Field Estimation and Signal Calibration
of RF Guns without Field Probe

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(Dated: January 26, 2007)

The field inside a resonator is proportional to the vector sum of the forward and the reflected wave in front of the power coupler. The cavity field derived from the measured amplitudes and phases of these waves can be used for low level RF (LLRF) control. This approach is required for cavities without field probe but offers also additional diagnostics and possibilities for cross-checks in case a probe is present. A precise field estimate requires a relative calibration of the forward and reflected waves in amplitude and phase. This article introduces a simple online calibration method which is applicable to any resonator that is operated close to steady state.

PACS numbers:

I. INTRODUCTION

Inside photoinjectors, electrons are liberated from a cathode (e. g. Cesium Telluride) by means of the photoeffect. In order to keep space charge effects small, the electrons are exposed to a very high field of $> 45\,\mathrm{MV/m}$ in a small volume of the resonator. The electrons are accelerated to a relativistic velocity on a path of the length of only a few centimeters.

The RF gun of the FLASH photoinjector is a 1.5 cell normalconducting cavity operated at $1.3\,\mathrm{GHz}$. It is designed to operate at high duty cycle with an average heat load of up to $50\,\mathrm{kW}$ ($\sim 200\,\mathrm{kW/m}$). This high heat load demands for an optimized placement of cooling channels for the heat-transport. Focussing solenoids put additional space restrictions and led to the decision to build the cavity without RF probe and to measure the forward and the reflected wave with a high precision directional coupler instead.

For field stability requirements of 1% in amplitude and $1°$ in phase it is not sufficient to stabilize the forward traveling wave alone. An amplitude distortion of 1% is already present if the resonator is detuned by 1% of its half-bandwidth, which is $650\,\mathrm{Hz}$ for a resonator of half-bandwidth $\omega_{1/2} = 65\,\mathrm{kHz}$. The dependency of the temperature of the resonator (e. g. $20\,\mathrm{kHz/°C}$), would require long term stability of $\frac{1}{30}°\mathrm{C}$. Due to the high heat load during a pulse, additional short term detuning caused by surface heating will lead to significant detuning angles of a few degrees ([1]) which can not be compensated by temperature control.

Field stability therefore can only be achieved by including the reflected power signal. From resonator theory, one can conclude that the field inside a cavity is the sum of the complex amplitudes of the forward and reflected waves. In a real system, all signals are subject to amplitude and phase errors, which demand for a precise

calibration.

In the following sections, U is used for the field inside the resonator, while U_for and U_ref are used for the the forward and reflected wave. $U, U_\mathrm{for}, U_\mathrm{ref} \in \mathbb{C}$ and therefore contain amplitude and phase.

All numbers throughout this article are taken from or calculated for the FLASH L-band high duty cycle photoinjector, [2].

II. RESONATOR THEORY

Energy is conserved at the high-power coupler in front of the RF-gun, the power P_for of the forward traveling wave is the sum of the transmitted power P_trans and the reflected power P_ref. The complex amplitudes of these waves fulfill the equation $U_\mathrm{trans} = U_\mathrm{for} + U_\mathrm{ref}$ (with the sign of U_ref properly defined). The power of the transmitted wave goes into field increase and into dissipation. The complex amplitude of the transmitted wave is directly proportional to the compex amplitude of the cavity field.

The complex amplitude U_trans is connected with the complex amplitude of the forward traveling wave via the properties of the resonator itself. Figure 1 shows U_trans, U_for and U_ref in the complex plane. The reflected wave is a superposition of a reflection due to frequency mismatch (detuning) and a reflection due to impedance mismatch. In figure 1, U'_ref is the fraction of the reflection that is caused by detuning. The impedance mismatch is characterized by Γ_0, the reflection coefficient for zero detuning and steady state and is determined by the coupling coefficient β,

$$\beta = \frac{1 + \Gamma_0}{1 - \Gamma_0} \quad \left(\Leftrightarrow \Gamma_0 = \frac{\beta - 1}{\beta + 1} \right). \tag{1}$$

The coupling coefficient is the ratio of the impedances of the transmission line and the cavity on resonance, $\beta = Z_\mathrm{ext}/R$. Generally, the reflection coefficient Γ is the ratio of the complex amplitudes of forward and reflected waves, $\Gamma = U_\mathrm{ref}/U_\mathrm{for}$.

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A resonator of half-bandwidth $\omega_{1/2}$ and detuning $\Delta\omega$ is formally described by the so-called envelope equation

$$\omega_{1/2}(1 + \Gamma_0)U_{\text{for}} = (\omega_{1/2} + i\Delta\omega)U_{\text{trans}} + \frac{\text{d}}{\text{d}t}U_{\text{trans}}. \quad (2)$$

The envelope-equation is derived (and approximated) from the more general second-order bandpass-equation

$$2\omega_0\omega_{1/2}(1+\Gamma_0)U_{\text{for}} = \frac{\text{d}^2}{\text{d}t^2}U_{\text{trans}}+2\omega_{1/2}\frac{\text{d}}{\text{d}t}U_{\text{trans}}+\omega_0^2 U_{\text{trans}}. \quad (3)$$

The envelope-approximation is justified by the assumption that $U_{\text{trans}}/e^{\omega_0 t}$ is changing slowly compared to the center frequency ω_0 of the resonator and that the detuning $\Delta\omega$ is small compered to ω_0. A derivation of the envelope-approximation can be found in [3]. In steady state, the envelope-equation becomes

$$U_{\text{trans}} = (1 + \Gamma_0)\frac{\omega_{1/2}}{\omega_{1/2} + i\Delta\omega}U_{\text{for}}. \quad (4)$$

This equation directly implies that U_{trans} is moving along a circle as depicted in figure 1 for different values of the detuning $\Delta\omega$ and constant U_{for}.

Since the steady state case scales linearly with U_{for}, it is useful to measure U_{ref} relative to the forward amplitude U_{for}, which is identical to measuring the reflection coefficient Γ.

An interesting way of looking at the reflected power signal is the following: in principle, the high power coupler is a field probe itself. The signal that is coupled out of the high power is the cavity field (or the transmitted wave) but is superimposed with the forward traveling wave and hence measured as the reflected wave.

The next sections identify U_{trans} with the accelerating cavity voltage U, which is directly proportional to U_{trans}. The field directly at the high power coupler is the superposition of the complex amplitudes of the forward and the reflected wave. The amplitude of the cavity field can be calculated from the power P_{trans} of the transmitted wave as $U = \sqrt{RP_{\text{trans}}}$ where R is the shunt impedance of the cavity.

A more elaborate introduction to resonator theory and microwave measurements can be found in [4].

III. PRECISION REQUIREMENTS

This section will give an analytic approach to quantify the calibration requirements. The relevance of the relative calibration between forward and reflected power has initially been shown in [1]

The field that is determined in the absence of an antenna is

$$U = U_{\text{for}} + U_{\text{ref}}. \quad (5)$$

U_{for} and U_{ref} are the amplitudes of the forward and the reflected wave measured in units of the cavity field, U. In practice, constant calibration errors, $(\Delta A/A)$ (as a relative quantity) in amplitude and $\Delta\varphi$ in phase, are made when determining the observables after the calibration. For simplicity, first the influence of a constant calibration error in amplitude, $(\Delta A/A)$, of the observables on the field estimate is considered. Figure 2 demonstrates how a constant calibration error in amplitude of the reflected field can lead to a time-varying error of the estimated cavity field phase. It shows the complex plane with solid arrows representing the true values as well as dashed arrows representing measurements by the controller, e. g. a digital signal processor (DSP). It is important to realize that for LLRF control, constant errors on the determined cavity field U are irrelevant. Due to time-varying detuning, constant errors on the observables U_{for} and U_{ref} can lead to a time-varying error on the field U, which is of relevance for LLRF control.

In order to estimate the error of the cavity field phase, a zoomed region is presented in figure 2. The error in phase is, in small-angle approximation, just the ratio between the length of a (see figure) and the field amplitude, $|U|$. The law of sines yields $|U_{\text{ref}}|\sin\alpha = |U|\sin\psi$. Therefore, the phase error of the cavity-field is

$$\begin{aligned} \Delta\varphi'_{\text{cavity}} &= \frac{a}{|U|} \\ &= \frac{\left(\frac{\Delta A}{A}\right)|U_{\text{ref}}|\sin\alpha}{|U|} \\ &= \left(\frac{\Delta A}{A}\right)\sin\psi \quad (6) \end{aligned}$$

where $(\Delta A/A)$ is the relative amplitude calibration error of the reflected wave. In reality, both forward and reflected wave will have statistically independent amplitude calibration errors. This can be included in the presented calculation by just multiplying with a factor $\sqrt{2}$. Its interpretation is that the measured fields are transformed into a coordinate system where the forward field has no calibration error. The calibration error of the forward wave is reflected in that transformation and increases the error of the reflected wave signal.

$$\Delta\varphi_{\text{cavity}} = \sqrt{2}\left(\frac{\Delta A}{A}\right)\sin\psi \quad (7)$$

Similarly, an approximation of the contribution of a constant phase calibration error $\Delta\varphi$ on the measured cavity field amplitude can be found,

$$\left(\frac{\Delta A}{A}\right)_{\text{cavity}} = \sqrt{2}\Delta\varphi\sin\psi \quad (8)$$

The contribution from a phase calibration error to a phase error and from amplitude calibtration to amplitude error is of second order and is neglected here. Also, the change in cavity field amplitude due to detuning in connection with a change of the detuning angle ψ is of second order and therefore neglected.
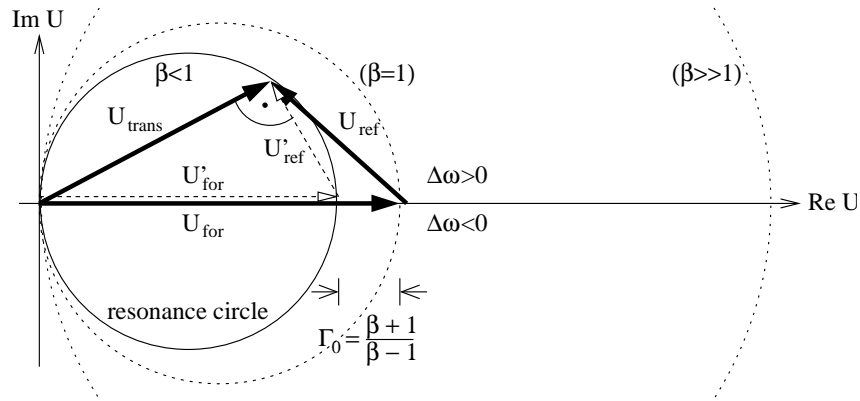
FIG. 1: Resonance circle. The dotted circles are the valid circles for critical coupling ($\beta = 1$) and a high overcoupling ($\beta >> 1$).
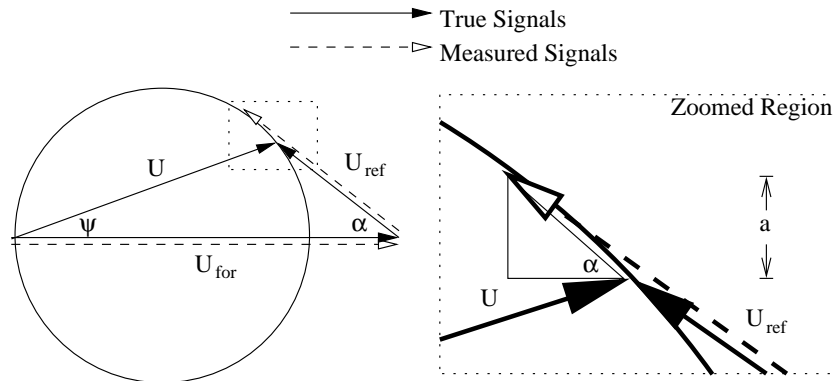

FIG. 2: Vector constellation in a complex plane for approximating the errors on the constructed cavity field. The dashed arrows represent the measurements made by the controller, e. g. a DSP. The solid arrows are the real values. In this simplified example, the measured forward vector coincides with the real forward vector, while the measured reflected vector has an error in amplitude compared to the real reflected vector.

A typical value for the detuning over a pulse is $\psi = 3°$, [1]. A calibration error of 5% in amplitude and 5° in phase will allow to measure the field up to 0.6% in amplitude and 0.2° in phase, which is sufficient for controlling the cavity with a precision of 1% and 1°.

It should be noted that not only constant calibration errors produce time-varying field-errors. Any additional quasi-constant error of the order of 5° in phase or 5 % in amplitude will contribute to significant errors on the reconstructed cavity field. Examples are amplitude-compression of elements of the sensor-chain, time-varying offsets, cross-talk between forward and reflected power sensors and general cross-talk from any other source. Additionally, sensors can have detector-specific errors. Industrial I/Q-modulators have shown the phase differences between the I and the Q channels to be different than 90 °.

For LLRF control, it is of importance to avoid time-varying errors on the reconstructed field signal rather than constant errors. The problem with reconstructing the field signal from two secondary signals (forward and reflected signal) is, that constant errors on the secondary channels induce time-varying errors on the reconstructed field.

IV. SIGNAL-CALIBRATION WITHOUT ANTENNA

Prerequisite for the calibration is that the measured forward and reflected signal need to be corrected only for quasi-constant distortions,

$$
\begin{aligned}
U_{\text{for}} &= A U_{\text{for}}^* + B U_{\text{ref}}^* \\
U_{\text{ref}} &= C U_{\text{for}}^* + D U_{\text{ref}}^*
\end{aligned} \tag{9}
$$

with $A, B, C, D \in \mathbb{C}$ representing the amplitude and phase corrections that need to be applied on the measured signals (indicated by the *) in order to get the calibrated signals. The signals are usually measured by directional couplers close to the cavity. In the case $B, C \neq 0$, (9) compensates for linear crosstalk between the forward and the reflected channel.

If the non-diagonal elements B, C are neglected, the reflection-coefficient for the measured signals $\Gamma^* = U_{\text{ref}}^*/U_{\text{for}}^*$ differs from the true reflection coefficient $\Gamma = U_{\text{ref}}/U_{\text{for}}$ by a factor D/A corresponding to a rotation and a scaling. It is possible to determine this factor because the reflection coefficient Γ is known for one point. Far away from the resonance frequency, the reflection coefficient is $\Gamma_{\infty=} - 1$. Usually, it is hardly possible to completely detune a resonator in order to measure Γ_∞. However, if just detuned by one half-bandwidth, the field inside the cavity already changes its phase by $45°$. That means that already a quarter ($90°$) of a whole circle is covered by detuning by a half-bandwidth in each direction. This is enough for fitting a circle with radius r and center c ($c \in \mathbb{C}$) through the measured points by minimizing

$$\chi^2 = \sum_i (|\Gamma_i^* - c| - r)^2 \qquad (10)$$

for all measured Γ_i^*. The uncalibrated reflection coefficient for maximum detuning can be identified with c and r as

$$\Gamma_\infty^* = (|c| + r)\frac{c}{|c|}. \qquad (11)$$

Since $\Gamma_\infty = -1$, the factor D/A is just $-1/\Gamma_\infty^*$. With this calibration, additional useful information can be obtained:

$$\begin{aligned} \Gamma_0 &= -\frac{D}{A}(|c| - r)\frac{c}{|c|} \\ &= -\frac{(|c| - r)}{(|c| + r)} \end{aligned} \qquad (12)$$

and from this the coupling $\beta = (1 + \Gamma_0)/(1 - \Gamma_0)$.

V.   WAYS OF DETUNING A CAVITY

There are several ways to detune the cavity in order to cover a significant fraction of the resonance circle. An obvious way is the temperature scan, where the temperature of the resonator is varied over time. The resonance frequency of the gun changes with typically $20\,\text{kHz}/°\text{C}$. $3°$ in temperature de-tune the electron gun by a half-bandwidth (e. g. $\sim 65\,\text{kHz}$ at a center frequency of $\omega_0 = 1.3\,\text{GHz}$). The disadvantage of the temperature-scan is the fact that it interrupts the machine operation and is probably affected by temperature transient effects. In addition, it is slow due to the high heat capacity of resonators.

Another way of detuning a cavity is to change the frequency of the drive signal rather than changing the center-frequency of the resonator. Since the cavities are typically locked to a frequency reference (master oscillator), this can be achieved by replacing the reference with an adjustable frequency generator. This approach usally is not applicable during normal operation, too.

An interesting way of covering fractions of the resonance circle is to induce the variation in frequency at the drive signal by digital frequency synthesis. Practically speaking, this is nothing but creating a linear phase sweep on the envelope of the drive signal. In a pulsed environment, this can be achieved completely transparently by applying a second, low-power pulse directly after the main pulse. The slope of the phase (which is the frequency offset) of this secondary pulse is varied from shot to shot. The length of the secondary pulse can be of the order of a few time-constants of the resonator itself to assure steady state. For a $65\,\text{kHz}$-resonator, this would be a few $10\,\mu\text{s}$.

VI.   EXPERIENCE AT FLASH

In order to exploit the properties of the resonance circle for an online calibration we implemented a feature with the following properties into the RF gun controller, [5] and [6]. Directly after the primary pulse for beam acceleration, a secondary pulse is produced. It is not controlled by feedback (only feedforward), regardless if feedback for the primary pulse is turned on or not and it is not influenced by changes in the amplitude or phase of the primary pulse. Furthermore, its amplitude is usually chosen to be significantly smaller than the amplitude of the primary pulse in order not to impose too large changes on the heat-load of the cavity. Figure 3 shows a typical result of the calibration routine. The left side shows the amplitudes of primary and secondary pulses. The phase of the primary pulse is held constant by feedback, while the phase of the secondary pulse has a slope corresponding to some detuning, usually between $-60\,\text{kHz}$ and $+60\,\text{kHz}$ in steps of a few kHz. For a certain detuning, the reflection coefficient is calculated for every single sample in the secondary pulse and averaged over all samples. Every cross in the right plot of figure 3 represents an averaged reflection coefficient. The collection of reflection coefficients forms a fraction of a circle, from which the algorithm automatically determines center and radius. The calibration coefficients are derived from the results as described in the sections above.

Application of the new feature is completely transparent to the operators and takes less than $30\,\text{s}$. It is used for long-term drift measurements as well as for fast checks of the calibration parameters after a shutdown. It is also used for tracking changes in the coupling coefficient β after a cathode-exchange.

VII.   SUMMARY

Modern photoinjectors are exposed to field requirements which can not be achieved by just stabilizing the incoming wave. Heat load due to high duty cycles lead to a time-varying detuning over the pulse, which demands for an inclusion of the reflected wave when con-
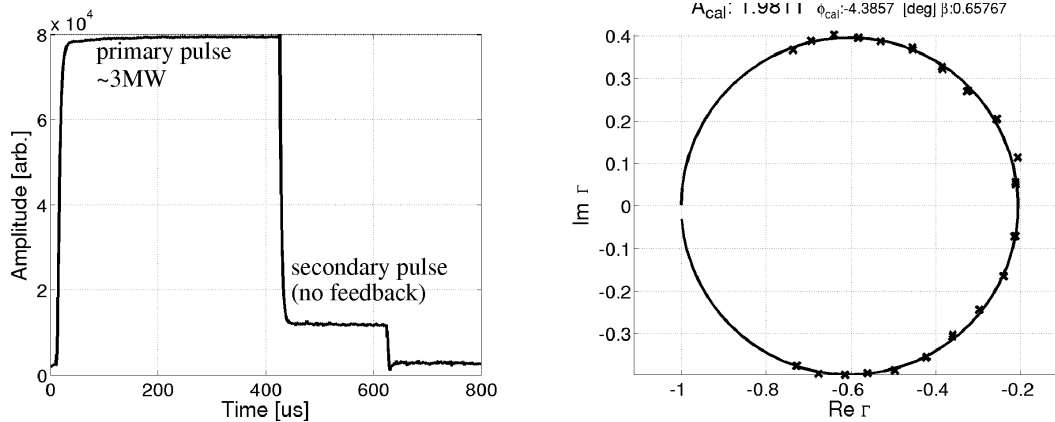
FIG. 3: Practical example for a calibration performed with the introduced algorithm at FLASH. The small step at the end of the primary pulse on the left picture is used for the calibration. The right picture shows the reflection coefficients in the complex plane after calibration together with all calibration coefficients.

trolling the cavity. While constant errors are irrelevant for LLRF control, time-varying errors have a significant impact on the control. It was shown that the cavity field, if composed from forward and reflected waves is subject to time-varying errors, even if the individual observables only have constant errors. Offsets, non-linearities and cross-talk of the sensor-chain can make LLRF-control without field probe impossible, especially if exposed to tight field requirements. Further, a calibration scheme was introduced that can be executed completely transparent to the operators. It allows permanent monitoring of the calibration as well as the coupling coefficient β of the cavity.

VIII.   ACKNOWLEDGMENTS

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