

Surface Roughness Study for the TESLA-FEL

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Introduction

This report summarizes the surface roughness wake field study for the TESLA FEL transfer line and the undulator. At present different models have been developed by K.Bane [1], G.Stupakov [2], A.Novokhatski, M.Timm, T.Weiland [3] to study the effect of a random surface roughness in a beam pipe, and by K.Bane, A.Novokhatski [4], G.Stupakov [5], M.Dohlus [6] to calculate the wake in beam pipes with periodic surface structure. The effects of surface structures with small and large transverse dimensions are estimated separately by two periodic models. The rms amplitudes and wavelengths of the corrugations are computed from a real measured surface function [7] after low- and high-pass filtering respectively. The longitudinal wake potentials of rectangular and sinusoidal periodical surfaces and the longitudinal resistive wall wake are calculated numerically and analytically respectively.

1 Short summary of different models.

1.1 Bane's model [1]

The roughness is represented by bumps or hollows of simple form (hemisphere, cube, prism etc.), which are located randomly on the internal surface of the pipe. For a pipe radius b , large compared to the radius of a hemisphere r ($b \gg r$), the contribution to the impedance from a single hemisphere in the low-frequency range is given by [8]:

$$Z_1(\omega) = -i\omega \frac{Z_0 r^3}{4\pi c b^2} \quad (1)$$

where ω is the frequency, $Z_0 = 377\Omega$ and c is the speed of light. For a small single roughness of varying form, the above expression needs to be multiplied by a form factor f , the numerical value of which, for some simple objects, varies between $f=0.6$ (rotated half cube) and $f=10.8$ (cube) [1, 9]. The contribution from a set of bumps (contribution from hollows is considerably smaller and is neglected) is given by the sum of the individual bump contributions to the impedance. The complete impedance per unit length is then

$$\frac{Z(\omega)}{L} = -i\alpha f k \frac{Z_0 r}{4\pi b} \quad (2)$$

Here α is a packing factor equal to the relative area on the surface occupied by the bumps and L is the length of the pipe. The longitudinal potential of a Gaussian bunch with rms longitudinal size σ_z can be written as:

$$W_z(s) = -\frac{Z_0\alpha frc}{4\pi^2b} \int_0^\infty k e^{-k^2\sigma_z^2/2} \sin(ks) dk = -\frac{Z_0\alpha frc}{2^{1.5}\pi^{1.5}\sigma_z^3b} s e^{-s^2/2\sigma_z^2} \quad (3)$$

Here $\widetilde{\lambda_{cg}}(k) = e^{-k^2\sigma_z^2/2}$ is the Fourier image of the longitudinal charge distribution of a Gaussian bunch.

The loss-factor in this a model is equal to zero. The rms value of the longitudinal potential is given by

$$\sigma_\epsilon = \frac{Z_0\alpha fc}{3^{3/4}2^{3/2}\pi^{3/2}b\sigma_z^2} r \quad (4)$$

Note, that the above formula differs from that given in [9]: the factor 3 in the denominator is raised to the power of 0.75 instead of 0.25 in [9].

The extremes of the function (3) are equal with opposite sign at $s = \pm\sigma_z$. The peak-to-peak energy spread is then expressed as:

$$\Delta W_p = \frac{Z_0\alpha fc}{\sqrt{2}e\pi^{3/2}b\sigma_z^2} r \quad (5)$$

The model considered is valid for a uniform distribution of identical bumps, when their characteristic cross section (radius at the base) is equal to their height.

1.2 The Stupakov model [2]

The model [2, 9] is based on the statistical description of roughness. The basic assumption of the model is that the angle between the normal to the rough surface and the radial direction is small in comparison to unity. It is also assumed that the height of the roughness and the average size of the cross section are small with respect to radius of the pipe.

The impedance of a statistically non-uniform rough cylindrical surface is determined by a two dimensional Fourier transformation of the surface correlation function $R(\tilde{k}_x, \tilde{k}_z)$ and in the general case is given by

$$Z(\omega) = -\frac{ikZ_0L}{2\pi b} \hat{S}, \quad \hat{S} = \int d\tilde{k}_x d\tilde{k}_z R(\tilde{k}_x, \tilde{k}_z) \frac{\tilde{k}_z^2}{\sqrt{\tilde{k}_x^2 + \tilde{k}_z^2}} . \quad (6)$$

Depending on the representation of the correlation function (i.e. the type of roughness) various expressions for the impedance are obtained.

In particular, for a rough surface with a Gaussian spectrum

$$R(\tilde{k}) = \frac{l_c^2 d^2}{2\pi} e^{-\tilde{k}^2 l_c^2 / 2}, \tilde{k} = \sqrt{\tilde{k}_x^2 + \tilde{k}_z^2} \quad (7)$$

the impedance is given by

$$\frac{Z(\omega)}{L} = -\frac{\sqrt{\pi}}{4\sqrt{2}} \frac{ikZ_0 d^2}{l_c b} \quad (8)$$

where d is the rms height of roughness, l_c the correlation length that is given by the characteristic size of the base of a roughness. The longitudinal wake potential is then expressed as

$$W_z(s) = -\frac{c}{16} \frac{Z_0 d^2}{l_c b \sigma_z^3} s e^{-s^2 / 2\sigma_z^2} \quad (9)$$

with the rms σ_ϵ and peak-to-peak ΔW_p energy spread given by

$$\sigma_\epsilon = \frac{c Z_0 d^2}{3^{3/4} 16 l_c b \sigma_z^3}; \quad \Delta W_p = \frac{c}{8\sqrt{e}} \frac{Z_0 d^2}{l_c b \sigma_z^2} \quad (10)$$

In the case of a fractal surface ($R(\tilde{k}) = A e^{-q}$ for $\tilde{k} > \tilde{k}_0$ and $R(\tilde{k}) = 0$ for $\tilde{k} < \tilde{k}_0$), the normalized impedance is expressed in terms of the parameter q and the characteristic wave number \tilde{k}_0 as

$$\frac{Z(\omega)}{L} = -\frac{ikZ_0}{4\pi b} \frac{q-2}{q-3} d^2 \tilde{k}_0 \quad (11)$$

where $\tilde{k}_0 \sim \pi/l_c$. Thus smaller values of q give more "spiky" profiles [2, 9].

The longitudinal potential of a Gaussian bunch is then written as

$$W_z(s) = -\frac{c}{4\sqrt{2\pi}} \frac{Z_0 d^2}{l_c b \sigma_z^3} \frac{q-2}{q-3} s e^{-s^2 / 2\sigma_z^2} \quad (12)$$

with rms σ_ϵ and peak-to-peak ΔW_p energy spread given by

$$\sigma_\epsilon = \frac{c}{3^{3/4} 4\sqrt{2\pi}} \frac{Z_0 d^2}{l_c b \sigma_z^3} \frac{q-2}{q-3}; \quad \Delta W_p = \frac{c}{2\sqrt{2\pi e}} \frac{Z_0 d^2}{l_c b \sigma_z^2} \frac{q-2}{q-3} \quad (13)$$

As has been shown in [9], the coincidence of the Bane and Stupakov models takes place when the rough surface consists of identical bumps randomly scattered over the surface. The corresponding relation between the parameters of the two models follows from the comparison of formulas (3) and (9) for Gaussian spectrum roughness, and formulas (3),(12) for fractal roughness. In particular

$$\alpha f r = \mu \frac{d^2}{l_c} \quad (14)$$

for the Gaussian spectrum roughness and

$$\alpha f r = \frac{\pi}{2} \frac{d^2}{l_c} \frac{q-2}{q-3} \quad (15)$$

for fractal roughness.

1.3 The NTW model [3]

It is shown in [3, 4], that the presence of roughness is equivalent to a pipe with a thin dielectric layer or periodic corrugations on the smooth wall of a pipe. In both these cases the resonator impedance model applies. The monopole resonant longitudinal and dipole transverse wake functions $w_z(s)$ and $w_r(s)$ for a single particle are presented as follows:

$$w_z(s) = 2\kappa_0 \cos ks, \quad w_r(s) = 4\kappa_0 \sin ks, \quad (16)$$

with $\kappa_0 = Z_0 c / 2\pi b^2$ and $s > 0$. The resonance wave number k for the dielectric layer and for the periodic rectangular corrugations is equal to

$$k = \sqrt{\frac{2\varepsilon}{(\varepsilon - 1)b\delta'}} \quad k = \sqrt{\frac{2p}{b\delta g}} \quad (17)$$

respectively (similar results were also presented by Stupakov [10] and Bane, Stupakov [11] for a round pipe with small periodical sinusoidal and rectangular corrugations). In the first case $\varepsilon \approx 2$ is the dielectric constant and δ' is the typical depth of the corrugations. In the second case δ is the depth, g is the gap and p is the period of the corrugations. For the Gaussian bunch distribution the longitudinal wake potential is presented as follows:

$$W_z(s) = -\frac{\kappa_0}{2} e^{-s^2/2\sigma_z^2} \left\{ \xi \left(\frac{-is - k\sigma_z^2}{\sqrt{2}\sigma_z} \right) + \xi \left(\frac{-is + k\sigma_z^2}{\sqrt{2}\sigma_z} \right) \right\} \quad (18)$$

where $\xi(z) = e^{-z^2} \operatorname{erfc}(-iz)$ is a complex error function. In the case of $k\sigma_z \gg 1$ one obtains from (18):

$$W_z(s) = -\sqrt{\frac{2}{\pi}} \frac{\kappa_0 \sigma_z}{s^2 + k^2 \sigma_z^4} s e^{-s^2/2\sigma_z^2} \quad (19)$$

and for $s \ll k\sigma_z^2$, the comparison with the Bane model (Eq.3) gives

$$\alpha f r = \frac{g}{p} \delta. \quad (20)$$

1.4 Surface Impedance of a Corrugated Pipe

The effect of a random surface roughness is modelled by a pipe with the surface function

$$r(z) = b - \delta r \cos zk_1 \quad (21)$$

with $\lambda_{cg} = 2\pi/k_1$ the period of the corrugation, and δr its amplitude. In [5] the impedance is derived for a shallow corrugation $ak_1 \ll 1$ and for frequencies $\omega < k_1 c_0 / 2$ as

$$Z(\omega) = -\frac{1}{2\pi b} \frac{Z_s(\omega)}{1 + ik_0 \frac{b}{2} \frac{Z_s(\omega)}{Z_0}} \quad (22)$$

with the surface impedance

$$Z_s(\omega) = ik_0 Z_0 \left(\frac{\delta r \pi}{\lambda_{cg}} \right)^2 \left(\frac{j}{k_+} + \frac{j}{k_-} \right) \quad (23)$$

and $k_{\pm} = \sqrt{k_0^2 - (k_0 \pm k_1)^2}$. This approach can be used to calculate the wake potential of gaussian bunches with rms length σ and rms frequency $\omega^\sigma = c_0/\sigma$ if the condition $\omega < k_1 c_0/2$ is fulfilled for $\omega < 3\omega^\sigma$ or $\lambda_{cg} < \sigma\pi/3 \simeq \sigma$. In [6] the model is generalized for the high frequency regime, so that it can be used to calculate the wake potential of long surface periods. Therefore the surface impedance Z_s in Eq. (22) has to be calculated by

$$Z_s(\omega) = ik_0 Z_0 \left(\frac{\delta r \pi}{\lambda_{cg}} \right)^2 \left(\frac{J_1(k_+ b)}{J_0(k_+ b)k_+} + \frac{J_1(k_- b)}{J_0(k_- b)k_-} \right). \quad (24)$$

To calculate the wake potential the impedance function is replaced by a pole expansion:

$$Z(\omega) = - \sum_{\nu} \frac{i\omega 2k_{\text{loss},\nu}}{\omega_{\nu}^2 - \omega^2} \quad (25)$$

with ω_{ν} the poles of Eq. (22) and $k_{\text{loss},\nu}^{-1} = \left. \frac{d}{d\omega} \frac{1}{\text{Im}\{Z(\omega)\}} \right|_{\omega=\omega_{\nu}}$ the inverse pole strengths. The longitudinal wake potential is given by the convolution integral

$$W_z(s) = \int_0^{\infty} w(x) \frac{g((s-x)/\sigma)}{\sigma} dx \quad (26)$$

with $w(s) = -2 \sum_{\nu} k_{\text{loss},\nu} \cos(s\omega_{\nu}/c_0)$ and $g(s)$ the gaussian normal distribution. It has to be mentioned that only pole coefficients ω_{ν} , $k_{\text{loss},\nu}$ have to be taken into account for which the beam spectrum is not negligible.

The normalized longitudinal wake for $b = 5\text{mm}$, $\sigma = 25\mu\text{m}$ can be seen in Figs. 1 and 2 for $\lambda_{cg} = 10, 50, 100$ and $200\mu\text{m}$ and $\delta r/\lambda_{cg} = 0.01$ and 0.005 . The shape of the wakes varies with the wavelength of the surface corrugation, but it scales in the investigated parameter range essentially with $\delta r^2/\lambda_{cg}$. For $\lambda_{cg} < \sigma$ only the first pole of Eq. (25) is significantly stimulated by the bunch spectrum. For this case the model described by Stupakov [5] is appropriate. For larger values of λ_{cg} further resonances ω_{ν} are stimulated, which are closely related to the cutoff frequencies of TM_{0n} waveguide modes by $k_-(\omega_{\nu}) = \omega_{c,\text{TM}0\nu}/c_0 + O(b^{-1})$. Therefore the lowest loss parameter $k_{\text{loss},1}$ describes the stimulation of the synchronous mode which is taken into account by the NTW model and the higher loss parameters $k_{\text{loss},\nu>1}$ describe the excitation of waveguide modes. As the sum of **all** loss parameters is $k_{\text{norm}} = Z_0 c_0 / 2\pi b^2$ the wake is dominated by the higher resonances if $\lambda_{cg} > 3\sigma$ and $k_{\text{loss},1}/k_{\text{norm}} \ll 1$.

2 Surface Roughness Wakefield Effects in the TESLA-FEL

Fig. 3 shows the surface function δr of one of the first probes measured at DESY [7]. Although δr is small compared to the bunch length it has transverse structures which are small compared to the bunch length as well as contributions with larger dimension. The models [1, 2, 3] are based on the assumption that all dimensions of the surface structure are small. Therefore we use the model of a periodical surface to estimate the wake field effects. To take into account the contribution of long and short surface wavelengths we split δr into a slowly varying part δr_1 and a fast varying part $\delta r_2 = \delta r - \delta r_1$ as shown in Figs. 3 and 4. The slowly varying part δr_1 is obtained by a convolution with a gaussian sampling function of the rms width $15\mu\text{m}$. (This is equivalent to a low pass filtering with a gaussian filter function.) The effects of both parts are estimated separately by the wakes of periodic surfaces δr_{p1} , δr_{p2} with the same rms roughnesses $\delta 1$, $\delta 2$ and similar transverse dimensions to the original functions. The properties of the original [7] and the periodic surface functions are listed in the following table.

function	rms amplitude	wavelength	comment
δr_1	$0.5\mu\text{m}$		slow random variation
δr_{1pr}	$0.5\mu\text{m}$	$100\mu\text{m}$	rectangular periodical
δr_{1ps}			sinusoidal periodical
δr_2	$0.1\mu\text{m}$		fast random variation
δr_{2p}	$0.1\mu\text{m}$	$10\mu\text{m}$	sinusoidal periodical

The longitudinal wake potential of the slowly varying part is calculated by two methods: for a direct numerical integration by MAFIA [12] a rectangular periodical surface function δr_{1pr} is used; the calculation based on Eq. (22) with Eq. (24) assumes a sinusoidal shape $y\delta r_{1ps}$. As the MAFIA model still includes some fast variations, the fast varying part δr_{2p} is calculated only for a sinusoidal surface (cf. Fig. 5).

The longitudinal wake potentials of the surfaces with δr_{1pr} , δr_{p2} and of a smooth surface with the conductivity of aluminum ($\sigma = 3.65 \cdot 10^7 \Omega^{-1}\text{m}^{-1}$) and their sum are plotted in Fig. 6 for the undulator pipe (radius $b = 5\text{mm}$). In Fig. 7 the wakes can be seen for the sinusoidal surface function δr_{p1s} . The peak, average and rms values of these wakes are summarized for the undulator pipe (radius $b = 5\text{mm}$) and the pipes of the transfer line (radius $b = 12\text{mm}$, radius $b = 12\text{cm}$) in the following tables.

b=5mm	min(w) V/pCm	max(w) V/pCm	$\langle w \rangle$ V/pCm	rms(w) V/pCm
$w_{\delta r_{1p}}$ rect (cos)	-60.3 (-21.3)	54.9 (20.8)	-9.6 (-8.8)	41.1 (12.4)
$w_{\delta r_{2p}}$	-8.8	8.8	0	6.4
$w_{\text{resistive}}$	-82.3	45.0	-40.4	39.6
$\sum w$	-143.1 (-108.8)	99.0 (68.8)	-50.0 (-49.2)	82.3 (56.2)

b=12mm	min(w_z) V/pCm	max(w_z) V/pCm	$\langle w_z \rangle$ V/pCm	rms(w_z) V/pCm
$w_{\delta r_{1p}}$ rect (cos)	-26 (-9.2)	24 (9.0)	-4 (4.1)	18 (5.2)
$w_{\delta r_{2p}}$	-4.2	4.3	0	3.1
$w_{\text{resistive}}$	-35.2	23.7	-20.0	15.3
$\sum w$	-57.6 (-45.1)	39.9 (33.6)	-24.1 (-24.1)	31.1 (21.4)

b=12cm	min(w_z) V/pCm	max(w_z) V/pCm	$\langle w_z \rangle$ V/pCm	rms(w_z) V/pCm
$w_{\delta r_{1p}}$ rect (cos)	-2.65 (-1.48)	2.38 (2.08)	-0.41 (-0.96)	1.80 (0.47)
$w_{\delta r_{2p}}$	-1.09	1.1	-0.23	0.36
$w_{\text{resistive}}$	-1.55		-0.97	0.48
$\sum w$	-3.56 (-3.29)	1.33 (1.35)	-1.61 (-2.17)	1.67 (1.02)

3 Conclusion

At present no models are available to describe the wake field effects of a 3d random surface with transverse surface structures small **and** large compared to the bunch dimension. To obtain a worst case estimation we use the model of a corrugated pipe. The effects of surface structures with small and large transverse dimensions are estimated separately by two corrugation models. The rms amplitudes of the corrugations are computed from a real measured surface function after low- and high-pass filtering respectively. The wavelengths of the periodic functions slightly overestimate the variation of the filtered function which leads to a more pessimistic estimation of the wake field effects. The same is true for the rectangular periodic model compared to a sinusoidal corrugation.

Our worst case estimation of the rms value of the complete wake in the undulator pipe und the small transfer line ($b = 12\text{mm}$) is about twice as large as the rms value of resistive wall wake alone. For the wide transfer line ($b = 12\text{cm}$) this factor is 3.5 because the rms value of the resistive wall is over-proportionally reduced.

Our estimations do not take into account the nonlinear interaction between the different contributions which may or may not be synergic. On the other hand they include a large safety margin: the rectangular periodic model includes more and sharper surface perturbations than the measured. Much further work

needs to be done to understand all effects so that less pessimistic estimations are justified.

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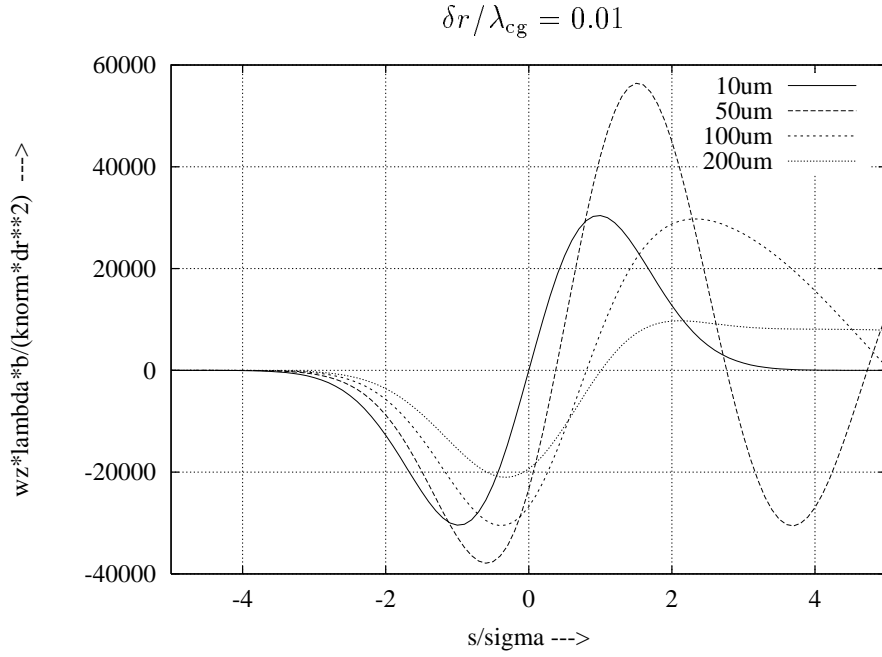


Figure 1: Normalized longitudinal wake for $b = 5\text{mm}$, $\sigma = 50\mu\text{m}$, $\delta r / \lambda_{cg} = 0.01$ and $\lambda_{cg} = 10, 50, 100, 200\mu\text{m}$. The wake is normalized to $k_{\text{norm}} \delta r^2 / b \lambda_{cg}$ with $k_{\text{norm}} = Z_0 c_0 / 2\pi b^2$.

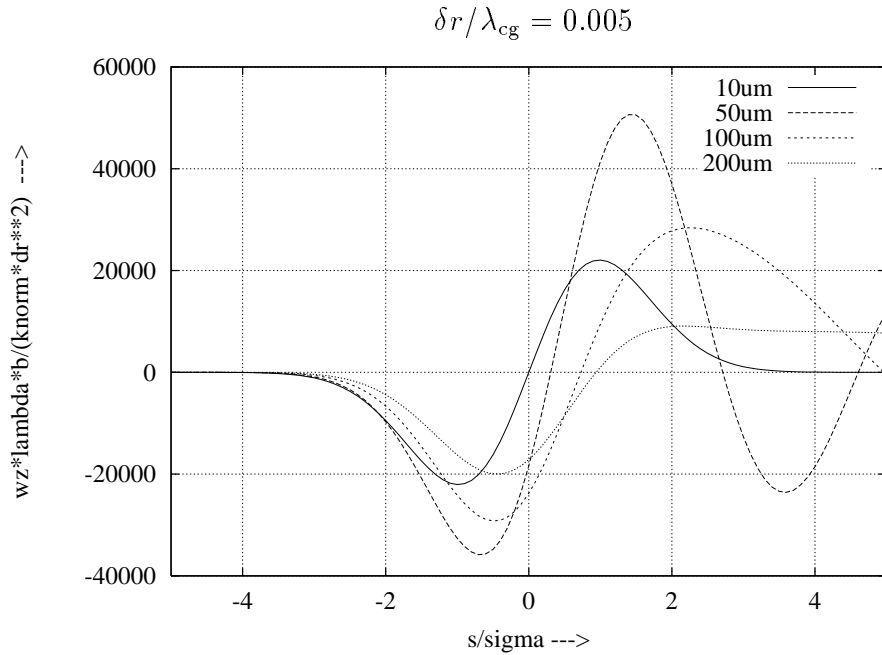


Figure 2: Same parameters as in Fig. 1 with exception of $\delta r / \lambda_{cg} = 0.005$

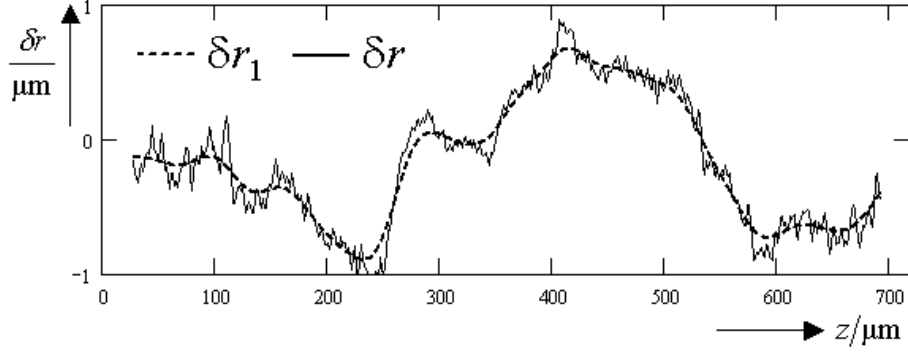


Figure 3: Measured surface function δr and the slowly varying contribution δr_1 .

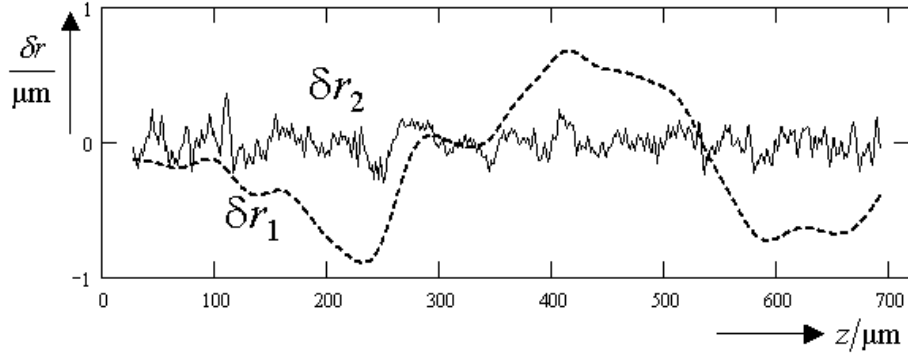


Figure 4: The slowly and fast varying contributions δr_1 and δr_2 .

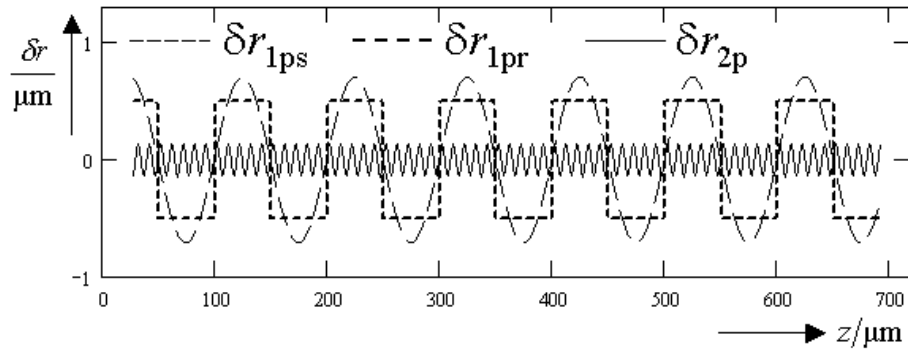


Figure 5: Periodic approximations δr_{1ps} , δr_{1pr} and δr_{2p} of the slowly and fast varying contributions of the surface function. δr_{1ps} and δr_{2p} are sinusoidal functions, δr_{1pr} is a rectangular function.

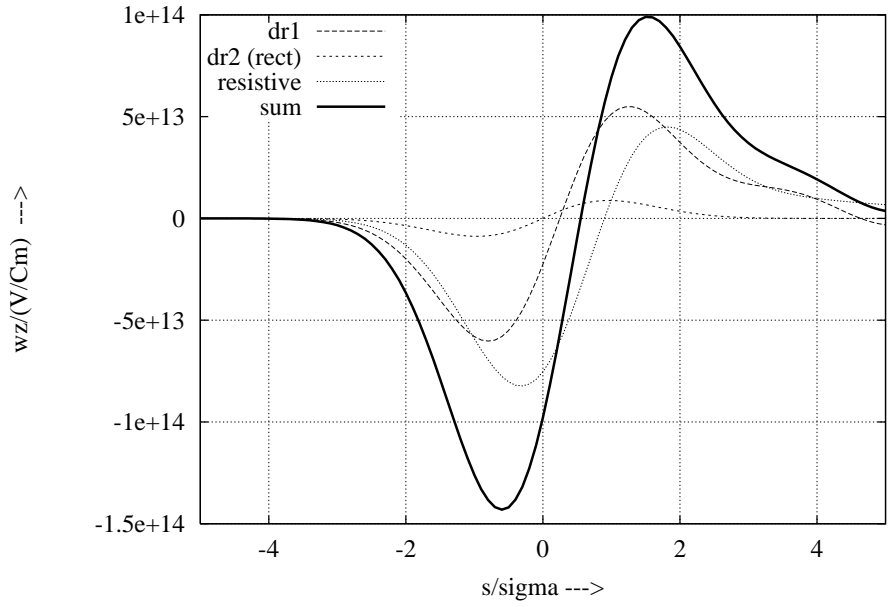


Figure 6: Longitudinal wake potentials for the undulator pipe ($b=5\text{mm}$). The wakes labeled dr1, dr2(rect) and resistive are caused by the rectangular periodic surface function δr_{1pr} , the sinusoidal surface function δr_{2p} and the surface resistivity respectively. The sum of all contributions is labeled sum.

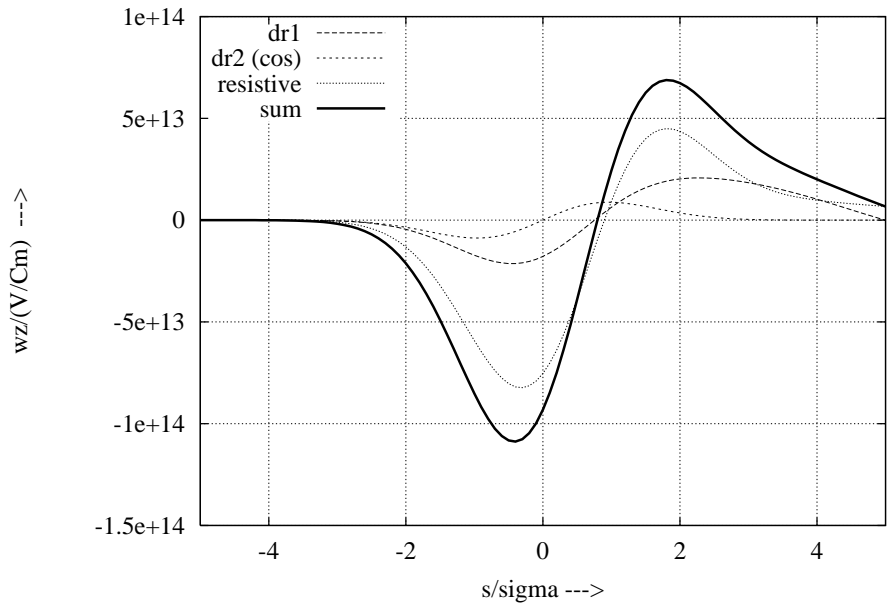


Figure 7: Same case as in Fig. 6 but for the sinusoidal surface function δr_{1ps} .