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Impedances of Collimators in European XFEL

Martin Dohlus, Igor Zagorodnov, Olga Zagorodnova*

Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, 22603 Hamburg, Germany

We consider the longitudinal and the transverse coupling impedances of collimators in the European XFEL beam line. We take into account the finite conductivity and consider the dependence of the integral wake parameters from the geometric dimensions of the collimators. The emittance growth due to the collimators is estimated as well. The longitudinal impedance of the collimators is compared with the total impedance budget of the beam line.

* Corresponding author. Tel.:+49-040-8998-1966; fax: +49-040-8998-4305
E-Mail address: olga.zagorodnova@desy.de

I. INTRODUCTION

In the European X-ray Free Electron Laser [1] the short bunches with a high peak current will be transported through the beam line and the undulators. The collimators are important sources of strong short range wakefields for such bunches. These wakefields increase the energy spread in the bunch and spoil its emittance.

II. OPTICAL APPROXIMATION

In recent paper [2, 3] a method to estimate the high frequency impedances of short transitions was developed.

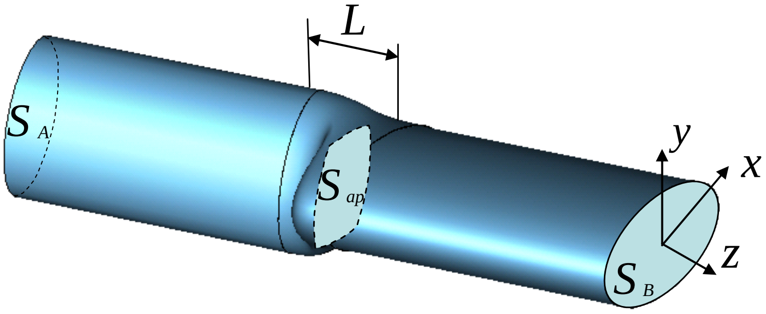

FIG 1. The geometry of transition.

Consider a short transition with aperture S_{ap} between two pipes with apertures S_A and S_B as shown in Fig. 1. Let a is a characteristic size of the aperture S_{ap}. If the bunch has a short rms length σ, $\sigma << a$, and the transition length L between the ingoing pipe aperture S_A and the outgoing pipe aperture S_B is short, $L << a^2/\sigma$, then the high frequency longitudinal impedance is a constant which can be calculated by relation

$$Z_{||}(\mathbf{r}_1,\mathbf{r}_2) = \frac{Z_0}{8\pi^2}\left[\int_{S_B} \nabla\varphi_B(\mathbf{r}_1,\mathbf{r})\nabla\varphi_B(\mathbf{r}_2,\mathbf{r})ds - \int_{S_{ap}} \nabla\varphi_A(\mathbf{r}_1,\mathbf{r})\nabla\varphi_B(\mathbf{r}_2,\mathbf{r})ds\right] \qquad (1)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are offsets of the leading and the trailing particles, correspondingly, and φ_A, φ_B are the Green functions of the Laplace operator in the ingoing and the outgoing pipe cross-sections

$$\Delta\varphi_A(\mathbf{r}_i,\mathbf{r}) = -\varepsilon_0^{-1}\delta(\mathbf{r}-\mathbf{r}_i),\ \mathbf{r}\in S_A,$$
$$\Delta\varphi_B(\mathbf{r}_i,\mathbf{r}) = -\varepsilon_0^{-1}\delta(\mathbf{r}-\mathbf{r}_i),\ \mathbf{r}\in S_B,\ i=1,2.$$

The above equations can be applied to find high-frequency impedance of a step collimator of arbitrary aperture.

In the following we consider only rotationally symmetric collimators. Other 3D examples can be found in [3, 4].

III. GEOMETRICAL WAKE FIELDS OF THE ROUND STEP COLLIMATOR

The longitudinal impedance of the round collimator at high frequency reads [5]

$$Z_{||} = \frac{Z_0}{\pi} \ln \frac{b}{a} .$$

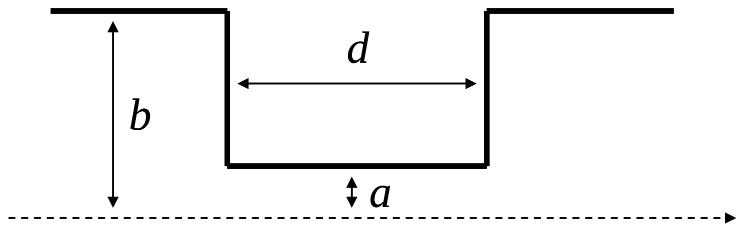

FIG 2. The collimator geometry.

The longitudinal wake function for the point charge can be written as

$$w_{||}(s) = Z_{||} c \delta(s) . \tag{2}$$

The bunch with longitudinal charge profile $\lambda(s)$ introduces wake potential given by relation

$$W_{||}(s) = \int_{-\infty}^{s} \lambda(\xi) w_{||}(s-\xi) d\xi = Z_{||} c \lambda(s) . \tag{3}$$

The energy loss of the bunch can be expressed in terms of an integral over the wake function

$$k_{||} = \int_{-\infty}^{\infty} W_{||}(s)\lambda(s) ds = c Z_{||} \int_{-\infty}^{\infty} \lambda^2(s) ds .$$

The loss factor and rms energy spread of a Gaussian bunch with rms length σ are approximately given by

$$k_{||} = \frac{c}{2\sqrt{\pi}\sigma} Z_{||} , \tag{4}$$

$$k_{||}^{rms} = \sqrt{\int_{-\infty}^{\infty} \left(W_{||}(s) - k_{||} \right)^2 \lambda(s) ds} = k_{||} \sqrt{\frac{2}{\sqrt{3}} - 1} \approx \frac{k_{||}}{2.542} \approx 0.4 k_{||} . \tag{5}$$

The transverse impedance of very short collimator ($L << a^2/\sigma$) is defined by [6]

$$Z_{\perp} = \frac{Z_0 c}{2\omega\pi} \left(\frac{1}{a^2} - \frac{a^2}{b^4} \right) \tag{6}$$

and for quite long collimator ($L \geq a^2/\sigma$) by [7]

$$Z_{\perp} = \frac{Z_0 c}{\omega\pi} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) . \tag{7}$$

The transverse wake function for the point charge can be written as

$$w_{\perp}(s) = \theta(s) \omega Z_{\perp} ,$$

where $\theta(s)$ is the Heaviside step function and we have used the fact that in the optical approximation the product $\omega Z_{\perp}$ is a constant independent from ω.

The bunch with longitudinal charge profile $\lambda(s)$ introduces the transverse wake potential

$$W_{\perp}(s)=\int_{-\infty}^{s}\lambda(\xi)w_{\perp}(s-\xi)d\xi=\omega Z_{\perp}\Lambda(s), \qquad \Lambda(s)=\int_{-\infty}^{s}\lambda(\xi)d\xi.$$

The transverse kick factor for an arbitrary normalized bunch shape $\lambda(s)$ can be found as

$$k_{\perp}=\omega Z_{\perp}\int_{-\infty}^{\infty}\lambda(s)\int_{-\infty}^{s}\lambda(\xi)d\xi ds=\frac{\omega}{2}Z_{\perp}, \tag{8}$$

where we have used the relation

$$\int_{-\infty}^{\infty}\lambda(x)\int_{-\infty}^{x}\lambda(y)dydx=\frac{1}{2}\left(\int_{-\infty}^{\infty}\lambda(x)dx\right)^{2}.$$

The spread of the kick for the Gaussian bunch can be found as

$$k_{\perp}^{rms}=\frac{k_{\perp}}{\sqrt{3}}\approx 0.58k_{\perp}. \tag{9}$$

IV. THE RESISTIVE WAKE OF THE ROUND PIPE WITH AN OXIDE LAYER AND ROUGHNESS

The steady state longitudinal resistive impedance of the round pipe can be written as [8, 9]

$$Z_{\parallel}=\frac{Z_s}{2\pi a}\cdot\frac{1}{1+j\dfrac{\omega}{c}\dfrac{a}{2}\dfrac{Z_s}{Z_0}},$$

where the surface impedance is given by relations

$$Z_s(\omega)=Z_s^{\kappa}(\omega)=\sqrt{\frac{j\omega\mu_0}{\kappa(\omega)}}, \quad \kappa(\omega)=\frac{\kappa_0}{1+j\omega\tau}.$$

The effect of the oxide layer and the roughness can be taken into account through the inductive surface impedance

$$Z_s(\omega)\approx Z_s^{\kappa}(\omega)+j\omega L,$$

with the inductance [10]

$$L=\mu_0\left(\frac{\varepsilon_r-1}{\varepsilon_r}d_{oxid}+0.01d_{rough}\right),$$

where d_{oxid} is the depth of the oxid layer and d_{rough} is roughness rms parameter.

The steady state transverse resistive impedance of the round pipe can be written as

$$Z_{\perp}=\frac{2}{a^2}\frac{c}{\omega}Z_{\parallel}.$$

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V. THE LONGITUDINAL AND TRANSVERSE WAKES OF THE ROUND XFEL COLLIMATOR

In the European XFEL 4 collimators with parameters $b = 20.25\,\text{mm}$, $a \sim 3\,\text{mm}$, $d \sim 50\,\text{cm}$ will be installed.

The geometrical longitudinal high frequency impedance of the *round* collimator is independent from its length. The resistive longitudinal wake is proportional to the collimator length d. Fig. 3 shows dependence of the loss parameter and the energy spread from the aperture a of the round step collimator of length $d = 50\,\text{cm}$. In this report we consider material "TIMETAL 6-4 alloy" with conductivity $\kappa_0 = 0.6e6\,\Omega^{-1}m^{-1}$, roughness $d_{rough} = 1\,\mu\text{m}$ and oxide layer thickness $d_{oxid} = 5\,\text{nm}$.

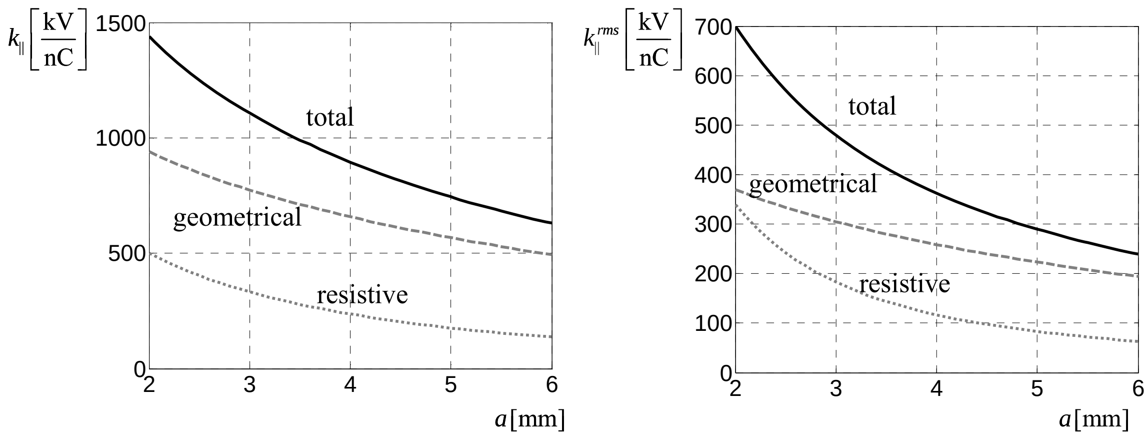

FIG 3. The energy loss and the energy spread vs. the collimator aperture.

The geometrical transverse high frequency impedance of the round collimator depends on its length. Fig.4 shows dependence of the kick factor from the collimator length d for the fixed aperture $a \sim 3\,\text{mm}$. These results are obtained through direct numerical simulations with code ECHO [11].

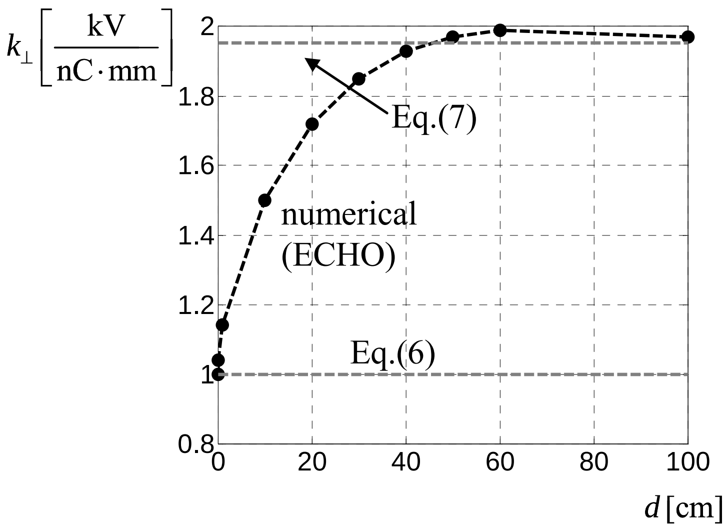

FIG 4. The dependence of the geometrical transverse kick from the collimator length.

It can be seen that for $d = 50\,\text{cm}$ Eq. (7) has to be used. The resistive transverse wake is proportional to the collimator length d. Fig.5 shows dependence of the kick and the kick spread from the aperture a of the round step collimator of length $d = 50\,\text{cm}$.

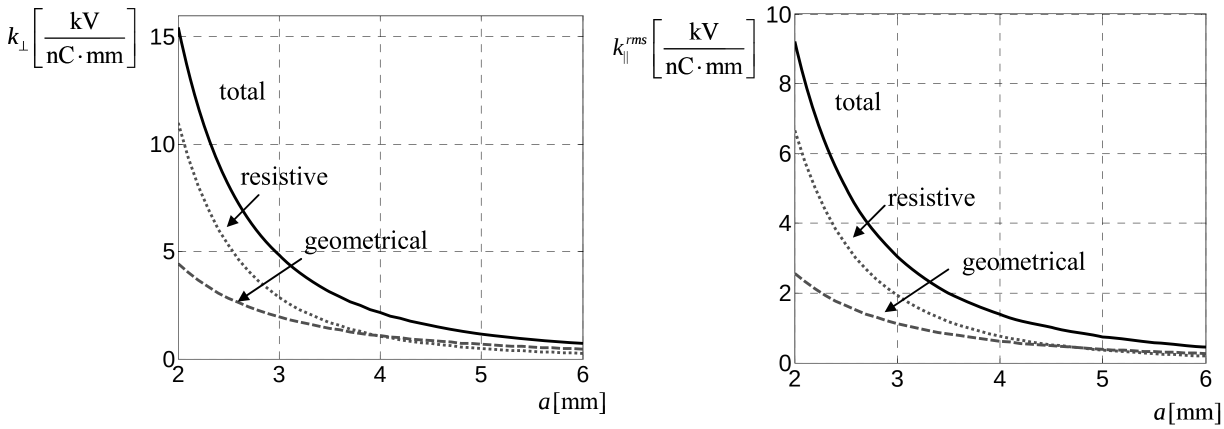

FIG 5. The transverse kick and the kick spread vs. the collimator aperture.

VI. TAPERING OF THE COLLIMATOR

The tapering of the collimator reduces the geometrical wake of the collimator considerably [12]. Consider the round collimator of radius $a = 3\,\text{mm}$ with beam pipes of radius $b = 20.25\,\text{mm}$. The results of 2D simulation for investigation the effect of tapering for the similar structure can be found, for example, in [13]. The significant reduction in the wake for short bunches will happen for the taper angle smaller than θ_0, where $\tan\theta_0 \sim \sigma_z / a$ [12]. In our case $\theta_0 \sim 0.55^\circ$ for the Gaussian bunch with $\sigma = 25\,\mu\text{m}$. Hence, the collimator has to be too long (>4m).

As an alternative solution we consider the "step + taper" geometry of the collimator shown in Fig. 6.

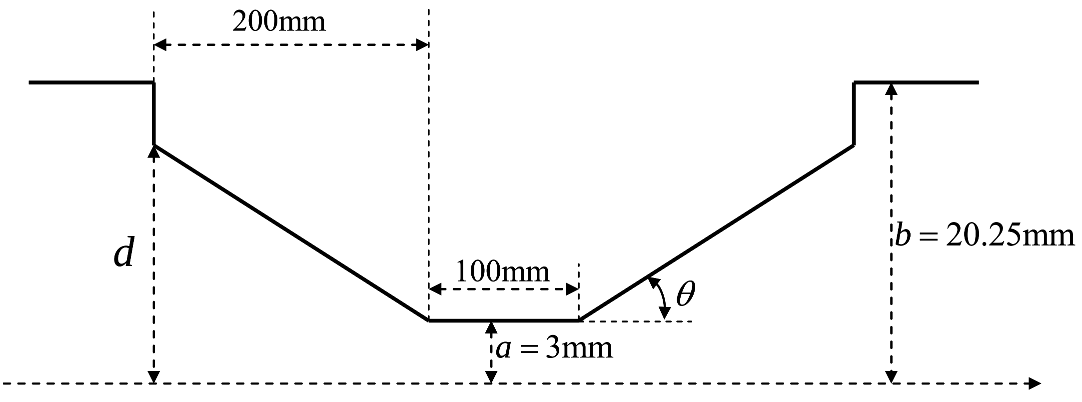

FIG 6. Geometry of the "step + taper" collimator.

The effectiveness of such kind of geometry was proven in [14]. The calculations are carried out for the Gaussian bunch with $\sigma = 25\,\mu\text{m}$.

Fig. 7 shows dependence of the loss factor, energy spread (left) and kick factor, kick spread (right) on the parameter d (see Fig. 6). As we see all functions have minimums and the value $d = 6\,\text{mm}$ can be taken as the optimum. For this value of parameter d angle of tapering θ is equal to $\approx 1^{\circ}$.

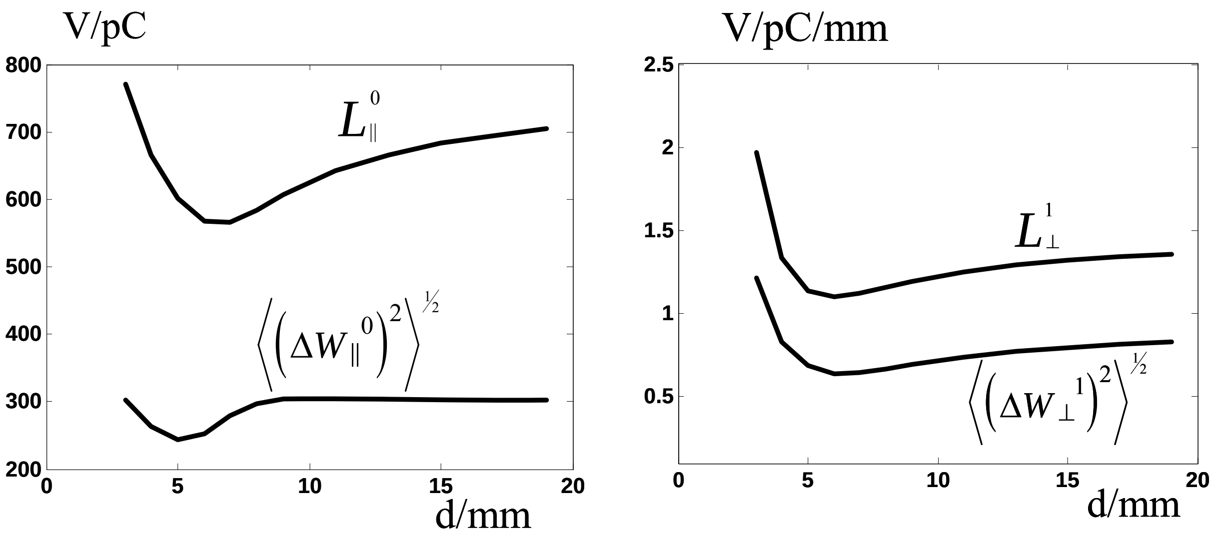

FIG 7. Collimator geometry optimization.

Table I shows comparison of loss factors and energy spreads and also kick factors and kick spreads for both model of collimator: without tapering and for the "step + taper" model with optimal value of $d = 6\,\text{mm}$.

TABLE I. Integral parameters of geometrical wake for the collimator of Fig. 6; $\sigma_z = 25\mu m$.

	$k_{\parallel}$, V/pC	$k_{\parallel}^{rms}$, V/pC	$k_{\perp}$, V/pC/m	$k_{\perp}^{rms}$, V/pC/m
d=0	770.41	302.09	1969.1	1216.8
d=6mm	568.06	253.34	1097.6	638.06

VII. EFFECT OF THE WAKES ON THE BEAM

The longitudinal and transverse short-range wake fields of collimator will induce an energy spread and emmitance growth. As shown in Appendix A, the increase of the projected emittance can be estimated as

$$\frac{\varepsilon_y - \varepsilon_{0y}}{\varepsilon_{0y}} = \sqrt{1 + \frac{\beta}{\varepsilon_{0y}} \left\langle y_c'^2 \right\rangle} - 1,$$

where the RMS centroids kick reads

$$\left\langle y_c'^2 \right\rangle^{1/2} = \left(\frac{E_0}{e} \right)^{-1} Q k_{\perp}^{rms} y .$$

Here E_0 is the design energy of the bunch at the location of the collimators, y is a bunch offset from the axis.

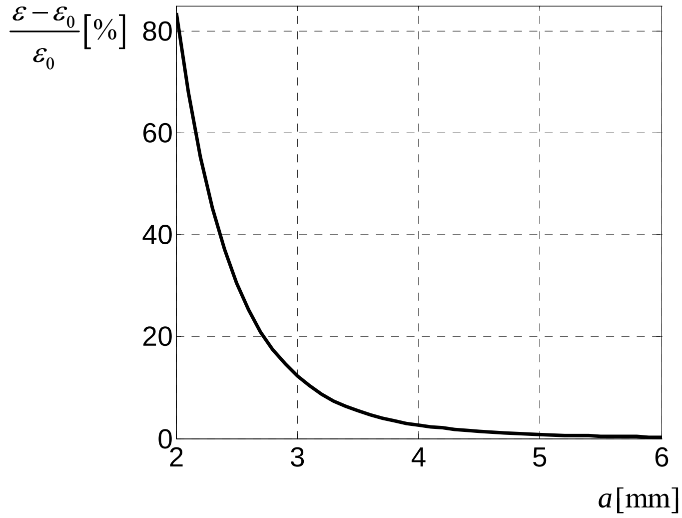

FIG 8. The projected emittance growth for the Gaussian bunch with offset 1 mm vs. the collimator aperture.

Let us consider the Gaussian bunch with charge $Q = 1$ nC, rms length $\sigma = 25\,\mu\text{m}$, energy $E_0 e^{-1} = 14\,\text{GeV}$, normalized emittance $\gamma\varepsilon_0 = 1\,\mu m$, the optical function $\beta = 200\,\text{m}$. For the bunch moving with offset $y = 1\,\text{mm}$ through the collimator of the length $d = 50\,\text{cm}$ the projected emittance growth is shown in Fig. 8. It is the result for only one collimator and we have to remember that in XFEL we will have four items.

The energy change along the bunch can be found as

$$\Delta E(s) = eQW_{\|}(s).$$

The total energy loss from the cathode up to the undulator without the collimators is about 28 MeV for the bunch with charge $Q = 1\,\text{nC}$ [15]. The energy spread for the same case is about 11 MeV. If we include the wakes of the collimators then the contribution of the four collimators is shown in Fig. 9.

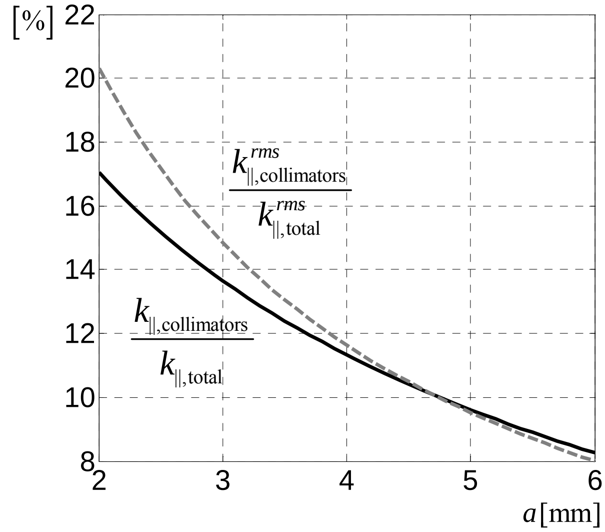

FIG 9. Contribution of the collimator wakefields to the longitudinal impedance budget vs. the collimator aperture.

VIII. CONCLUSION

In this report we have calculated geometrical and resistive wakefields of the round collimator for different apertures. Additionally, we have estimated effect of the tapering. It allows reduction of the kick by factor 2. The projected emittance growth was estimated as well.

APPENDIX: ANALYTICAL ESTIMATION OF THE EMITTANCE GROWTH

In the conditions of the optical approximation the wake potential of an arbitrary bunch profile $\lambda(s)$ can be found as

$$W_y(s) = 2k_y \Lambda(s), \quad \Lambda(s) = \int_{-\infty}^{s} \lambda(s')ds' . \tag{A.1}$$

The kick to the bunch can be found as

$$\Delta y'(s) = \frac{\Delta p_y}{p_z} = \frac{eQW_y(s)}{\beta_z^2 E} = 2S\Lambda(s), \quad S = \frac{eQk_y}{\beta_z^2 E}.$$

Let us consider a transverse Gaussian particle distribution

$$\rho_0(y, y', s) = \frac{1}{2\pi\varepsilon_{0y}} \exp\left(-\frac{\gamma y^2 + 2\alpha yy' + \beta y'^2}{2\varepsilon_{0y}}\right)\lambda(s)$$

with an arbitrary longitudinal profile $\lambda(s)$.
After the kick the distribution has the form

$$\rho = \frac{\lambda(s)}{2\pi\varepsilon_{0y}} \exp\left(-\frac{\gamma y^2 + 2\alpha y(y' + \Delta y'(s)) + \beta(y' + \Delta y'(s))^2}{2\varepsilon_{0y}}\right).$$

The projected distribution after the kick does not depend on the $\lambda(s)$

$$\bar{\rho} = \int_{-\infty}^{\infty} \rho(y, y', s)ds = \int_0^1 \frac{1}{2\pi\varepsilon_{0y}} \mathrm{e}^{-\frac{\gamma y^2 + 2\alpha y(y' + 2S\Lambda) + \beta(y' + 2S\Lambda)^2}{2\varepsilon_{0y}}} d\Lambda =$$

$$= \frac{Erf\left(\dfrac{y\alpha + (2S + y')\beta}{\sigma_{0y}\sqrt{2}}\right) - Erf\left(\dfrac{y\alpha + y'\beta}{\sigma_{0y}\sqrt{2}}\right)}{4\sqrt{2}\pi S\sigma_{0y}} \mathrm{e}^{-\frac{y^2}{2\sigma_{0y}^2}},$$

where $\sigma_{0y} = \sqrt{\varepsilon_{0y}\beta}$.
The projected emittance can be calculated analytically

$$\varepsilon_y = \sqrt{\langle y^2\rangle\langle y'^2\rangle - \langle yy'\rangle} = \sqrt{\varepsilon_{0y}^2 + S^2 \frac{\varepsilon_{0y}\beta}{3}} \approx \varepsilon_{0y} + S^2 \frac{\beta}{6} \tag{A.2}$$

Hence, the relative emittance growth reads

$$\frac{\varepsilon_y - \varepsilon_{0y}}{\varepsilon_{0y}} = \sqrt{1 + S^2 \frac{\beta}{3\varepsilon_{0y}}} - 1 \approx S^2 \frac{\beta}{6\varepsilon_{0y}}. \tag{A.3}$$

For the transverse wake of arbitrary shape we can proceed as described in [16]. The moments of the kicked bunch in Eq. (A.2) can be written as

$$\left\langle y^2 \right\rangle = \varepsilon_{0y}\beta \,,\; \left\langle y'^2 \right\rangle = \left\langle y_c'^2 \right\rangle + \varepsilon_{0y}\gamma \,,\; \left\langle yy' \right\rangle = -\varepsilon_{0y}\alpha \,,$$

where

$$\left\langle y_c'^2 \right\rangle^{1/2} = \left(\frac{E_0}{e} \right)^{-1} Q k_{\perp}^{rms} y \,.$$

Hence, the emittance growth can be estimated as

$$\frac{\varepsilon_y - \varepsilon_{0y}}{\varepsilon_{0y}} = \sqrt{1 + \frac{\beta}{\varepsilon_{0y}} \left\langle y_c'^2 \right\rangle} - 1 \,. \tag{A.4}$$

For the case when the wake potential is given by Eq. (A.1) the general case equation (A.4) reduces to Eq. (A.3).

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