Klystron Instability of a Relativistic Electron Beam in a Bunch Compressor

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Abstract

In this paper we consider a klystron-like mechanism of amplification of parasitic density modulations in an electron bunch passing a magnetic bunch compressor. Analytical expressions are derived for the small-signal gain. The main emphasis is put on analysis of coherent synchrotron radiation (CSR) effects.

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1 Introduction

Magnetic bunch compressors are designed to obtain short electron bunches with a high peak current for linac-based short-wavelength FELs [1–5] and future linear colliders [5–7]. The basic principle of compression is very simple. A relativistic electron bunch accumulates energy chirp while passing RF accelerating structures off-crest and then gets longitudinally compressed due to an energy-dependent path length in the magnetic compressor (for instance, in a chicane). Since, however, electron bunches are very short and intensive, collective effects like coherent synchrotron radiation (CSR) [8] can seriously influence beam dynamics in compressors [9].

In the recent experiments with bunch compressors [10–12] and in numerical simulations [13] the strong high-frequency perturbations of longitudinal phase space has been observed. The self-consistent simulations [14] of beam dynamics in the TESLA Test Facility (TTF) bunch compressor chicane (BCC), taking into account CSR effects, have also shown phase space fragmentation. It has been explained by strong enhancement of CSR effects due to the locally peaked (non-Gaussian) density distribution created during compression process because of RF nonlinearity (see also [15]).

It has been mentioned in [14] that another mechanism can be considered which is also relevant for the ideal linear RF modulation (or, even without modulation). Namely, high-frequency components of the beam current spectrum (higher than typical inverse pulse duration) cause energy modulations at the same frequencies due to wakefields. The energy modulation is converted into an induced density modulation while the beam is passing the bunch compressor. If the wakefields are strong enough, the induced modulation can be much larger than the initial one. In other words, the system can be treated as a high-gain klystron-like amplifier. The general tendency is that higher frequencies (to some extent) are going to get amplified stronger so that they may become much better pronounced in comparison with the case of undisturbed compression. Thus, the charge distribution and, more generally, the longitudinal phase space can be essentially modified.

In this paper we study such a mechanism analytically in linear approximation. Since it is difficult to measure (simulate) small high-frequency perturbations in the initial state of the beam, one cannot exactly predict its final state. Thus, our goal is to calculate (estimate) the gain as a function of frequency. If the gain is large then one may expect significant modifications of longitudinal phase space in the bunch compressor, and vice versa. In section 2 we study the dynamical aspect of the problem assuming linear energy chirp along the beam and the given amplitude of parasitic energy modulation at some frequency. In section 3 we consider the case when these energy perturbations are created due to wakefields upstream of bunch compressor. In section 4 we study CSR in the bunch compressor chicane. In section 5 we estimate effective density modulations due to the shot noise in electron beam.

2 Compression of the beam with linear energy chirp and superimposed sinusoidal modulation

In this paper we consider 1-D model of the electron beam. An undisturbed phase space distribution of the beam with dc current, linear energy chirp along the beam and Gaussian energy spread can be described with the following function:

$$f(z, \delta\gamma) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma} \exp\left[-\frac{(\delta\gamma - h\gamma_0 z)^2}{2\sigma_\gamma^2} \right] , \tag{1}$$

where z is the coordinate along the beam (particles with positive values of z are placed behind the particle with $z = 0$), $\gamma_0 = \mathcal{E}_0/(mc^2)$ is the nominal energy in units of the rest energy, m is electron's mass, $\gamma_0 \gg 1$, c is the velocity of light, $\delta\gamma = (\mathcal{E} - \mathcal{E}_0)/(mc^2)$ is the energy deviation from the nominal value, $\sigma_\gamma = \sigma_\mathcal{E}/(mc^2)$ is the rms local energy spread, $h = \mathrm{d}(\delta\gamma)/(\gamma_0 \,\mathrm{d}\, z)$ describes linear energy chirp along the beam, I_0 is the beam current. Normalization is chosen in such a way that after integration over $\delta\gamma$ we get the current. We assume γ_0 to be large and consider small energy deviations $\delta\gamma \ll \gamma_0$ although formally we let $\delta\gamma$ extend from $-\infty$ to ∞. The model of dc current allows us to exclude edge effects from consideration and to deal with small sinusoidal modulations.

To describe phase space transformation in the bunch compressor we assume a linear dependence of path length in the compressor on $\delta\gamma/\gamma_0$ described by the element of a linear transfer matrix:

$$R_{56} = \frac{\partial z}{\partial(\delta\gamma/\gamma_0)} \ .$$

Then a particle position in the beam before and after compression, z_{i} and z_{f}, are connected by

$$z_{\mathrm{f}} = z_{\mathrm{i}} + R_{56}\frac{\delta\gamma}{\gamma_0} \ .$$

Therefore, to describe the final state of the beam we should substitute z in (1) by $z - R_{56}\delta\gamma/\gamma_0$. Then the new distribution will have the form of (1) where I, σ_γ, and h are substituted by CI, $C\sigma_\gamma$, and Ch, respectively. Here C is the compression factor:

$$C = \frac{1}{1 + hR_{56}} \ .$$

For compression one should provide $hR_{56} < 0$. For instance, $R_{56} < 0$ for the chicane so that h has to be positive in this case. In addition, in this paper we restrict our consideration by the condition $1 + hR_{56} > 0$, i.e. the beam is undercompressed.

Now let us consider an energy modulation at some frequency ω on top of the linear chirp. In front of the bunch compressor the phase space distribution has the form:

$$f(z,\delta\gamma) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma}\exp\left\{-\frac{[\delta\gamma - h\gamma_0 z + \Delta\gamma\sin(kz)]^2}{2\sigma_\gamma^2}\right\} \ ,$$

where $k = \omega/c$ and $\Delta\gamma$ is the amplitude of energy modulation. As it was done above, we substitute z by $z - R_{56}\delta\gamma/\gamma_0$ to describe the change of distribution function in the bunch compressor. Then we integrate over $\delta\gamma$ in order to get current as a function of z:

$$I(z) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma}\int\limits_{-\infty}^{\infty}\mathrm{d}\,\delta\gamma\exp\left\{-\frac{[\delta\gamma(1 + hR_{56}) - h\gamma_0 z + \Delta\gamma\sin(kz - kR_{56}\delta\gamma/\gamma_0)]^2}{2\sigma_\gamma^2}\right\}$$

After change of variables $x = [\delta\gamma(1 + hR_{56}) - h\gamma_0 z]$ the integral takes the following form:

$$I(z) = \frac{CI_0}{\sqrt{2\pi}\sigma_\gamma}\int\limits_{-\infty}^{\infty}\mathrm{d}\,x\exp\left\{-\frac{[x + \Delta\gamma\sin(Ckz - CkR_{56}x/\gamma_0)]^2}{2\sigma_\gamma^2}\right\}$$

The integral of such a form is known to describe the process of density bunching starting from initial sinusoidal energy modulation but without linear energy chirp (see, for instance, [16]). Making integration and Fourier expansion, one gets:

$$I(z) = CI_0 \left[1 + 2 \sum_{n=1}^{\infty} J_n \left(nCkR_{56} \frac{\Delta\gamma}{\gamma_0} \right) \exp \left(-\frac{1}{2} n^2 C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2} \right) \cos(nCkz) \right] \ . \quad (2)$$

Here J_n is the Bessel function of nth order. Without compression ($h = 0$, $C = 1$) the expression (2) is reduced to the well-known one [16].

Analyzing (2) we see that the frequency range (of initial modulation), in which the beam can be effectively bunched, is limited by $k \le (CR_{56}\sigma_\gamma/\gamma_0)^{-1}$. Within this range the condition $Ck|R_{56}|\Delta\gamma/\gamma_0 \ge 1$ means that the beam is completely bunched and the phase space is fragmented. For $k \simeq (CR_{56}\sigma_\gamma/\gamma_0)^{-1}$ this happens when $\Delta\gamma \ge \sigma_\gamma$. It is worth mentioning that σ_γ always stands for the initial energy spread (before compression).

In this paper we will use linear approximation assuming that $Ck|R_{56}|\Delta\gamma/\gamma_0 \ll 1$. This leaves us with only the first harmonic of the beam current ($J_1(X) \simeq X/2$):

$$I(z) \simeq CI_0 \left[1 + \rho_{\mathrm{ind}}\ \mathrm{sgn(R_{56})} \cos(\mathrm{Ckz}) \right] \ , \quad (3)$$

where $\mathrm{sgn(R_{56})}$ is the sign of R_{56} and ρ_{ind} is the amplitude of the first harmonic in the final state of the beam:

$$\rho_{\mathrm{ind}} = Ck|R_{56}| \frac{\Delta\gamma}{\gamma_0} \exp \left(-\frac{1}{2} C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2} \right) \ . \quad (4)$$

We have considered here the model of infinitely long beam. The results of this paper can be used for a bunch with finite length σ as soon as the following condition is satisfied:

$$k\sigma \gg 1 \ . \quad (5)$$

3 Wakefields upstream of a bunch compressor

Let us assume that upstream of the bunch compressor there is a small density perturbation ρ_i at some frequency:

$$I(z) = I_0 \left[1 + \rho_\mathrm{i} \cos(kz)\right] \ . \tag{6}$$

Due to some wakefields upstream of compressor the beam gets modulated in energy at the same frequency with the amplitude $\Delta\gamma$. Describing the action of wakefields by longitudinal impedance $Z(k)$ we can connect the amplitudes of energy and density modulations as follows:

$$\Delta\gamma = \frac{|Z(k)|}{Z_0} \frac{I_0}{I_\mathrm{A}} \rho_\mathrm{i} \ , \tag{7}$$

where $Z_0 = 377\ \Omega$ is the free-space impedance and $I_\mathrm{A} = 17$ kA is the Alfven current. Then, using (4) we calculate the amplitude of the induced density modulation at the end of bunch compressor. In general case, to find final density modulation ρ_f one should sum up induced modulation and (transformed to the end of compressor) initial one, taking care of phase relations. But in this paper we use approximation

$$\rho_\mathrm{i} \ll \rho_\mathrm{ind} \ll 1 \ .$$

In other words, $\rho_\mathrm{f} \simeq \rho_\mathrm{ind}$ and the gain in density modulation

$$G = \frac{\rho_\mathrm{f}}{\rho_\mathrm{i}} \simeq \frac{\rho_\mathrm{ind}}{\rho_\mathrm{i}}$$

is assumed to be high, $G \gg 1$ (otherwise the effect, considered in this paper, is not of great importance). Under this approximation the gain depends neither on phase of $Z(k)$ nor on sign of R_{56} and is equal to

$$G = Ck|R_{56}| \frac{I_0}{\gamma_0 I_\mathrm{A}} \frac{|Z(k)|}{Z_0} \exp\left(-\frac{1}{2} C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2}\right) \tag{8}$$

For broadband nonresonant wakefields the product $k|Z(k)|$ is usually a growing function of k. For such cases the maximal gain is achieved at

$$k_{\mathrm{opt}}^{-1} \simeq \frac{\sigma_\gamma}{\gamma_0}|R_{56}|C \ . \tag{9}$$

The optimal final frequency (when the beam is compressed) roughly does not depend on compression factor C. A crude estimate for the maximal gain is

$$G_{\max} \simeq \frac{I_0}{\sigma_\gamma I_{\mathrm{A}}}\frac{|Z(k_{\mathrm{opt}})|}{Z_0} \ . \tag{10}$$

The term $I_0/(\sigma_\gamma I_{\mathrm{A}})$ is proportional to a longitudinal brightness (particles density in longitudinal phase space). In practice the phase space distribution can be of complex shape. We note that the local energy spread should be taken for estimations of amplification effect.

In the considered case the influence of beam emittance ϵ on longitudinal dynamics is a second order effect and is negligible when

$$k\epsilon S_{\mathrm{c}}/\beta \ll 1 \ , \tag{11}$$

where S_{c} is the length of a path through compressor and β is the beta-function. This condition is always met in practice.

The formulas of this section can be used when the effect of wakefields in front of compressor is much stronger than any collective effects inside the bunch compressor.

4 CSR in a bunch compressor chicane

Wakefields can also exist inside bunch compressors. We consider here coherent synchrotron radiation which is an intrinsic feature of magnetic compressors. CSR effects can be minimized there but not avoided. A CSR-induced beam instability in storage rings has been investigated in [17]. That instability develops continuously, in small increments, like most instabilities of relativistic electron beams. We analyze here quite different klystron-like mechanism of the instability in a bunch compressor.

4.1 The model

In this section we consider a simplified case when the beam is not compressed (no energy chirp: $h = 0$, $C = 0$). While the formulae of the previous section are pretty general and do not depend on a type of the bunch compressor, in this section we have to choose a specific model. We consider a symmetric three-dipole chicane[1] where the first and the last dipoles have the length L_{d}, and the middle one is as long as $2L_{\mathrm{d}}$. The bending angle in the first dipole θ is small, $\theta = L_{\mathrm{d}}/R \ll 1$ (R is the bending radius), and the distance ΔL between two subsequent dipoles is much larger then the dipole length:

$$\Delta L \gg L_{\mathrm{d}} \ . \tag{12}$$

The R_{56} can then be expressed in a simple form:

$$R_{56} \simeq -2\Delta L \theta^2 . \tag{13}$$

To describe CSR we use the steady-state model neglecting transient effects. The domain of validity of this model can be estimated as:

$$L_{\mathrm{d}} \gg (24R^2/k)^{1/3} \ . \tag{14}$$

We neglect the influence on CSR of transverse beam size and of the screening effect of the vacuum chamber requiring that [9,19]

$$(R/b^3)^{1/2} \ll k \ll (R/\sigma_\perp^3)^{1/2} \ , \tag{15}$$

where b is the transverse size of vacuum chamber and $\sigma_\perp$ is that of electron beam. Since we analyze the dynamics inside bunch compressor we cannot use the condition (11) for neglecting emittance effect on longitudinal dynamics. The relevant condition will be presented below. We also neglect here collective transverse forces [18].

[1] Under limitations, accepted in this section, all the results are valid for a four-dipole chicane, too.

Under the conditions (14) and (15) the module of CSR impedance in the first and the last dipoles can be expressed as [19]

$$\frac{|Z(k)|}{Z_0} = \frac{|Z_{\mathrm{l}}(k)|L_{\mathrm{d}}}{Z_0} = \frac{2\Gamma(2/3)}{3^{1/3}}\frac{L_{\mathrm{d}}k^{1/3}}{R^{2/3}} \ , \tag{16}$$

and in the middle dipole it is two times larger. Here $Z_{\mathrm{l}}(k)$ is the impedance per unit length and $\Gamma(...)$ is the complete gamma-function, $\Gamma(2/3) \simeq 1.354$.

One can use the impedance (16) to calculate energy modulation only if the relative change of density modulation is small on the scale of formation length $(24R^2/k)^{1/3}$ (see [20] for more details). In our model the density changes on the scale of L_{d} and this condition is met (see (14)).

4.2 "Cold" electron beam

Let us first consider the case when the energy spread can be neglected. In the framework of the accepted model we consider the two-stage amplification in the bunch compressor. The gain is assumed to be large in each stage.

An initial density perturbation (6) causes energy modulation in the first dipole (see (7), (16)). This modulation increases linearly inside the dipole:

$$\frac{\mathrm{d}\,\Delta\gamma}{\gamma_0\,\mathrm{d}\,s_1} = \frac{|Z_{\mathrm{l}}(k)|}{Z_0}\frac{I_0}{\gamma_0 I_{\mathrm{A}}}\rho_{\mathrm{i}} \ , \tag{17}$$

where s_1 is a coordinate along the reference trajectory, $s_1 = 0$ at the entrance to the first dipole. The amplitude of the induced density modulation in the second dipole can then be calculated using simple generalization of formula (4) for cold beam:

$$\rho_2(s_2) = -k\int\limits_0^{L_{\mathrm{d}}} \mathrm{d}\,s_1 \frac{\mathrm{d}\,\Delta\gamma}{\gamma_0\,\mathrm{d}\,s_1} R_{56}(s_1 \to s_2) \ , \tag{18}$$

where $s_2 = 0$ at the entrance to the second dipole and $R_{56}(s_1 \to s_2)$ connects energy kick at s_1 and a change of position z at s_2. Under the condition (12) it is well approximated by

$$R_{56}(s_1 \to s_2) \simeq -\frac{\Delta L}{R^2}(L_\mathrm{d} - s_1)s_2 \ . \tag{19}$$

Then we get

$$\rho_2(s_2) = \frac{1}{2}\frac{|Z_\mathrm{l}(k)|}{Z_0}\frac{I_0}{\gamma_0 I_\mathrm{A}}\frac{k\Delta L L_\mathrm{d}^2}{R^2}\rho_\mathrm{i} s_2 \ . \tag{20}$$

Since we assumed that the induced modulation ρ_2 is much larger than the initial one (gain is large), the CSR-induced energy modulation in the second dipole is

$$\frac{\mathrm{d}\,\Delta\gamma}{\gamma_0\,\mathrm{d}\,s_2} = \frac{|Z_\mathrm{l}(k)|}{Z_0}\frac{I_0}{\gamma_0 I_\mathrm{A}}\rho_2(s_2) \ , \tag{21}$$

with negligible initial value $\Delta\gamma(s_2 = 0) \approx 0$.

Then we do similar calculations for the third dipole ($s_3 = 0$ at its entrance):

$$\rho_3(s_3) = -k \int\limits_0^{2L_\mathrm{d}} \mathrm{d}\,s_2 \frac{\mathrm{d}\,\Delta\gamma}{\gamma_0\,\mathrm{d}\,s_2} R_{56}(s_2 \to s_3) \ , \tag{22}$$

where

$$R_{56}(s_2 \to s_3) \simeq -\frac{\Delta L}{R^2}(2L_\mathrm{d} - s_2)s_3 \ . \tag{23}$$

Finally, using (20)-(23) we obtain the total gain as a ratio between final and initial amplitudes of density modulation:

$$G = \frac{\rho_3(s_3 = L_\mathrm{d})}{\rho_\mathrm{i}} = \frac{2}{3}\frac{|Z_\mathrm{l}(k)|^2}{Z_0^2}\left(\frac{I_0}{\gamma_0 I_\mathrm{A}}\right)^2 \frac{k^2 \Delta L^2 L_\mathrm{d}^6}{R^4} \ .$$

With the help of (13) and (16) we rewrite the expression for the gain in a different form:

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}}\left(\frac{I_0}{\gamma_0 I_\mathrm{A}}\right)^2 \frac{k^{8/3}|R_{56}|^2 L_\mathrm{d}^2}{R^{4/3}} \ . \tag{24}$$

When deriving this expression we neglected density bunching inside a dipole caused by the energy modulation induced in the same dipole. In other words, we ignored a self-consistent process inside a dipole which may eventually lead to an exponential growth of the modulation. Thorough analysis shows that the expression (24) is accurate as soon as

$$L_{\mathrm{d}} \leq L_{\mathrm{g}} \ ,$$

where

$$L_{\mathrm{g}} = \left(\frac{\gamma_0 I_{\mathrm{A}}}{I_0}\right)^{1/4} k^{-1/3} R^{2/3} \ .$$

In the case when $L_{\mathrm{d}} \gg L_{\mathrm{g}}$ one could expect an exponential growth inside magnets with the growth rate about L_{g}^{-1}. In practice, however, for any reasonable set of electron beam parameters, this regime cannot be achieved because the frequency range will be limited by the energy spread and/or emittance. The exponential growth (with a different scaling for the growth rate) inside dipoles of a bunch compressor has been predicted in [21] but the results of that paper are incorrect [2].

4.3 Gaussian energy spread

Let us first make a simple estimation of the gain. If we assume that energy modulation happens only in the first dipole, we can use (with $C = 1$) the relations (9), (10) and (16) to estimate the gain at $G_{\max} \simeq g_0$, where

$$g_0 = \frac{I_0}{\sigma_\gamma I_{\mathrm{A}}} \left(\frac{\gamma_0}{\sigma_\gamma}\right)^{1/3} \frac{L_{\mathrm{d}}}{(R^2|R_{56}|)^{1/3}} \ . \tag{25}$$

Actually, we have two-stage amplification and should expect the dependence $G_{\max} \simeq g_0^2$. The accurate calculations confirm this estimate. Leaving out the details of calculations we present here the final result for the gain

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} g_0^2 f(\hat{k}) \ . \tag{26}$$

[2] An incorrect equation of longitudinal motion has been used.

Here

$$\hat{k} = \frac{\sigma_\gamma}{\gamma_0}|R_{56}|k \ ,$$

and the function f is

$$f(\hat{k}) = 3\hat{k}^{2/3}\exp(-\hat{k}^2/2)\left[1 + \frac{\sqrt{\pi}}{2}\frac{\hat{k}^2-2}{\hat{k}}\exp(\hat{k}^2/4)\mathrm{erf}(\hat{\mathrm{k}}/2)\right] \ , \qquad (27)$$

where

$$\mathrm{erf(x)} = 2\pi^{-1/2}\int\limits_0^{\mathrm{x}} \mathrm{d\,t}\exp(-\mathrm{t}^2)$$

is the error function. For $\hat{k} \ll 1$, when the influence of energy spread is negligible, we get $f(\hat{k}) \simeq \hat{k}^{8/3}$ and (26) is reduced to (24).

The maximal value of f is achieved at $\hat{k}_{\mathrm{opt}} = 2.15$ and is equal to 1.98. Thus, the maximal gain is

$$G_{\max} = 1.16 g_0^2 \qquad (28)$$

in agreement with a simple estimate. The accurate result (28) differs by only a few per cent from the result of ref. [22], although an incorrect assumption of constant density modulation inside dipoles has been used in [22].

4.4 Estimation of emittance effect

One of the limitations due to emittance (15) was already presented. More stringent limitation, however, comes from the longitudinal motion inside dipoles [3]. Indeed, a particle with an offset x from the reference orbit moves along the beam:

$$\frac{\mathrm{d}\,z}{\mathrm{d}\,s} = -\frac{x}{R} \ .$$

[3] The net effect through the whole compressor is of the second order, see (11).

For a typical offset $x \simeq \sqrt{\epsilon\beta}$ the maximal change of coordinate z can be estimated at $\sqrt{\epsilon\beta}L_{\rm d}/R$. Therefore, the above presented results for the gain can be used if

$$\frac{k\sqrt{\epsilon\beta}L_{\rm d}}{R} \ll 1 \ . \qquad (29)$$

The energy spread cuts off the gain at $k \simeq \sigma_{\gamma}^{-1}\gamma_0|R_{56}|^{-1}$. Using (13) we estimate that formula (26) is valid in the entire wavelength range when

$$\sqrt{\epsilon\beta} \ll \frac{\sigma_{\gamma}}{\gamma_0}\Delta L\frac{L_{\rm d}}{R} \ . \qquad (30)$$

In other words, transverse size, defined by emittance, should be smaller than dispersion-induced transverse size in the middle dipole. In opposite case the cut-off will be defined by the condition $k \simeq R/(\sqrt{\epsilon\beta}L_{\rm d})$. To roughly estimate maximal gain one can substitute this expression for k into formula (24).

5 Estimation of shot noise effect

In this paper we considered a bunch compressor as a high-gain linear amplifier of klystron type. It amplifies initial density modulations within some frequency band. For broadband wakefields (like CSR) we have a broadband amplifier with $\Delta\omega \simeq \omega_0$, where ω_0 is some optimal frequency defined by the energy spread (eventually by emittance). Even without macroscopic density modulation the initial signal for such an amplifier always exists because of the shot noise in electron beam.

Let us consider the beam consisting of randomly emitted electrons with an average current I_0. Its spectrum is a "white" noise which will be filtered by the amplifier. The time domain fluctuations of the current at the amplifier entrance within a frequency band $\Delta\omega$ are defined by Schottky formula:

$$< i^2 >= \frac{eI_0\Delta\omega}{\pi} \ . \qquad (31)$$

Since $\Delta\omega \simeq \omega_0$ in the considered case, we can write down the following expression for relative initial fluctuations:

$$< \rho_{\rm i}^2 >_{\rm sh} = \frac{< i^2 >}{I_0^2} \simeq \frac{e\omega_0}{\pi I_0} \simeq \frac{1}{N_\lambda} \ , \qquad (32)$$

where N_λ is a number of particles per wavelength $\lambda = 2\pi c/\omega_0$. Thus, effective initial density modulation due to the shot noise is

$$(\rho_{\rm i})_{\rm sh} \simeq \frac{1}{\sqrt{N_\lambda}} \ . \qquad (33)$$

The coherence length for such a broadband system is about $l_{\rm c} \simeq 2\pi c/\omega_0 = \lambda$. Thus, the effective shot noise bunching is about inverse square root of number of particles per coherence length.

Relative amplitude of ac current behind the bunch compressor is then of the order of

$$\rho_{\rm f} \simeq \frac{G_{\rm max}}{\sqrt{N_\lambda}} \ . \qquad (34)$$

Due to the lack of coherence this current will constitute irregular, spiky structure on a time scale of ω_0^{-1}.

For typical parameters of electron beams one can estimate $(\rho_{\rm i})_{\rm sh}$ at 10^{-4}. Thus, for a sufficiently large gain (especially in a chain of bunch compressors) the shot noise bunching can become important.

Finally, we note that when simulating bunch compressors with the help of macroparticles approach, one should eliminate artificial noise connected with a relatively small number of macroparticles. If, for instance, one uses $N_{\rm m}$ macroparticles in a bunch and they are distributed in phase space randomly, then the noise effect will be overestimated by a factor $\sqrt{N_{\rm b}/N_{\rm m}}$, where $N_{\rm b}$ is actual number of particles in the bunch.

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