Features and Usage of the Extended COMFORT Beam Optics Code with Space Charge

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1. Introduction

For electron beams with high phase space density, space charge effects are known to yield an important contribution to the particle dynamics in the lower and medium energy range. This is a matter of much concern in particular for the design of low-emittance RF-guns such as used for Free Electron Laser facilities. Several computer simulation codes have been developed to investigate in great detail the beam dynamics in this low-energy range (typically up to a few MeV). However, the space charge forces remain strong enough up to energies in the 100MeV range to be non-negligible, as can easily be seen from an estimate of the betatron Q-shift per unit length for typical FEL-bunch parameters ($Ne=10^{10}$, σ_z=1mm, $\varepsilon=10^{-6}$ m):

$$\Delta Q / \Delta s \approx -\frac{N_e r_e}{(2\pi)^{3/2}\sigma_z \varepsilon \gamma^2} \approx -0.044 m^{-1} / (E/100MeV)^2 \tag{1}$$

The first phase of operation of the TTF-FEL takes place at energies of ~200MeV, so that from the above estimate we can conclude that space charge effects must be taken into account over the entire length of the beam line. In order to provide a fast and easy-to-use tool for beam optics calculations, we extended the existing COMFORT code to include space charge in linear approximation. While this can provide only a first order approximation, it is nevertheless very helpful in allowing to provide beam optics matching for different assumptions on the bunch parameters, as well as a fast tool for re-calculating the actual beam optics from the known accelerator parameter settings in the control room. In fact, the extended COMFORT code has been routinely in use already for some time at the TTF and with this note we would like to provide a brief summary of how the space charge extension has been implemented in the code as a reference for present and future users.

2. Space Charge Model in COMFORT

The linear defocusing strengths due to space charge are given by:

$$k_{x,y}(s) = \frac{2N_e r_e}{(2\pi)^{1/2}\sigma_z \sigma_{x,y}(\sigma_x + \sigma_y)\gamma^3} \quad \text{with} \quad \sigma_{x,y} = \sqrt{\varepsilon_{x,y}\beta_{x,y}/\gamma} \tag{2}$$

This expression applies to the defocusing strength in the core of a beam with 3-dimensional Gaussian distribution. With other assumptions on the distribution function, one has to adjust the bunch charge used in the calculation in order to yield the same linear space charge effect. For example, in a bunch with homogeneous longitudinal distribution the charge density is equal to the one in the center of a Gaussian bunch of same σ_z, if the ratio of Gaussian to homogeneous bunch charge is $Q_G/Q_H = (\pi/6)^{1/2}$.

In a beam optics code, the beta functions are calculated from some initial values by applying the linear transfer matrix method. The transfer matrices are calculated from the properties of the lattice elements (quads, drifts, accelerating structures, etc.). In our case, of course the linear defocusing due to the space charge force must be taken into account as well. This is done here in the following approximate procedure.

Each beam line element is cut into two halves and a thin-lens quad element is inserted in the middle. The integrated strengths of the quad, $k_{x,y}{\cdot}l$, are calculated from eq. (2) by using the β-functions which have been previously determined without space charge. A new transfer matrix (including the thin lens) is calculated for the element and the modified beta-functions (and their derivatives) are determined. Proceeding from element to element yields the beam optics for the entire beam line. This procedure is iterated in a loop, each time using the beta-functions of the previous step, in order to converge to the "exact", i.e. self-consistent solution. It should be noted that the accuracy of the final result depends on the length of the elements. It is therefore advisable to cut longer elements (e.g. drift spaces) into shorter pieces already in the COMFORT input deck. There is no check of the convergence included in the code.

3. Treatment of a Bunch Compressor

The effect of a bunch compressor can also be approximately taken into account. The code calculates the momentum compaction matrix element R56 for dipole magnets. By specifying a coherent and incoherent energy spread in the input deck (see below), the evolution of bunch length in the compressor section is calculated in linear approximation. The space charge calculation in this case works as described above, non-inertial fields are not included. The contribution of the dispersion function to the beam size is taken into account by adding the term $D{\cdot}(\Delta p/p)_{incoh}$ in quadrature. The incoherent part of the energy spread, $(\Delta p/p)_{incoh}$, is adjusted with decreasing bunch length such that $\sigma_z{\cdot}(\Delta p/p)_{incoh}$ remains constant. The evolution of bunch length and energy spread in the bunch compressor is shown in the output listing of the program.

4. Commands in the Input Deck

The space charge option is activated by specifying the command SPC (for a general description of the COMFORT input file syntax, please consult the user manual [1]).
The SPC-command must be accompanied by the following parameters (as usual in COMFORT, the parameter values must be given in the line below the keywords):

```
SPC       ENX   ENY   QB   SIGZ  DEE  DEDZ
```

Here, ENX, ENY denote the horizontal and vertical normalized emittances in mm*mrad, QB the bunch charge in nC, SIGZ the bunch length in mm, DEE and DEDZ the incoherent and coherent relative energy spread (a value DEDZ<0 means lower energy in bunch head, as required for a chicane-type compressor). Before the SPC command, the initial beam energy must be specified as parameter P0 (in GeV) in the NUM command.

The definition of a standing wave resonator element has been added to this updated version of the code, in order to be able to take into account the rf-focusing which can be quite strong in the initial part of the TTF linac. The syntax for specifying one or more of these elements is:

```
ACC        CAV1  (CAV2  CAV3....)
          Gradient
           Length
           Phase,
```

where the gradient is given in MV/m, the length in m and the phase in degree (zero phase means on-crest acceleration). Since the transfer matrix for a cavity depends on the beam energy at the entrance, the internal structure of COMFORT requires that for each resonator an individual element must be specified, even if the gradients and phases are equal for a group of cavities. Note that the influence of ACC elements on the bunch energy spread is not taken into account.

In the code, the ACC transfer matrix elements are re-normalized to remove the adiabatic damping from the beta-function transformation (det{MACC} = 1). Beam sizes are therefore easily calculated from a COMFORT optics listing by using the absolute emittances at the respective position in the beam line. When calculating the orbit caused by steering elements from an optics calculation, one must not forget to take the adiabatic damping due to acceleration into account.

The executable module of the updated COMFORT program is publicly available on the AFS system [2].

5. TTF Example

In order to illustrate the usage of COMFORT, we give here an example of optics calculations for the TTF linac [3]. We assume an initial beam energy of 15MeV after the capture cavity, a normalized emittance of 3mm*mrad and a bunch length of 1.5mm. The uncorrelated and correlated energy spread is set to $2.5 \cdot 10^{-4}$ and $7.5 \cdot 10^{-3}$, respectively, to yield a bunch length of 0.3mm after the compressor. The accelerating gradient is set to 13MV/m in both modules to achieve a final beam energy of 230MeV. The beam optics with and without space charge are shown in Figs. 1 and 2, respectively. The differences in the beta-functions between the two cases is very significant, which clearly shows the necessity to take the space charge into account for an accurate matching into the undulator section.

The linear optics calculation also allows to estimate the growth in projected beam emittance caused by the spread of the space charge defocusing strength over the length of the bunch. By calculating the Twiss parameters for different QB, one obtains the variation of $\beta_{x,y}$ and $\alpha_{x,y}$ at the end of the beam line, from which the projected emittance is obtained by:

$$\varepsilon_{proj} = \varepsilon_0 \cdot \sqrt{\langle \beta(z) \rangle \cdot \left\langle \frac{1+\alpha^2(z)}{\beta(z)} \right\rangle - \langle \alpha(z) \rangle^2} \tag{3}$$

The averaging <...> is performed over longitudinal position z in the bunch with a Gaussian $\exp(-z^2/2\sigma_z^2)$ as weight function. For our TTF example we obtain an increase of the projected emittance w.r.t. ε_0 of 9% in the horizontal and 7% in the vertical plane. The variation of the initial β's and α's versus position z in the bunch has not been taken into account here. It can lead to an additional increase or a reduction of the final projected emittance.

Another source of emittance growth is the dispersion mis-match due to space charge in the bunch compressor (clearly visible in Fig. 1). This yields an approximately 10% dilution for the horizontal emittance, but here it must be stressed that this estimate is inaccurate because of the neglect of non-inertial fields.

6. Conclusion

The incorporation of linear space charge effects in the COMFORT code has proven as a useful tool for preparing and investigating beam optics for the TTF/FEL facility. The approximate treatment inevitably can not accurately determine all beam dynamics effects related to space charge, so that improved and more accurate codes are still desirable, in particular in view of more stringent beam quality demands for the future operation of the FEL.

[1] The COMFORT documentation is available as text file ~brinkman/public/comfort/docnew.

[2] The executable COMFORT module is available as ~brinkman/public/comfort/comfort_exe.

[3] The input file for this example can be found in ~brinkman/public/comfort/ttf_example.

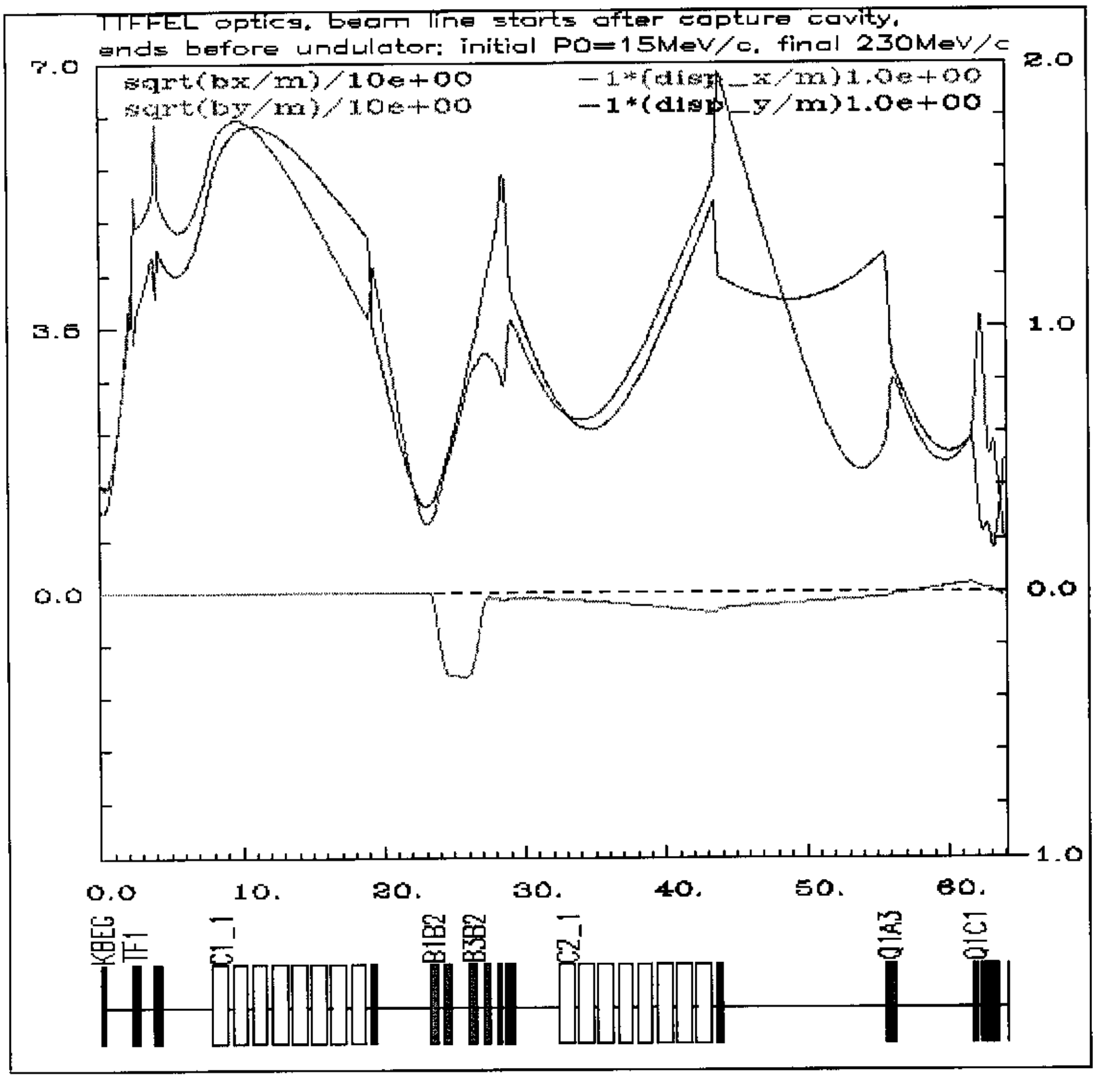

Figure 1: Beam optics for the TTF linac calculated with COMFORT including space charge (parameters see text).