E_{int}, P_{out}, etc
- LOW TUNABILITY AND FLEXIBILITY
- TECHN. PROBLEMS → COLLECTOR, WINDOWS, B_0 etc etc.

Tr. 44

CARM
(Cyclotron AutoResonance Maser)

- Exclusive case: free plane wave along B_0:

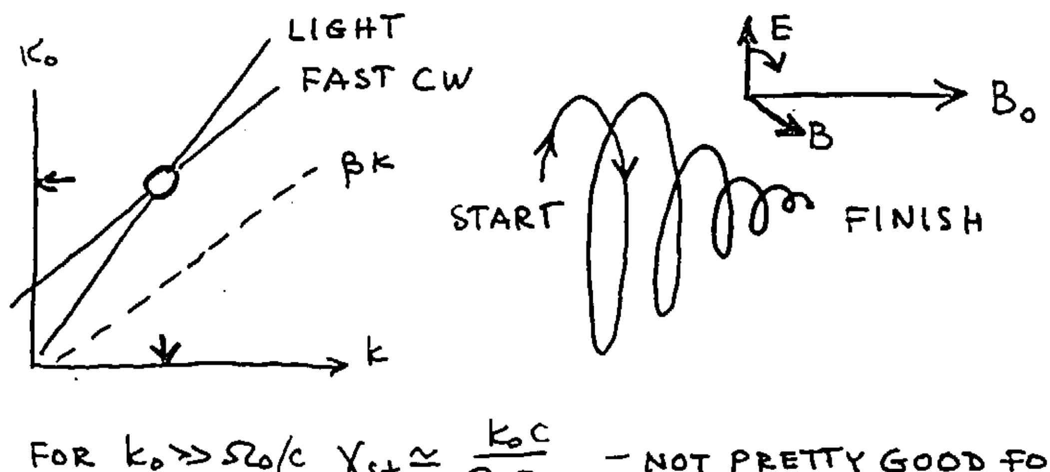
$$k = k_0 \, ; \qquad k_0 = \frac{\Omega/c}{1-\beta} = \frac{\Omega_0/c}{\gamma(1-\beta)} \, ; \; \Omega_0 = \frac{qB_0}{mc}$$

But $\gamma(1-\beta) = \gamma - p_z$ is an <u>exact</u> integral of motion because $\Delta\gamma - \Delta p_z = -\frac{\hbar\omega}{mc^2} + \frac{\hbar k}{mc} \equiv 0$

- 3-D cyclotron resonance is sustained inspite of energy, Larm. rad., and long. momentum change. The "3-D" cyclotron <u>is</u> suitable for relativ. particles

- A wave with $k_0 \lesssim \frac{\Omega_0/c}{\gamma(1-\beta)}$ will ~~be~~ self amplified

- „Transv." energy (Larmor rad.) and long. momentum vanish simultaneously → 100% efficiency ?



- For $k_0 \gg \Omega_0/c$ $\gamma_{st} \simeq \frac{k_0 c}{2\Omega_0}$ – not pretty good for short waves

- Laser acceleration as an inverse process ?

If one is satisfied with $k_0 = 6 \times 10^{11} B_0\,[\mathrm{T}]\,\mathrm{cm}^{-1}$ the choice of $I = 1$ can be recommended. Then starting from an arbitrary energy γ with the initial angle $\theta = \arccos\sqrt{(\gamma - 1)/(\gamma + 1)}$ the resonance will be kept up to $\gamma = 1$ which means the 100% energy transfer (in cases of $I < 1$ $\quad \gamma_f = I^{-1}$). Unfortunately, an attemt to use very large γs meets a necessity of large electric fields to realize this potentially high efficiency at acceptable lengths. It is worth to note that an inversed autoresonance scheme was originally suggested for particle acceleration in laser beams.

Lecture 5

Free Electron Lasers

Now we are discussing possibilities of generation and amplification of very high frequency waves, i.e., so called free electron lasers. The considerations above show two conditions to be met:

– Interaction with plane free waves with phase velocity =1. The normal Doppler effect is to be used with high energy beams.

– Transverse oscillations of electrons with the highest possible frequency must be provided in the lab frame. The larger is the frequency the less severe are requirements to the particles energy.

Free oscillations, that is, oscillations induced at injection can hardly have a sufficiently high frequency. Actually, the operational frequency in a longitudinal magnetic field was estimated at the previous lecture for CARMs and was found to be limited by available field strengths. External quasi-elastic forces or transverse beam coherent oscillations also seem hardly adequate because their frequencies are small for heavy relativistic particles too. Tr.45

Forced oscillations induced by external transverse fields alternating in space along the beam trajectory look better from this viewpoint. Their spatial period which determines the frequency can be made small enough, especially if one uses free electromagnetic "pumping" waves for this purpose. The term stresses the relation of the scheme to traditional lasers. Moreover, stationary fields (both magnetic and electric) of large strengths can be used. The term "pumping" then still can be used at least in the rest frame of the beam.

As far as we deal with two waves (pumping and emitted ones) and with non-interacting particles the process can be considered as wave Compton scattering or two wave process. The (k_0, k) diagram showing the corresponding conservation laws is presented in the figure. Tr.46 One can see that the reflected wave frequency really may exceed the pumping wave one if the particle gets an essentially relativistic recoil momentum.

Induced scattering. We shall start with pumping produced by an incident free electromagnetic wave of a comparatively low frequency ω'. Being directed oppositely to the beam the wave shifts particles transversally with the frequency $(1+\beta)\omega'$ in the lab frame. Hence, they radiate along z a wave of the high frequency $\omega'(1+\beta)/(1-\beta) \approx 4\gamma^2\omega'$. The effect is quite recognizable as a reflection of the incident wave from a moving relativistic mirror. Calling it Compton scattering we mean this frequency shift, although the effect has not much common with real quantum Compton scattering which reveals at much larger frequencies than those of interest.

Of course, the scattering cross-section of an individual particle is so miserable in practice that one may expect only numbered back scattered photons to be observable. However, the induced effects should cause stimulated scattering meaning that the beam is self modulated in density and re-radiates the high frequency wave coherently. In a way, it looks like a Bragg mirror with high selective reflection coefficient. As was discussed above the radiation length must be less than the interaction length for this purpose.

As a rule, one can not neglect the short-range interaction of particles involved in the process. So a wave interpretation of this stimulated effect is of a general physical interest

TR. 45

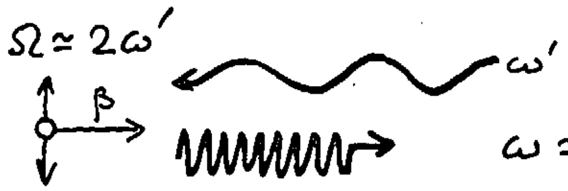
FORCED OSCILLATIONS

- INITIALLY EXCITED TRANSV. OSC-NS (CRM) ARE EXHAUSTED DUE TO RADIATION AND HAVE TO POSSESS SMALL ENERGY TO SUSTAIN STRONG DOPPLER EFFECT NECESSARY FOR SHORT-WAVE RADIATION
- POSSIBLE SOLUTION — TO PUMP THEM CONTINUOUSLY AT A DESIRABLE LEVEL:
  - a) BY AN ADDITIONAL LOW-FREQ "PUMPING" WAVE
  - b) REPUMPING LONG. MOMENTUM TO TRANSVERSE ONE BY MEANS OF TRANSV. MAGN. FIELD (UBITRON)

a) INDUCED SCATTERING

- SINGLE ELECTRON — REFLECTION FROM A MOVING MIRROR

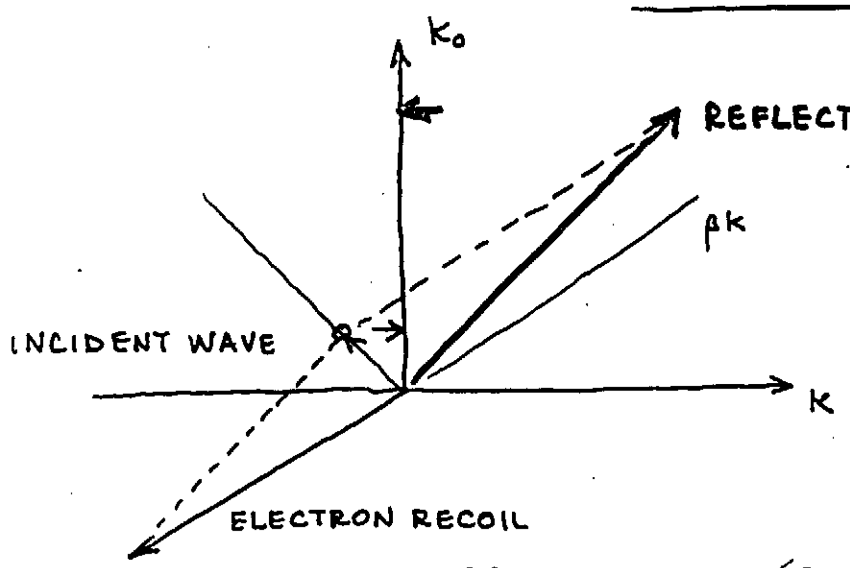
„COMPTON" SCATTERING



$$\omega \simeq 2\gamma^2 \Omega \simeq 4\gamma^2 \omega'$$

TR 46

2-WAVES PROCESS



CROSS-SECTION (REST FRAME) $\sim 10^{-27}\,cm^2$

$N_{ref}/N_{inc} \lll 1$ IF NON-COHERENT

INDUCED SCATTERING

- NONINTERACTING PARTICLES ($k_p^2 \to 0$): PHASING (LONG-L) IN THE SCATTERED WAVE → MOVING SCATTERING LATTICE → → BRAGG (COHERENT) REFLECTION OF THE PUMP WAVE

- FOR INTERACTING PARTICLES: SPACE-CHARGE WAVE RECOIL MOMENTUM ESSENTIAL IF $k_p^* \simeq k_{pump}$; 3-WAVES PROCES (HIGH CURRENT BEAMS, 0.1 — 3 cm WAVELENGTH)

- SATURATION OF PLASMA WAVES MAY LIMIT THE PROCESS

- FOR $k_p^* \uparrow$ EFFICIENCY INCREASES BUT GAIN $\sim I^{1/4}$

and deserves several words. For interacting particles the recoil momentum belongs to a space charge wave as well as to an individual particle. This obviously changes the picture at least when the plasma frequency is of the order of magnitude of the pumping wave frequency. Moreover, in dense beams the space charge waves frequency splitting becomes essential so that the gain dependence on the current is closer to $I^{1/4}$ rather than $I^{1/3}$.

Different people use different names for this collective stimulated scattering. In FEL physics it is usually called a 3-wave regime bearing in mind the incident wave, the scattered one, and a slow space charge wave which plasmon takes the recoil nomentum. In radiotechnics they say about parametric excitation of the system with an eigenfrequency. Perhaps, the most adequate to lasers is an optical language where the phenomenon is known as a stimulated Raman scattering process.

The next figure illustrates the analogy with the optical Raman effect. This is a dispersion diagram where $\Omega(k)$ represents an internal vibrational degree of freedom (plasma waves in the beam rest frame). There are two channels of the incident wave scattering including plasma waves. If a new plasmon accepts a part of the recoil momentum (a Stokes component) the conservation laws say that the reflected wave frequency is shifted down. Note that the new born plasmon must propagate the same direction as the incident wave does. The second possibility consists of dissapearance of an already existing plasmon (oppositely directed). Then the reflected wave frequency is shifted up (an anti-Stokes component in the Raman scattering process).

The same situation looks more complicated being cosidered in the lab frame because an electron momentum recoil leads there to an essential loss in energy. Except of the usual change in frequency due to a reflection from a moving object the delivery of a new plasmon produces the Stokes frequency shifted component. However, the sign of the shift depends on a kind of the new plasmon. If the latter is associated with the fast space-charge wave the shift is negative as above. But an appearance of a negative energy wave plasmon shifts the frequency down. The anti-Stokes component behaves oppositely but it is not of interest at the moment.

Now we can foresee a possibility of stimulated Raman scattering with exponential developement of both components if no initial plasmon exists. Being born for the account of the beam energy the slow plasmon produces some modulation of density which increases the incident wave reflection if its wavenumber in the plasmon rest frame (denoted by primes) satisfies the Bragg condition $k_p' = k_i'$. So the number of the plasmons grows and the intensity of the reflected wave grows as well up to a certain saturation level.

Simple calculations shown in the transparency lead to the following conclusions:

– A spatial lattice develops in the beam with

$$ck_p = \frac{\omega_i - \Omega}{1-\beta} \qquad \text{and velocity} \qquad \beta_p = \frac{\omega_i\beta - \Omega}{\omega_i - \Omega}\,.$$

Note that in a relativistic case (strong Doppler effect!) it moves a bit slower than the beam and has the spatial period about $2\gamma^2$ times smaller than that of the incident wave.

– The reflected wave frequency is

$$\omega_r = \frac{\omega_i(1+\beta)+\Omega}{1-\beta} \approx 4\gamma^2\omega_i.$$

The corresponding increase in quanta energy as well as the lattice potential energy are paid by the beam kinetic energy.

Relativistic ubitrons or undulator FELs. At last we came to the main scheme widely used for modern FELs. It is based on the stimulated emission of undulator radiation (Phillips) produced by relativistic beams (Madey). The scheme is well known and was described in numerous publications and monographs. I will stay only on its conformity with the considerations above. Tr.47

This stimulated emission of undulator radiation may be treated in various ways. The steady state undulator field might be also considered as a pumping wave (not free, of course!) with $\omega_i = 0 \quad k_i = k_u = 2\pi/l$ where l is its spatial period. Sometimes radiation is considered as a result of the pumping wave stimulated reflection (Compton or Raman one depending on the ratio $k\beta c/\Omega_p$).

In the case of the fixed spatial period of the transverse oscillations we can specify the parameter of phase motion sensitivity to energy deviations which is essential for the stimulated phasing mechanism. As far as the value $\Omega = \beta k_u c/l$ is now directly proportional to β, one has $\alpha \approx \gamma^{-3} \ll 1$. Thus, the phasing is rather lazy for relativistic particles and large radiation lengths are to be expected for high frequency devices.

Let me remind also that for a fixed desirable frequency and for the undulator coefficient $K \approx 1$ one more essential parameter remains, i.e. the radiation length which determines the mode of operation. If the beam current and the available length of the system are small one needs a cavity to increase the internal field. Then the system operates as an oscillator. For large currents one has a luxury to afford $L \gg L_r$ and to get an FEL amplifier. (However, some intermediate schemes like a regenerative amplifier have been suggested.)

Several general relations are repeated in the next transparency just to remind the previous treatment. Note that the mode composition of radiation changes along the system (within the synchronism condition frames, of course). At initial parts of the interaction space modes with the optimal positive detuning are more favorable and grow as z^3 untill the radiation length is reached. Then the synchronous modes growing up exponentially overtake them. A coexistence of various modes is assured only at the linear stage of the process. One may expect that the most successive mode will supress the others and then, at the suturation length, will decay because of non-linear processes. Tr.48

The frequency of the desirable mode can be controlled with two parameters, the beam energy and the undulator coefficient if the latter is large enough. This provides one of the most essential advantages of an undulator FEL – its large tunability. Tr.49

For an ideal FEL amplifier the frequency band is limited from below by the ratio of a so called emission length to the gain length. The former can be defined as a length at which the radiation wave overtakes a particle for one wavelength. This is, of course, the wavelength divided by $1-\beta$ factor, i.e., the undulator period l. One may say that the relative frequency band is just the inverse number of periods along the gain length. Of course, there are plenty of reasons why the real band can be wider than l/L_r. Some of them are listed in the transparency and are more or less obvious. I would like to comment just one thing related to the time structure of a high energy beam. If the beam is produced in a resonant linear accelerator, especially under condition of a small energy spread, it looks like a train of short bunches radiating independently, i.e., in a non-coherent way. In the amplification regime each bunch is accompanied then by its own flash of radiation which, of course, is faster than the bunch itself. However, the bunch in the rest frame as a rule is longer than the gain length meaning that $l_b \gg (1-\beta)L_r \approx L_r/2\gamma^2$, and the light emitted by tail electrons never reaches the head ones. So in most cases the finite length

Tr. 47

RELATIVISTIC UBITRON (PHILLIPS 1961)
(UNDULATOR FEL) (MADEY 1972)

- THE PUMP FIELD CAN BE ESSENTIALLY INCREASED IF IT IS A STEADY STATE MAGNETIC FIELD ALTERNATING IN SPACE

$$\omega_{pump} = 0 \,; \quad k_{pump} = \pm k_u \quad (0.1 \div 10 \ cm^{-1})$$

- CAN NOT PROPAGATE. MUST BE EVERYWHERE IN INT. REGION
- PRODUCES TRANSV. ALTERN. FORCE $\longrightarrow$ PUMPING WITH

$$\Omega = \beta k_u c$$

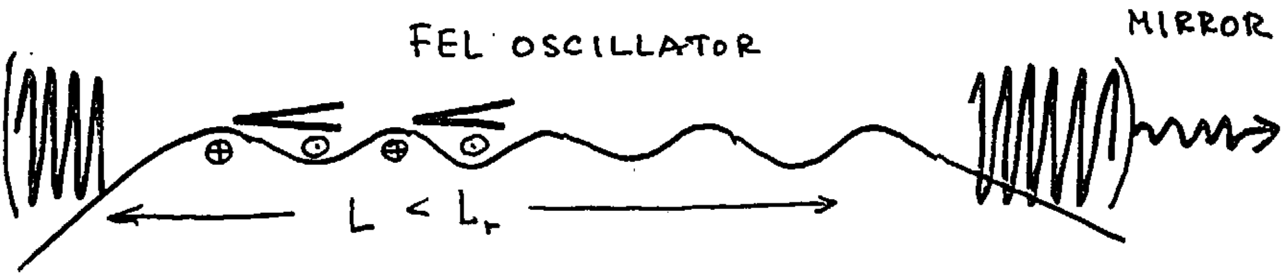
- LONGITUDINAL MOTION RESPONSIBLE FOR PHASING

$$\alpha = -\frac{\partial}{\partial \gamma}\left(\frac{k_0 \pm \Omega/c}{k_0 \beta}\right) = -\frac{\partial}{\partial \gamma}\frac{1}{\beta} = \frac{1}{\beta^3 \gamma^3}$$

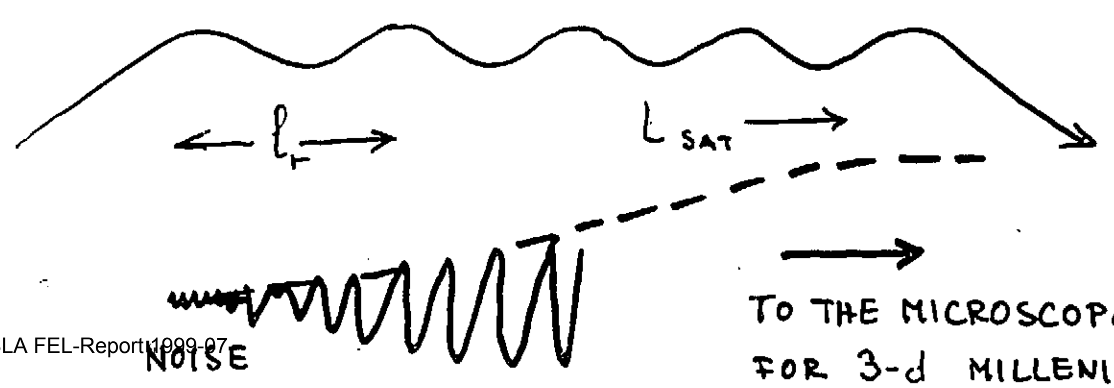
- PLANE OR ROTATING $\longrightarrow$ PLANE OR CIRC. POLARIZATION

$$\boxed{\text{THIS IS AN UNDULATOR (WIGGLER IF } K > 1)}$$

FEL OSCILLATOR



AMPLIFIER OR SASE

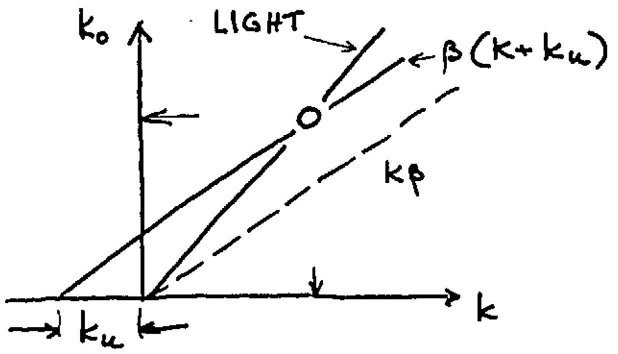


TO THE MICROSCOPE FOR 3-d MILLENIUM

TR. 48

FREQUENCY AND AMPLIFICATION

- UNDULATOR FIELD FORCES TRANSVERSE OSCILLATIONS $\Omega = \beta k_u$ AND DRIVES A FAST BEAM WAVE (TRANSV. POSITIVE ENERGY) WITH $k_0 = \beta k + \beta k_u$ WHICH IS SYNCHR. WITH E.M. WAVE ($k_0 = k$)



$$k_0 = k = \frac{k_u \beta}{1-\beta}$$

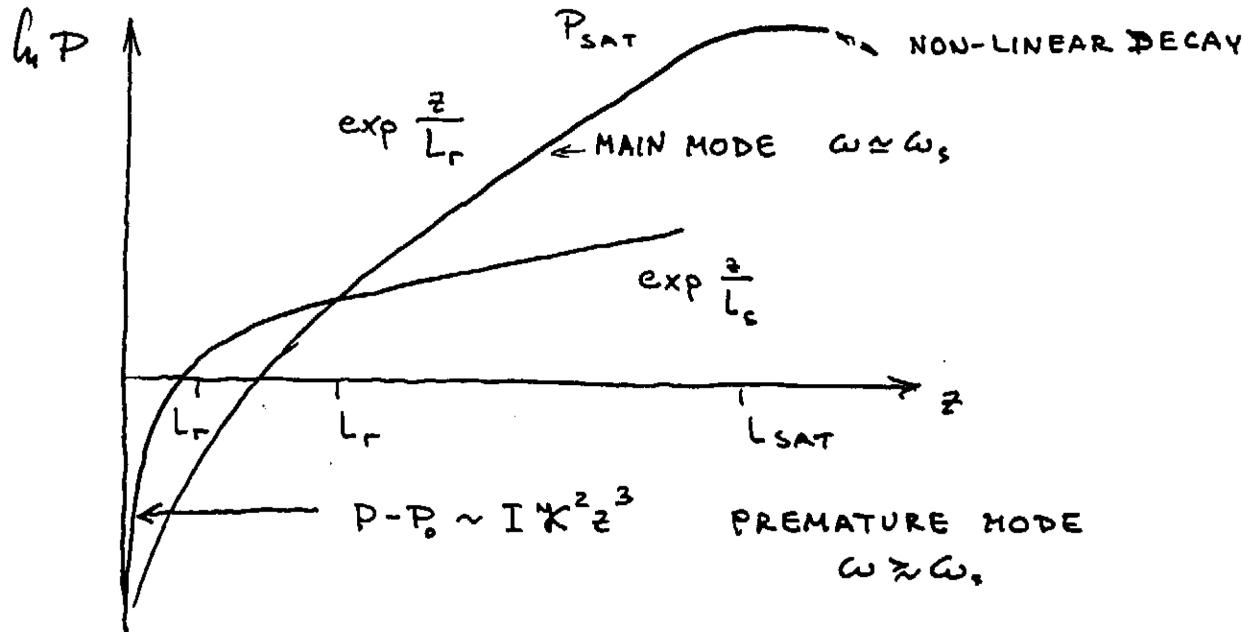
SUPPOSING $\beta_\perp = \text{const}$

$$\beta = \sqrt{1-\gamma^{-2}-\beta_\perp^2} \simeq 1 - \frac{1+\gamma^2\beta_\perp^2}{2\gamma^2}$$

SO:

$$k_0 = \frac{2\gamma^2 k_u}{1+\mathcal{K}^2} \quad ; \quad \mathcal{K} = \gamma\beta_\perp$$

- LIGHT PULSE ENERGY (POWER) VS DISTANCE



- $\boxed{\text{TUNABILITY CAN BE PROVIDED } (\mathcal{K} \text{ CONTROL})}$

Tr. 49

FREQUENCY BAND

- IDEAL (RADIATION WIDTH) $\frac{\Delta\omega}{\omega} \propto \frac{\ell}{L_r} = \frac{1}{N_\ell}$
- POSSIBLE WIDENING:
  - + TRANSVERSE NONUNIFORMITY IN UNDULATOR
  - + MODE COMPETITION (SASE)
  - + NONLINEARITY. FIRST APPEARANCE — SIDE BAND MODES DUE TO SYNCHR. OSCILLATIONS OF CAPT-D PARTICLES
  - + FINITE BUNCH LENGTH. USUALLY NON ESSENTIAL BECAUSE "BUNCH IS LONGER THAN UNDULATOR", MEANING THAT LIGHT FROM TAIL PARTICLES NEVER REACH HEADONES. IF IT DOES AN INITIAL MODULATION EXISTS AND WILL BE AMPLIFIED — INTERESTING COHERENT "SUPERRADIANCE".
  - + BEAM INSTABILITIES, WAKE FIELDS, etc, etc

SATURATION

- SAT. LENGTH — TO BE COMPUTED. PHYS. MEANING — MAXIMAL PHASE BUNCHING. DEPENDS ON INITIAL SIGNAL LEVEL, LINEAR GAIN, DIFFRACTION, SPACE CHARGE etc.
- SAT. POWER AND EFF-CY CAN BE ROUGHLY ESTIMATED STARTING FROM MAX. ENERGY DECREASE OF CAPT-D PARTICLES

$$\Delta\gamma_{max} = +\sqrt{2g/\alpha k} \qquad g = \frac{eE_0 K}{mc^2\gamma}$$

AND POWER BALANCE

$$P = E_0^2 c S/4\pi = I \Delta\gamma \frac{mc^2}{e}$$

$$\Delta\gamma = \left[\frac{16\pi K^2 I}{S k_u^2 I_0}\right]^{1/3};\ \eta = \frac{\Delta\gamma}{\gamma};\ P = \left[\frac{16\pi K^2}{S k_u}\right]^{1/3}\left(\frac{I}{I_0}\right)^{4/3} \cdot 9\ \mathrm{GW}$$

$$\text{"}\ \frac{m^2c^5}{e^2}$$

of a bunch does not influence the spectral width. However, in some cases of low gain it does and the beam structure is of importance.

Actually, the spontaneous radiation spectrum for a chain of short ($l_b \ll (1-\beta)L$) bunches has a width determined by l_b rather than by L. If one increases the gain so that L_r becomes comparable with L the width is of order of L^{-1}. So an observer sees the line narrowing with somewhat increasing intensity. This is an analogy of the optical effect known as "superradiance" which is the first manifistation of lasing.

I have mentioned several times the saturation length as one of the important FEL parameters. For a small gain regime it was defined as a length at which a particle makes about 1/4 of a phase oscillation in the radiation field. Unfortunately, in the high gain regime this estimate is not informative and the saturation length is to be computed many specific factors taken into account. Inspite of that, the saturation power level can be roughly estimated bearing in mind that the change in the particle energy is of order of the "energy size" of the separatrix in the phase plane. Then a simple energy balance shows that the saturation power drops with the mode cross-section as $(Sk_u^2)^{-1/3}$ and increases with current as $I^{4/3}$.

As for the radiation (gain) length, it also can be estimated assuming the operational mode concentrated within a cylinder of the cross-section S. It is proportional to $(S/I)^{1/3}$ although must be computed, of course, under particular conditions being a crucial FEL parameter. Note, by the way, that the mode cross-section does not coincide with the beam one and exceeds the latter because of diffraction.

Diffraction phenomena are a subject of a 2-D theory and very qualitative considerations only can be adduced in the frames of our simple model. In a low gain regime particles radiate within an angle of γ^{-1} which, as a rule, is larger than the beam angular aperture (transverse dimension a over the interaction distance L). So the beam side surface emits everywhere along z and the radiation field outside it drops as a/r, typically for a cylindrical wave. However, in a high gain regime the angular divergency of the really amplified modes is of order of $a/L_r < \gamma^{-1}$. So the outgoing radiation is rather weak as compared with the directed one and the radiation field is concentrated in the vicinity of the beam. This present of nature is called sometimes beam wave guiding. Tr 50

It was shown (Saldin, Schneidmiller, Yurkov) that in most cases the effect can be characterized by a single diffraction parameter B proportional to $I^{1/3}$. The effect of the field concentration is more or less independent of a transverse beam density distribution (see the computed curve in the transparency).

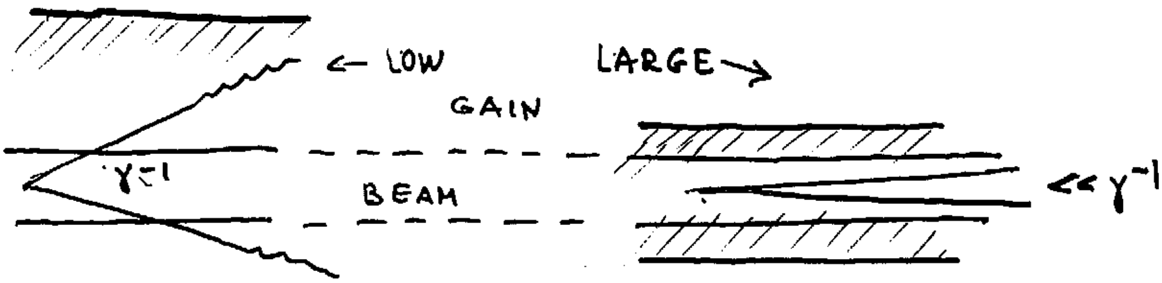
Beam quality. Let me say now a couple of words on a required beam quality. Calculation of tolerances is a separate and very serious problem for any particular project and includes a lot of compromises. It hardly can be enlightened in these brief notes. However, several estimations should be done just to show that the stimulated emission mechanism is not a magic stick and can produce and keep the fine spatial density modulation (with a period of λ!) in a well organized beam only. I conciously omit now numerous requirements to the beam alignment, to the undulator magnetic field, etc which are meaningless unless electron beam internal parameters are sufficiently precise. Tr.~~46~~ 51

There are two main beam parameters obviously influencing the process of interest – an energy spread and a transverse beam emittance. The first can be readily estimated using the condition of synchronism. If a particle does not satisfy the condition at the main mode frequency it is at least useless not taking part in the beam – wave interaction.

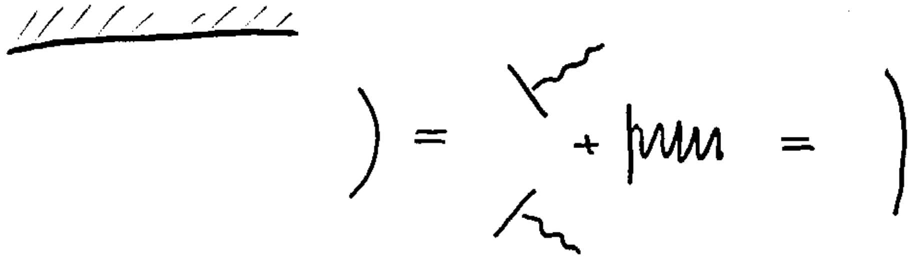
Tr. 50

DIFFRACTION (2D EFFECT)

- RADIATION EMISSION TAKES PLACE EVERYWHERE ALONG THE TRANSVERSALLY BOUNDED BEAM



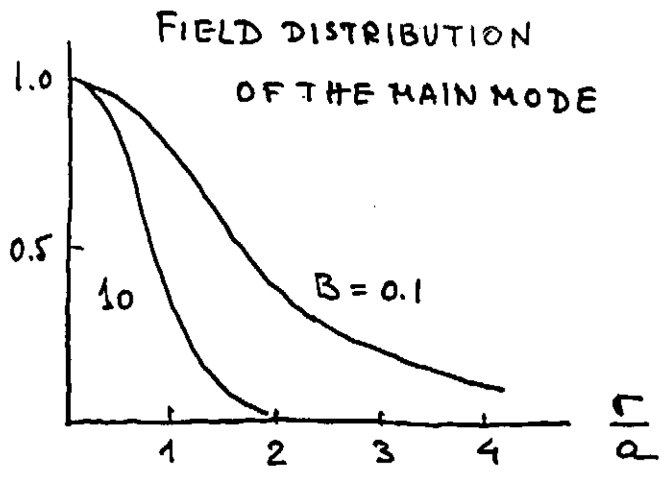
MODE TRANS. PROFILE



OUTGOING PLANE WAVES ARE POORLY AMPLIFIED AS COMPARED WITH FAVOURABLE ONE. THE MODE PHASE FRONT STRAIGHTENS AND THE FIELD IS CONCENTRATED IN THE BEAM VICINITY

THIS IS WAVE GUIDING

DIFFRACTION PARAMETER $B = 4a^2 k_u^2 \gamma^2 \left(\frac{I}{I_0} K \right)^{1/2}$



(SALDIN et al)

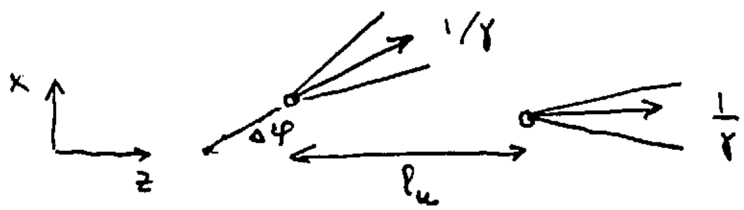
TR. 51

BEAM QUALITY (EMITTANCES)

- ENERGY SPREAD SHOULD NOT INFLUENCE THE DESIRED VALUE OF L_r. FROM THE SYNCHRONISM CONDITION

$$k \Delta\beta \cdot L_r < 2\pi \qquad \text{OR} \qquad \frac{\Delta\gamma}{\gamma} < \gamma^2 \frac{\lambda}{L_r} = \frac{1}{N_r} \quad (1)$$

- TRANSVERSE EMITTANCES



TO SUPPORT TRANSVERSE COHERENCY AT THE PHOTON FORMATION DISTANCE $\left(\lambda/(1-\beta) \simeq 2\gamma^2\lambda = l_u\right)$ ONE HAS TO PROVIDE BEAM RADIUS $a < l_u/\gamma$; $\Delta\varphi < \gamma^{-1}$ OR:

$$\boxed{\varepsilon \lesssim \frac{l_u}{\gamma^2} \simeq \lambda \quad (2)}$$

- NOTE: FOR $K \lesssim 1$ MAX. ANGULAR DEVIATION OF THE WIGGLING TRAJECTORY IS $\beta_\perp \simeq K/\gamma$

A POSITION DEVIATION FROM THE AXIS $\Delta x \sim K/\gamma k_u$

I.E. THE SAME ORDER OF MAGNITUDE

- EXTERNAL FOCUSING HELPS

(LENSES, PROFILED POLES etc)

The acceptable relative frequency deviation was estimated above as N_r^{-1} where N_r is a number of undulator periods per the gain length. Noting that an energy deviation $\Delta\gamma$ produces a longitudinal velocity deviation of order of $\Delta\beta \approx \Delta\gamma/\gamma^3$ one can evaluate an acceptable relative energy spread as N_r^{-1} with the coefficient of 1/2 depending on one's optimism.

Emittance estimates are more ambiguous. On one hand, to make an income to the common radiation field a particle angular deviation should not exceed at least the natural angle of γ^{-1}. On the other hand, the transverse off-set should be small enough to give a possibility to irradiate all particles at the next undulator period to support a transverse coherency. These arguments (rather misty, I admit) lead to a simple conclusion: the required 1-D transverse emittance is of order of the wavelength. Moreover, a plane undulator produces some focusing with a betatron oscillation wavelength of order of γ periods. The beam emittance should be matched with the undulator acceptance. This gives again the same order of magnitude. Actually, this is not a law and the situation can be essentially improved with additional external focusing. Anyway, one should be ready to pay for an extra emittance not to sacrify the gain.

By the way, the energy spread estimated above is nothing but the longitudinal beam emittance in some units. Bearing in mind that this is just the total 3-D emittance which is untouchable one can repump a part of a transverse phase volume to the longitudinal one or vice versa depending on the most stringent conditions. These schemes of "beam preparation" or emittance – aceptance matching give sometimes a dramatic increase in the gain for a fixed total current.

Certain improvements. In conclusion let me stay brifly on certain modifications of the main scheme which seem to be of a general physical interest. Tr.~~17~~52

The first one (Skrinsky and Vinokurov) was suggested to overcome the smallness of the phase sensitivity parameter $\alpha \approx \gamma^{-3}$ which leads to rather lengthy systems for comparatively small currents. The last situation is typical for FELs fed with storage rings beams. The idea consists of a special dispersion section placed between two sections of the undulator. As far as the time of flight through the section depends on the particle energy rather than on its velocity the bunching effect is essentially enhanced. The first undulator section then plays a role of a modulator, the dispersion section bunches the particles, and the final part serves as a radiator. There is an obvious analogy of this "optical klystron" with its radiotechnical namesake. Note that the increased phase to energy sensitivity automatically makes requirements to the beam energy spread more stringent. However, the spread is already small in storage rings because of intrinsic radiation cooling.

The second modification is more radical from a viewpion of general physics. Originally (Sprangle) it was directed to an increase in FEL efficiency which otherwise is regrettably low (of order of N_r^{-1}). By the way, one may have various opinions about the high efficiency necessity unless very large light pulse energies are required. The question is rather economical one. But, anyway, this is a bit offensive to accept that the highly organized beam energy can not be exploited and serves only for the Doppler effect maintenance.

The idea is to force the beam to radiate after the saturation occured and the particles were bunched inside a separatrix in the phase plane. Note that this situation is a starting one for resonant accelerators where the main aim is to force them to absorb the field energy. The prescription is well known: one should move the bucket up in energy space with a frequency time modulation as in proton synchrotrons or with a spatial modulation

CERTAIN IMPROVEMENTS

Tr. 52

1. OPTICAL KLYSTRON

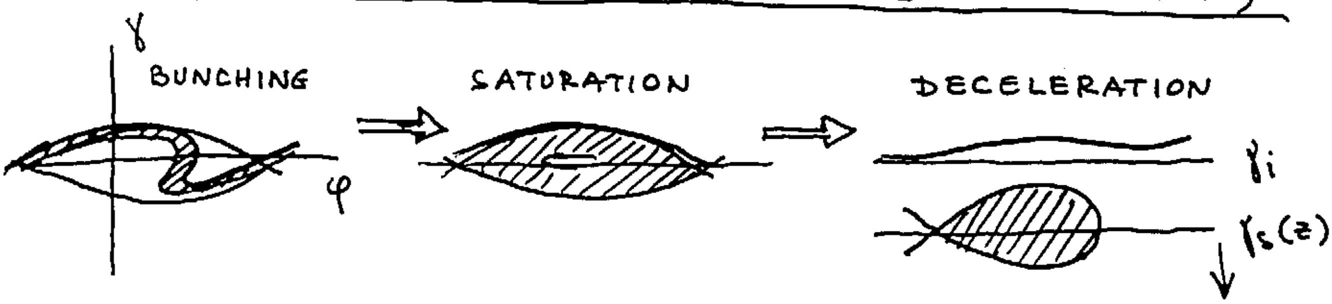
FOR FIXED k_w PHASE SHIFT IS DUE TO $\beta(\gamma)$ ONLY

$$\frac{d\varphi}{d\zeta} = \alpha(\gamma - \gamma_s) \quad ; \quad \alpha = -\frac{\partial}{\partial\gamma}\frac{1}{\beta} = \frac{1}{\beta^2\gamma^3} \ll 1$$

SMALL α MEANS LONG DISTANCE OF INITIAL BEAM BUNCHING.

TO INCREASE PHASE SLIP SENSITIVITY TO ENERGY DEVIATIONS:

2. HIGH EFF-CY NONLINEAR REGIME (CAPTURED BUNCH)



CAPTURED BUNCH FOLLOWS $\gamma_s(z)$ DELIVERING ENERGY TO E.M. FIELD OF THE FIXED FREQUENCY

$$k_o = \frac{2\gamma_s^2 k_w}{1 + K^2} = \text{const} \Longrightarrow \gamma_s \sim \left(\frac{1 + K^2(z)}{k_w(z)}\right)^{1/2}$$

EFFICIENCY CAN BE ESSENTIALLY INCREASED (exper. 30%)

$\left(\eta \approx \frac{\gamma_i - \gamma_f}{\gamma_i}\right)$ IF ONE CAN INCREASE $k_w(z)$

(TAPERED UNDULATOR)

of parameters (drift tubes length) as in linear proton accelerators. Our aim is an opposite one – to shift the resonance energy down for a fixed frequency. The phase stability mechanism does the rest if the rate of the forced energy variation is not too large.

Practically this can be done with variations of the undulator period along the particles path or, as they say, with a tapered wiggler. (The name originates from helical undulators with a common excitation current where a change of a perid length is conjugated with a change of the apperture.) Both theoretically and experimentally an efficiency of orders of tens per cent was proved at least in the infrared domain. Unfortunately, the scheme has two disadvantages. Firstly, the undulator period being a rather stressed parameter has to be decreased further. Secondly, the radiation field level must be matched with the period variation (like in proton linacs). This restricts the tunability which is one of the most valuable features of FELs. Let me just express my hope that it is not the last word in the history of this invention.