

# Influence of different focusing solutions for the TESLA X-ray FEL's on debunching of the electron beam

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## Abstract

For SASE-FELs the total undulator length increases with decreasing wavelength. In the X-ray wavelength range, the optimum  $\beta$ -function is of the order of 40 m, which makes a separate focusing structure the best choice. In principle, three periodic focusing structures can be considered, namely a singlet (FODO), doublet or a triplet structure. In this report, the three different types of focusing for the TESLA X-ray FEL parameters will be discussed.

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## 1. Introduction

The TESLA X-ray FEL facility is a proposed user facility in the wavelength range between 0.085 and a 6 nm. The longest wavelength is about the shortest wavelength produced by the TESLA Test Facility (TTF) FEL, under construction at DESY [1]. Similar to the TTF-FEL, the TESLA X-ray FEL employs the principle of Self Amplified Spontaneous Emission (SASE), in which the radiation is built up from the shotnoise produced by the electron beam in a single pass through the undulator. This scheme does not need high reflectivity mirrors which are not available in this wavelength range anyhow.

The layout of the X-ray FEL is shown in Fig. 1. The electron beam produced by the accelerator will have an energy between 15 and 30 GeV, distributed over two beam lines, one in the energy range from 15 to 25 GeV, a second one between 20 and 30 GeV. After the beam is collimated both in energy and size, a fast kicker at the end of both beamlines will make it possible to distribute the electron beam to two undulator beam lines each, i.e. four lines in total (see Fig. 1). For each energy, one undulator has a fixed gap, the second has a variable gap. The wavelength to be generated in the fixed gap device determines the electron beam energy, whereas the gap in the second undulator determines, at this same energy, the wavelength for a second user. Therefore, both users can have an independently tunable wavelength at their disposal. While the electron

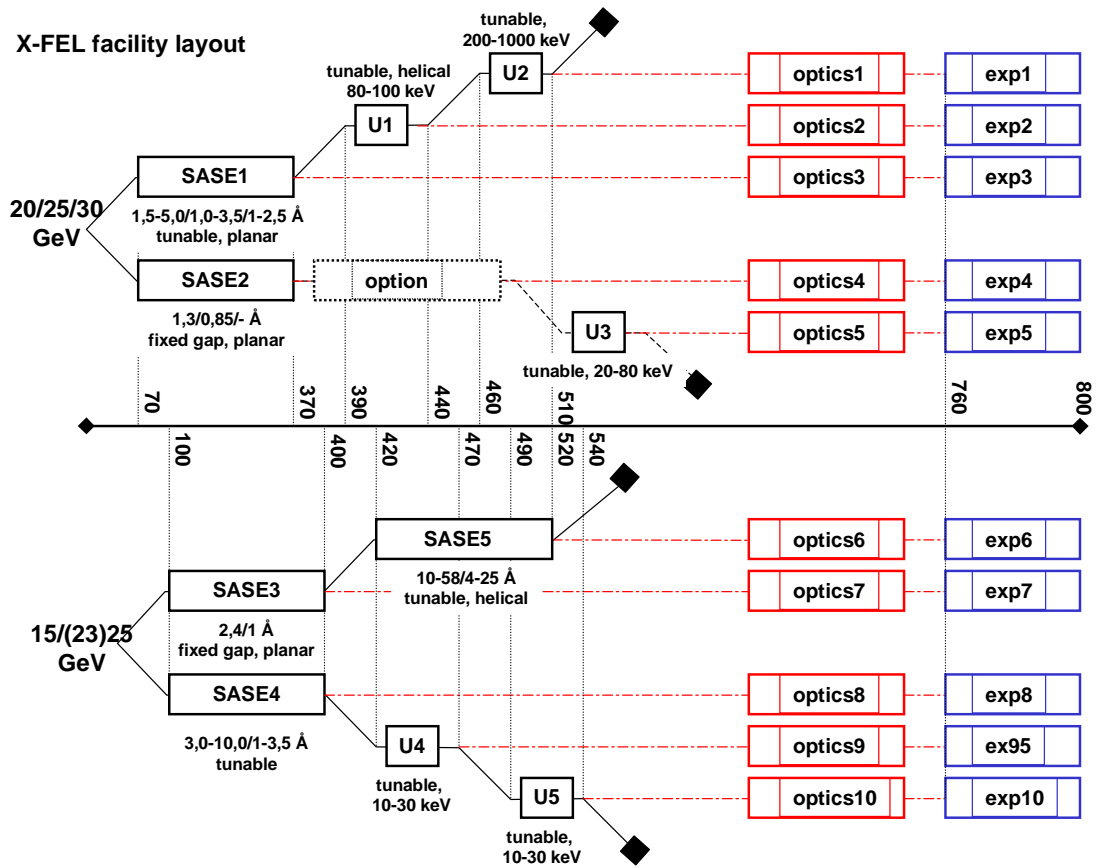


Fig. 1. Schematic layout of the TESLA X-ray FEL facility. The facility consists of four planar undulators in the short wavelength range (0.85 to 10Å), one helical undulator at longer wavelengths (4 to 58 Å) and five spontaneous radiators. The future optional extension for a two-stage FEL to reduce the bandwidth is already shown.

beam produces the SASE FEL radiation, its energy reduces due to emission of synchrotron radiation. At the same time, the energy spread increases. Therefore, the beam can no longer be used to generate SASE radiation at the same wavelength. At longer wavelengths, however, it is still possible to produce intense SASE light. In addition, broadband spontaneous radiation, which does not suffer from reduced beam quality, can also be produced. Therefore, behind one undulator beamline (SASE-3), a second undulator is placed to produce SASE radiation at a longer wavelength (SASE-5), behind three (SASE-1, SASE-2 and SASE-4), spontaneous radiators are placed. Behind SASE-2, there is also space reserved for a so-called two-stage FEL option. This principle allows to narrow the FEL radiation bandwidth without reducing the output power, thus increasing the spectral brilliance by close to three orders of magnitude [2].

Because saturation has to be reached in a single pass, the electron beam has to be of high quality. This means that both transverse emittance and energy spread have to be small. Especially the normalized emittance of  $1.6 \pi$  mm mrad is a challenge, since this value is at the limit of what has been achieved so far. Furthermore, because the peak currents needed for the FEL cannot be

<b>Electron beam parameters</b>	
Electron beam energy, $\mathcal{E}$	13 – 50 GeV
Peak current, $I$	5.0 kA
rms bunch length	25 $\mu\text{m}$
rms transverse beam size at 25 GeV	38 $\mu\text{m}$
rms normalized slice emittance, $\epsilon_n$	1.6 mm mrad
rms slice energy spread, $\sigma_E$	2.5 MeV
Macropulse duration	1070 $\mu\text{s}$
Bunch separation	93 ns
Number of pulses per train	11500
Repetition rate	5 Hz
<b>Undulator parameters for SASE-1</b>	
Period	60 mm
Gap	22 mm
$K_{\text{rms}}$ -parameter	2.62
Peak magnetic field	0.67 T
Unit length	5 m
Number of units	53

Table 1

Electron beam parameters at the entrance of the TESLA X-ray FEL undulators and parameters for the SASE-1 undulator tuned to a wavelength of 1 Å at a beam energy of 25 GeV, for which results are shown in the next sections.

achieved for any existing electron source, the beam has to be longitudinally compressed and transversely focused. For the beam parameters given in Table 1, the saturation length, and thus the minimum undulator length, has been calculated both analytically and with the use of simulation codes [3–5]. The saturation length is usually given in terms of magnetic undulator length. However, each undulator system with a total length of several hundred meters, consists of cells. Each cell is built up of an undulator segment, a phase shifter to match the phase of electrons and radiation field between segments, and a focusing element to keep the beam at a constant small diameter. The most commonly proposed focusing structure is a FODO structure, in which one needs one quadrupole per undulator segment, which is alternately focusing or defocusing in one direction. An alternative is a triplet focusing structure, in which 3 quadrupoles are needed per undulator segment. The segment length can be considerably longer in this case. A third alternative, which has been discussed for the TTF-FEL is a doublet structure [6]. In this case, two quadrupoles per undulator segment are needed with a segment length between the two options mentioned before. Because of the tight tolerances on the magnetic field and hence on the maximum allowable gap variation of at most 2  $\mu\text{m}$ , the maximum support length for an undulator unit is 5 m [7,8]. With a FODO structure, each unit has the appropriate end fields. For a doublet and triplet structure, with segments consisting of two and three 5 m long units respectively, only the outer ends need special sections. The three structures that are studied here are schematically shown in Fig. 2.

In this report, these three different undulator focusing options have been studied. The first option is the FODO-structure. For the FODO option, the undulator segments are typically 5 m long. The second option is a doublet-lattice. It has the advantage that it can even span over

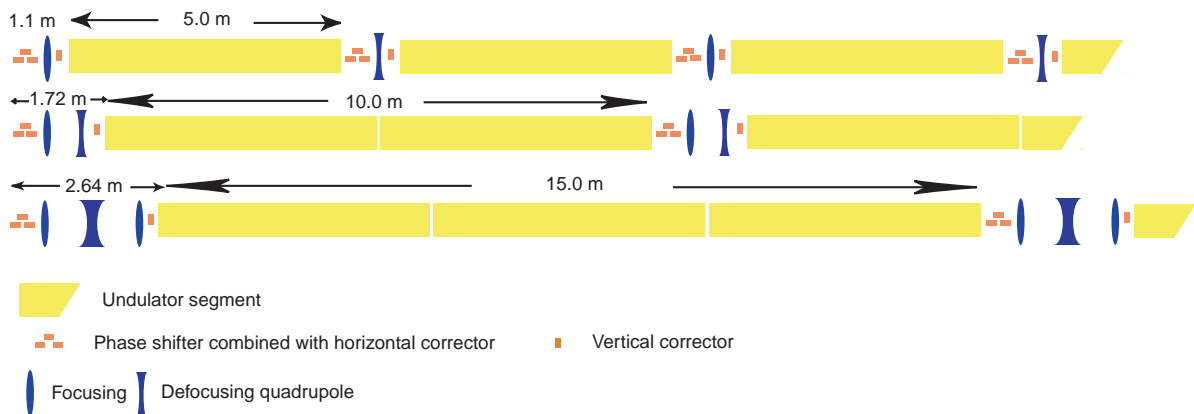


Fig. 2. Three different options of focusing for the TESLA X-ray FEL. The top figure shows a FODO structure, the middle a doublet structure and the bottom figure shows a triplet structure.

two undulator units with a total length of 10 m. For a triplet structure, the magnetic length between focusing elements is 15 m, which is enough to span a distance of three units. In all cases, however, the space needed for the phase shifters and correctors is 800 mm.

The next section discusses the influence of debunching for the three options. In Sec. 3, the simulation results comparing them are presented. The last section discusses the advantages and disadvantages and the reason for the final choice.

## 2. Debunching in a separate focusing structure

In any FEL, debunching occurs due to longitudinal velocity spread, i.e. due to energy spread and finite emittance of the electron beam and the betatron oscillation performed by individual electrons. The influence of energy spread effects and emittance has been well recognized and leads to limits on those two parameters for all FELs [9]. Also the influence of focusing has already been described before [10].

The additional problem occurring with the separate focusing is the pathlength difference due to different initial conditions for the individual electrons. An electron with zero transverse position and momentum will enter a next undulator segment at a different time than an electron with an initial angle and offset. Averaged over all electrons, this results in debunching in the space between undulator segments, and therefore a decrease in gain. If one looks at the difference in path length of an electron of initial offset and angle  $(x, x')$  compared to an on axis electron, this can be written as

$$\Delta\ell = \int_0^z (\sqrt{1 + x'^2(\zeta)} - 1) d\zeta \approx \frac{1}{2} \int_0^z x'^2(\zeta) d\zeta,$$

and similar in the  $y$ -direction. Averaged over all electrons, this results in

$$\frac{2\delta\ell}{\epsilon} = \frac{2\langle\Delta\ell\rangle}{\epsilon} = \beta_0 \int_0^z C'^2(\zeta)d\zeta + \gamma_0 \int_0^z S'^2(\zeta)d\zeta - 2\alpha_0 \int_0^z C'(\zeta)S'(\zeta)d\zeta, \quad (1)$$

with  $\alpha_0, \beta_0$  and  $\gamma_0$  the Twiss parameters and  $C'$  and  $S'$  the transfer functions (see for example [11]). The value for  $\delta\ell$  gives an average path length difference,  $\sigma_\ell = \langle\Delta\ell^2\rangle - \langle\Delta\ell\rangle^2$  gives the debunching. From Eq. (1) one can see that the effect becomes smaller for smaller emittance. An alternative would be to reduce the undulator segment length. This would make a smaller and more constant  $\beta$ -function possible, and thus reduce values of the Twiss parameters, Because the FODO-lattice has the shortest segment length, one would expect that debunching would be less than in the other two cases. In addition, because in the doublet and triplet structures, the (de)focusing quadrupoles counteract each other, the path length difference is expected to be larger than in case of a single quadrupole, as for the FODO structure.

It should be noted here that for a combined focusing structure like the one for the TTF-FEL [12], the same effect occurs. Inside the undulator, however, the debunching effect is counteracted by the bunching effect due to the potential well caused by the combination of the undulator and the radiation field, which is not present in the field free space between segments. Furthermore, the focusing is more equally distributed along the undulator segment and therefore the effect is smaller within a distance of for example a gain length in case of the TTF-FEL. For the TESLA X-ray FEL, with its gain length much longer than an undulator segment length, the debunching effect of the separate focusing is estimated to be comparable to that of the TTF-FEL with a combined focusing undulator, and therefore not a dominant effect, as seen in the next section.

### 3. Comparison of the three different focusing solutions

The first set of simulations has been done for the FODO lattice. The power and bunching along the undulator are shown in Fig. 3. As can be seen, both power and bunching increase steadily until saturation is reached. The influence of the intersections is a mere geometrical increase in saturation length. No influence of debunching is visible. The total length at which saturation is reached is 254 m, which corresponds to 210 m magnetic undulator length, as shown in Table 2.

For a doublet structure, the magnetic undulator length required to reach saturation is, within the accuracy of the simulation, the same as for a FODO structure as can be seen in Fig. 4. The total length needed is reduced by a few percent due to a slightly shorter intersection length. Some debunching, as described in the previous section, is visible in the undulator intersections. At this level, however, no gain reduction can be observed.

In Fig. 5, the power and bunching are shown for a triplet structure. The magnetic undulator length needed to reach saturation has increased compared to the two previous cases. This

	FODO	Doublet	Triplet
Number of quads. per cell	1	2	3
Number of correctors per cell (H/V)	3/1	3/1	3/1
Cell length [m]	6.10	11.72	17.64
Intersection length [m]	1.10	1.72	2.64
Max. quad. gradient [T/m]*	27	47	47
Filling factor (und./total length)	.82	.85	.85
$\bar{\beta}$ [m]	45	45	45
$P_{sat}$ [GW]	23	24	24
$L_{sat}$ [m]	210 (254)	211 (245)	220 (256)

\*The central quadrupole of the triplet has an optical length of 400 mm, all others have an optical length of 200 mm.

Table 2

Global information on the cell structure for a FODO, doublet and triplet focusing options for the SASE-1 undulator tuned to a wavelength of 1 Å for an electron beam energy of 25 GeV. For the saturation length both the magnetic undulator length and the total length (between brackets) required are indicated. The magnetic undulator length is about the same for all cases. The difference in total length is due to the difference in total intersection length.

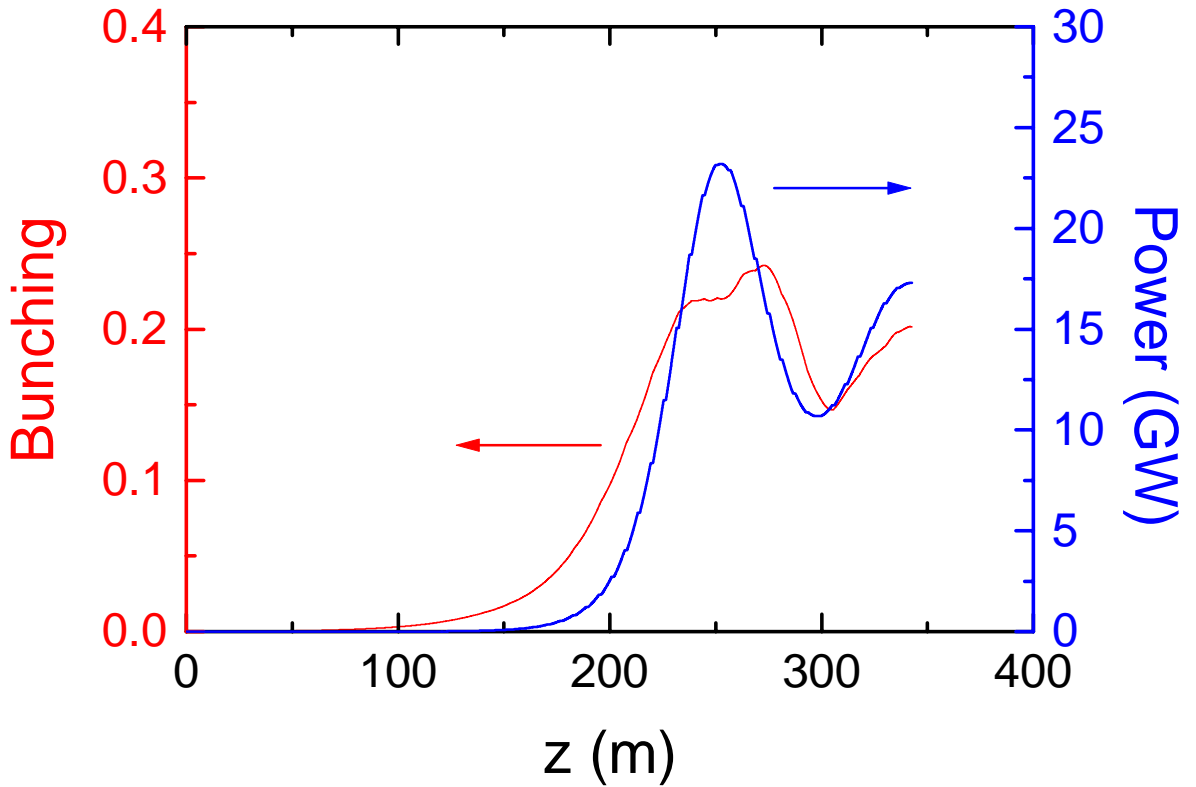


Fig. 3. Bunching (left) and power (right) along the undulator for a separated FODO structure. Parameters are as given in Table 1 for an electron beam energy of 25 GeV and a radiation wavelength of 1 Å. Specific parameters for the FODO structure are quoted in Table 2.

is caused by an increased debunching in the undulator intersection, which has become more pronounced. Because the filling is approximately the same as for the doublet system, the total length required has increased by the same amount.

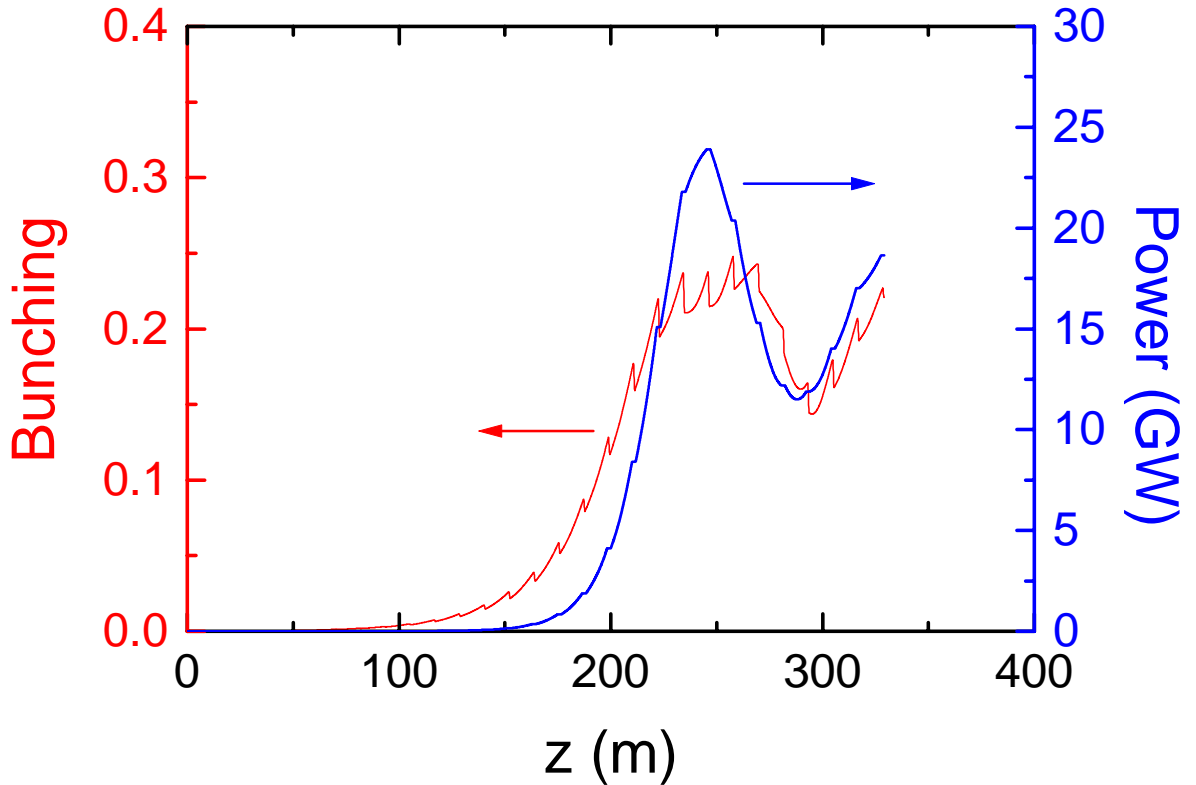


Fig. 4. Bunching (left) and power (right) along the undulator for a separated doublet structure. Parameters are as given in Table 1 for an electron beam energy of 25 GeV and a radiation wavelength of 1 Å. Specific parameters for the doublet structure are quoted in Table 2.

From these results one sees that the magnetic undulator length needed for the FODO and doublet structures are equal. The small difference in total length is just a geometrical effect due to the smaller intersection length in case of the doublet structure. The total length needed for a triplet structure is comparable to that for a FODO structure. However, more magnetic undulator length is needed. This is caused by debunching in the intersections. Both doublet and triplet structures need a maximum quadrupole gradient of about 50 T/m.

For the SASE-2 to SASE-4 undulators, results are similar. For the SASE-5 undulator, the  $\beta$ -function is 15 m instead of the 45 m for SASE-1 to SASE-4. In order to reach this value, the quadrupole gradient has to be increased to approximately 50 T/m in case of the FODO structure. For a doublet structure, this would lead to a gradient close to the limit for these quadrupoles of 100 T/m<sup>1</sup>. Because the saturation length is already comparable or even larger than in case of a FODO structure, a FODO structure is the optimum choice for the SASE-5 undulator.

So far, both FODO and doublet focusing structures show comparable performance. An important issue is how sensitive the three different structures are to quadrupole offsets. For a FODO

<sup>1</sup> For the beam based alignment procedure, the quadrupole gradient has to be varied around its nominal value by about 20%, leading to a maximum gradient of 120 T/m [13].

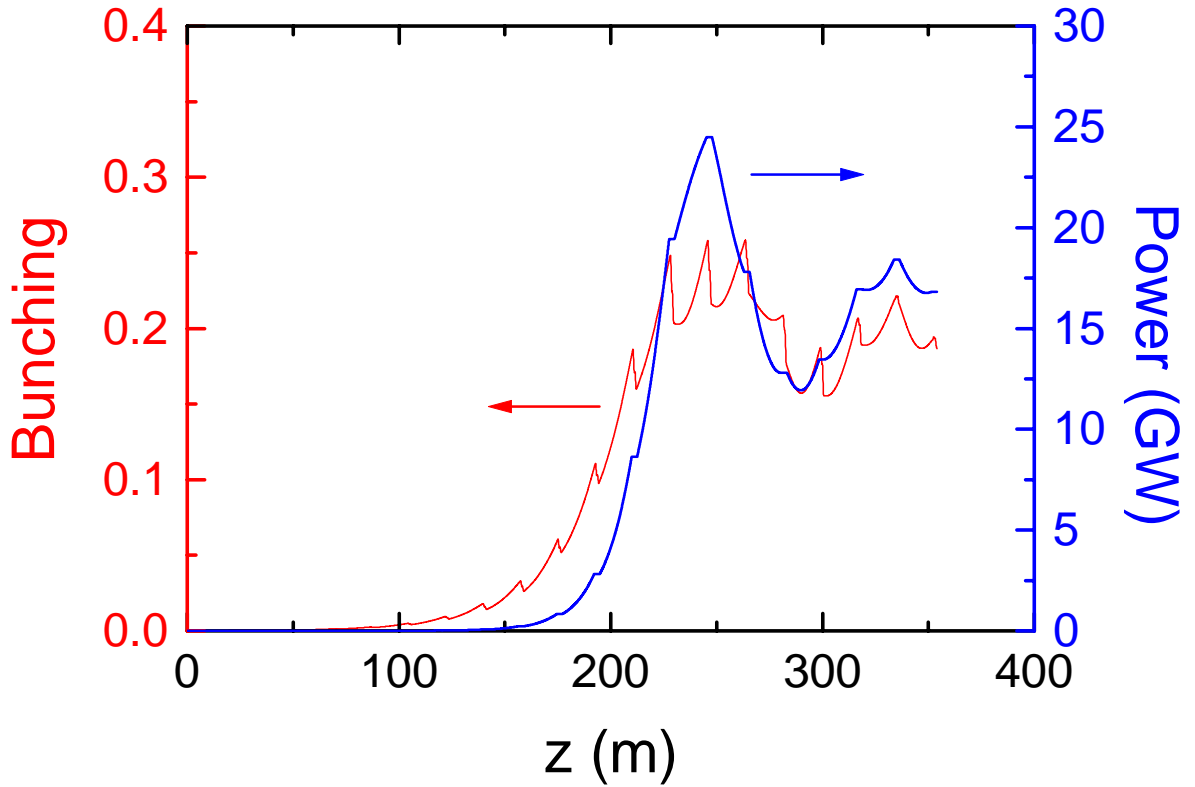


Fig. 5. Bunching (left) and power (right) along the undulator for a separated triplet structure. Parameters are as given in Table 1 for an electron beam energy of 25 GeV and a radiation wavelength of 1 Å. Specific parameters for the triplet structure are quoted in Table 2.

structure, the different sources of errors has been discussed in Ref. [8]. It has been shown there that one of the most critical issues is the overlap between radiation field and electron beam. An upper limit for the rms-value of the deviation of the electron beam from a straight line is approximately  $7 \mu\text{m}$ . The main source of orbit deviation is due to kicks by misaligned quadrupoles. For the SASE-1 undulator, the rms electron beam deviation as a function of quadrupole misalignment is shown for the three different focusing structures in Fig. 6. Each point in this figure is the result of 1000 different random distributions of quadrupole offsets within the range indicated. It can be clearly seen that the beam deviation scales linearly with the quadrupole offset. Furthermore, the sensitivity to quadrupole kicks is much larger for a doublet than for a FODO structure. The triplet structure gives the largest beam deviation for a given random quadrupole offset. In order to remain inside the  $7 \mu\text{m}$  tolerance level, the quadrupoles have to be aligned within  $0.4 \mu\text{m}$ ,  $0.8 \mu\text{m}$  and  $1.5 \mu\text{m}$  for respectively the triplet, doublet and FODO structures. Since residual kicks by the undulator are not included, the actual values are even smaller. The tolerance levels become very close to normal variations in position that are to be expected by ground motion above a 2 Hz frequency, i.e. 100 nm [14]. In addition, even in theory, it is difficult to align the quadrupoles to this level using beam based alignment [13]. Therefore, the FODO structure seems to be the most feasible structure that can be used.



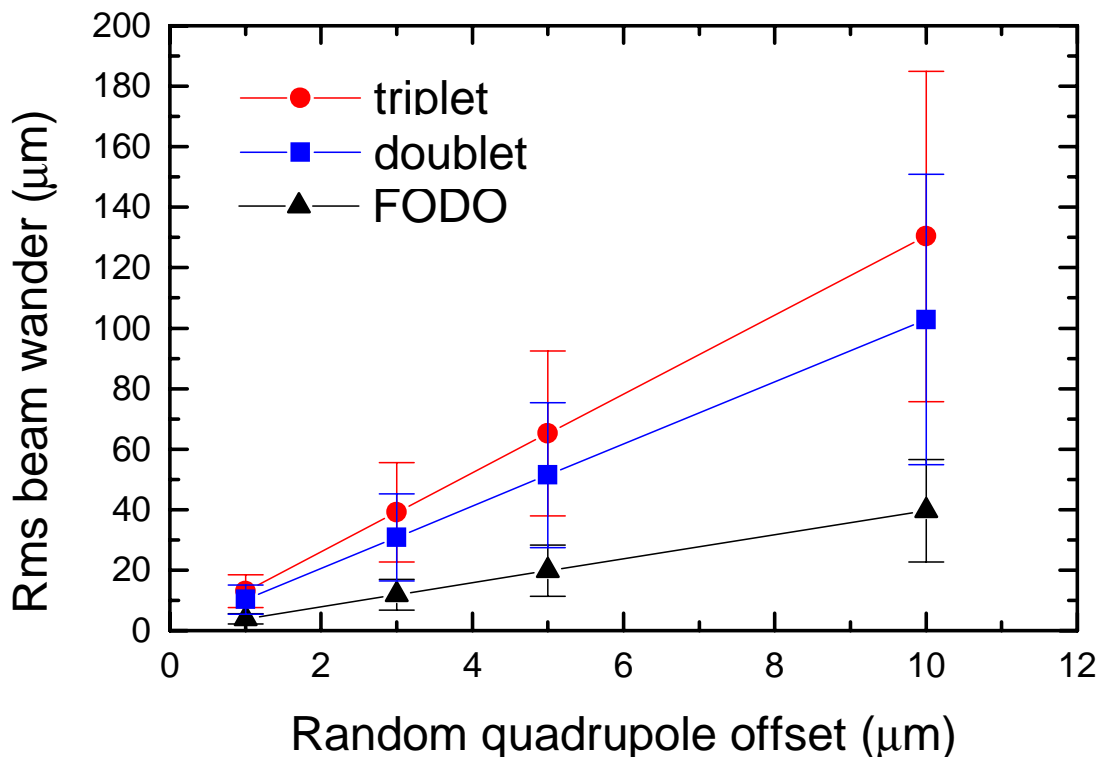


Fig. 6. Rms beam wander versus random quadrupole offset for FODO, doublet and triplet focusing structures. The maximum acceptable beam wander is approximately  $7 \mu\text{m}$ . The parameters for the three different structures used are quoted in Table 2.

#### 4. Discussion

Based on the results presented in the previous section, there is clear preference to use the FODO structure to focus the electron beam. The doublet structure requires a comparable magnetic undulator length. In case of the triplet structure, it is slightly longer. The quadrupole gradient needed for a doublet and triplet structure is twice the value needed for a FODO structure. Although such a gradient can be reached without any problems, it results in larger transverse kicks due to misalignment errors and thus to tighter tolerances that have to be achieved.

An additional problem is that for the SASE-5 undulator, with an average  $\beta$ -function of 15 m, the required gradient cannot be obtained with the same quadrupoles. As a consequence, one would need different quadrupoles for different undulators, which is not a desirable situation. The quadrupoles with gradients needed for a FODO structure, on the other hand, could be used for all undulators.

A second reason to choose a FODO structure is related to the assumed electron beam parameters. It is quite possible that within the timeframe in which the TESLA X-ray FEL is realized, new methods are found to improve the electron beam quality. Particularly if the transverse emittance can be reduced, the optimum  $\beta$ -function would be reduced simultaneously. The minimum

$\beta$ -function that can be achieved is given by geometrical considerations only, independent of energy. For a triplet structure, this minimum average  $\beta$ -function is simply  $\bar{\beta}_{\min} = L/\sqrt{3}$ , with  $L$  the distance between the triplets. In order to achieve this  $\beta$ -function, a triplet focal length of  $L/3$  is needed. The focal length for the inner and the two outer quadrupoles is  $f_- = -\sqrt{d \cdot L}/6$  and  $f_+ = \sqrt{2d \cdot L}/3$ , respectively. Here,  $d$  is the distance between quadrupoles in thin lens approximation. For the parameters used here ( $L \approx 18$  m and  $d \approx 0.75$  m), this gives  $\bar{\beta} \approx 10$  m and  $f_+ = 3$  m, corresponding gradient at 25 GeV of 140 T/m. Therefore, this minimum  $\beta$ -function can only be achieved with an increase in overall length to get the required focal length of the quadrupoles. For a FODO structure, a similar  $\bar{\beta}$  of 10 m can be achieved with a quadrupole gradient of less than 80 T/m.

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